

The Effectiveness of Using Scientific Calculator in Solving Non-Linear Equations by Newton- Raphson Method

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Abstract: *In this paper, we will report the result of the study on the effectiveness of using scientific calculator in solving non-linear equation by Newton- Raphson method from the aspect of students' marks and duration taken in solving question. A total of 38 mathematics students who enrolled for numerical analysis course in the second semester session 2008/2009 in the Sultan Idris Education University are involved in this research. The data taken from the pre test and post test were analyzed using the Statistical Package for Social Science (SPSS). The findings show that using scientific calculator gives positive effect on students' ability to obtain the correct solution as well as reduces time to solve the problems.*

1. Introduction

Calculator is a tool not only to perform mathematical computations, but it can be a tool for learning mathematics. Despite the myths of harmful consequences resulting from their use, calculator is a pedagogical tool of great value. Studies about the role of calculators had been conducted and Kissane [1] stated that students using calculators have direct access to representations of mathematical ideas which are conceptually powerful and vital for learning.

Students can now focus in understanding the mathematical concepts and not become distressed about computing the wrong answers with the aid of calculators. Indeed calculators are crucial to help students to understand the mathematical concepts but not to be feared due to its rote computations.

CALC button in Casio fx-570ES and fx-570MS scientific calculator provides the ability to input the function by using the variables and calculate for the values of the function. Tay [2], Guerrero-García and Santos-Palomo, Á. [3] developed a collection of non-trivial keystroke sequence in Casio fx-570MS scientific calculator to solve several numerical methods including Newton-Raphson Method [4].

2. Problem Statement

Despite of all their benefits and capabilities, scientific calculators will never replace the human mind when it comes to reading and understanding a problem situation, writing an appropriate equation for the problem, choosing which operation to use in order to solve the problem,

interpreting the solution correctly that is displayed on the calculator, and determining the appropriateness of the answer. Scientific calculators are only as effective as the information students enter into them. Scientific calculators, in combination with mental, paper-and-pencil, and estimate skills when appropriate, comprise the tools to help students work through the computations and manipulations necessary for solving problems.

The National Council of Teacher Mathematics [5] stated in the Technology Principle that calculators as well as computers are vital tools for teaching, learning, and doing mathematics. Electronic technologies provide visual images of mathematical ideas. It benefits the students by organizing and analyzing data. Hence, the students are able to compute efficiently and accurately. When technological tools are available, students can focus on making decision, reflecting, reasoning, and problem solving. Technology enriches the range and quality of investigations by providing a means of viewing mathematical ideas from multiple perspectives.

Scientific calculator technology paved a way for the students to gain a higher level of mathematical understanding, rather than giving up after being frustrated or bored by these tedious manipulations. This research will study the effects of using scientific calculators in finding the roots of non linear equations by the Newton- Raphson method.

3. Research Questions

This study aimed to determine the effectiveness of using the Casio *fx-570ES* scientific calculator in finding the roots of non-linear equations by Newton- Raphson method by answering the following research questions:

- i. Can students' achievement improve with the help of the Casio *fx-570ES* scientific calculator?
- ii. Can the Casio *fx-570ES* scientific calculator reduce the duration taken to answer question?

4. Research Hypothesis

- i. By using the Casio *fx-570ES* scientific calculator, students can improve their achievement in solving non-linear equations by Newton- Raphson method.
- ii. By using the Casio *fx-570ES* scientific calculator, students can reduce the duration taken to answer the question.

5. Methods

A total of 38 mathematics students who enrolled for numerical analysis course in the second semester session 2008/2009 in the Sultan Idris Education University were selected by simple random sampling to be the sample for this research. A workshop was conducted for the participants. They were first taught how to solve non-linear equation using Newton- Raphson method without teaching them how to use a calculator. A pre test was conducted after they had completed the exercises and were able to solve non-linear equation using the Newton- Raphson method. After that, the participants were taught how to use the calculator to solve non-linear equation using the Newton-Raphson method as described in Section 7. The total marks for both

pre test and post test is 10 and the maximum duration to answer both tests is 10 minutes. The scores for both tests and duration taken to answer both tests were analyzed using the Statistical Package for the Social Science (SPSS) to test whether there are improvement in the scores and reduction in the duration taken to answer question.

6. The Newton- Raphson Method

In numerical analysis, the Newton- Raphson method is one of the best known methods to approximate the roots of non-linear equations. Newton's method can often converge remarkably quickly; especially if the iteration begins "sufficiently near" the desired root.

Given a function $f(x)$ and its derivative $f'(x)$, we begin with a first guess x_0 and iterate using the following formula:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, \quad i = 0, 1, 2, \dots \quad (1)$$

Repeat the iterations until $f(x_i) = 0$, $|f(x_i)| < \varepsilon$ or $|x_{i+1} - x_i| < \varepsilon$.

7. Algorithm of the Casio fx570-ES Scientific Calculator for Newton-Raphson Method

Example: Find the root of $y = \cos(x) - 3x + 7$ with $x_0 = 2$ by using Newton-Raphson Method. Iterate until $|x_{i+1} - x_i| < 0.005$.

Solution: Knowing that the iteration formula for Newton- Raphson Method is given by;

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, \quad i = 0, 1, 2, \dots$$

Step 1: Change the setup into radian mode and fix mode into 3 decimal places.

Press **SHIFT** **MODE** **4** for radian mode since the function is in trigonometric function.

1:MthIO	2:LineIO
3:Deg	4:Rad
5:Gra	6:Fix
7:Sci	8:Norm

Press **SHIFT** **MODE** **6** for fix mode.

1:MthIO	2:LineIO
3:Deg	4:Rad
5:Gra	6:Fix
7:Sci	8:Norm

Press **SHIFT** **MODE** **3** for 3 decimal places.

Fix 0~9?

Step 2: Input the iteration formula $(x - \frac{\cos(x)-3x+7}{-\sin(x)-3})$ into the calculator which is as follows:

Press $\boxed{\text{ALPHA}} \boxed{)} \boxed{-} \boxed{\frac{\square}{\square}}$

Shift the cursor up to the numerator box and key in the function into it.

$$\frac{x - \square}{\square}$$

Press $\boxed{\cos} \boxed{\text{ALPHA}} \boxed{)} \boxed{)} \boxed{-} \boxed{3} \boxed{\text{ALPHA}} \boxed{)} \boxed{+} \boxed{7}$

$$\frac{x - \cos(x) - 3x + 7}{\square}$$

Shift the cursor down to the denominator box and key in the function into it.

Press $\boxed{\nabla} \boxed{(-)} \boxed{\sin} \boxed{\text{ALPHA}} \boxed{)} \boxed{)} \boxed{-} \boxed{3}$

$$\frac{x - \cos(x) - 3x + 7}{-\sin(x) - 3}$$

Step 3: Start by calculating the first iteration value by substituting the initial value, $x_0 = 2$.

Press $\boxed{\text{CALC}}$ and you will be asked to specify the value of x in order to be substituted into the function.

$$\frac{x - \cos(x) - 3x + 7}{-\sin(x) - 3}$$

X?
0.000

Press $\boxed{2} \boxed{=}$

$$\frac{x - \cos(x) - 3x + 7}{-\sin(x) - 3}$$

2.149

Step 4: Continue to calculate the next value by pressing $\boxed{\text{CALC}}$ $\boxed{\text{Ans}}$ $\boxed{=}$ buttons. Note that the last answer calculated is stored in answer memory. The answer memory can be recalled by pressing $\boxed{\text{Ans}}$ button.

Press $\boxed{\text{CALC}}$ $\boxed{\text{Ans}}$ $\boxed{=}$

$$\frac{x - \cos(x) - 3x + 7}{-\sin(x) - 3}$$

2.151

Step 5: Repeat Step 4 until $|x_{i+1} - x_i| < \varepsilon$.

SOLUTION:

Table 1 Iteration Table

i	x_i	$x_i - x_{i-1}$
0	2.000	
1	2.149	0.149
2	2.151	0.002

Since $|x_2 - x_1| < 0.005$, hence x_2 is the root. The root for $y = \cos(x) - 3x + 7$ is $x = 2.151$.

8. Result and Analysis

Students' scores

The students' mean marks in the pre test and post test are shown in table 2. The mean value for the pre-test is 8.6667, while the mean value for the post-test is 9.4722. Thus, there is a difference of 0.80556 where the mean marks increases in the post-test. Obviously, the students' performances in solving the non-linear equations increase after treatment was given to them.

Table 2 Mean and standard deviation for students' marks

		Mean	N	Standard deviation
Pair 1	Pre test	8.6667	36	2.00000
	Post test	9.4722	36	1.78063

We compared the pre test and post test marks by using the paired sample t-test analysis in order to analyze whether there is any improvement on the students' performances. The maximum mark for both tests is 10. The result of this analysis is shown in Table 3:

Table 3 Paired sample t-test for students' marks

	Paired Differences			t	df	Sig. (2-tailed)
	Mean	Standard Deviation	Standard Error Mean			
Pre test - Post test	-0.80556	2.58368	.43061	-1.871	35	0.070

Based on the result in Table 3, the test statistic value obtained is -1.871 and the p -value obtained for this test is 0.0350 where we divide 0.070 by 2 given that this is a one tailed test. The p -value obtained is less than the significance value, 0.05. Thus, we have enough evidence to support the hypothesis that the students' marks increases after they were taught how to use the calculator to solve the non-linear equation by Newton- Raphson Method.

Duration taken

As shown in Table 4, the mean value duration for the pre-test is 9.2222 minutes, while the mean value duration for the post-test is 5.3889 minutes. The mean duration taken in solving the non-linear equations decreases as much as 3.83333 minutes after the students were taught the technique to use the calculator. Hence, we can state that the duration taken by students in solving non-linear equations by Newton-Raphson method was reduced when the respondents solved the mathematical problems by using the scientific calculator.

Table 4 Mean and standard deviation for duration taken

		Mean	N	Standard deviation
Pair 1	duration_pre_test	9.2222	36	1.80651
	duration_post_test	5.3889	36	2.71796

We compared the duration taken by the participants in the pre test and post test using the paired sample t-test analysis in order to analyze whether there is any improvement on the students' efficiency on solving the problem. The maximum duration taken for both test is 10 minutes. The result of the analysis is given in Table 5:

Table 5 Paired sample t-test for duration taken

Paired Differences						
	Mean	Standard Deviation	Standard Error Mean	t	df	Sig. (2-tailed)
Pre test – Post test	3.83333	3.07525	0.51254	7.479	35	0.000

The result shown in Table 5 confirms that the test statistic value equals to 7.479 and the p -value is 0.000 where we divide 0.000 by 2 given that this is a one tailed test. The p -value obtained is less than the significance value, 0.05. Thus, we have enough evidence to support the hypothesis that there is a reduction in the duration taken by the students to answer the question after they were taught how to use the calculator to solve the non-linear equation by Newton- Raphson Method as described in Section 7.

Common Mistakes

Table 6 shows the number of respondents that had performed the common mistakes.

Table 6 Common mistakes done by the participants

Type of common mistakes	Pre Test	Post Test
1. Wrong calculation	7	2
2. Incomplete solution	3	0

Wrong calculation and incomplete solution are the common mistakes done by the participants in the pre-test. Both common mistakes are greatly reduced in the post test after the techniques of using the calculator were taught as given in Section 7. Three participants did not have the complete solution in the pre test which might be caused by their inability to solve the problem efficiently. However, there is no participant that faced this problem in post test. Thus, this shows that they can answer the question efficiently with the algorithm in Section 7.

9. Conclusion

The effectiveness of the scientific calculator in solving non-linear equations by the Newton-Raphson method has been verified in this study. The usage of the scientific calculator in solving non-linear equations by the Newton-Raphson method improves students' marks. Students' marks increased after the respondents were exposed to the method of solving the non-linear equations by the Newton-Raphson method by using scientific calculator. Time reduced tremendously when the

participants answered questions utilizing the Casio *fx-570ES* scientific calculator. This research also shows that the common mistakes made by the participants are reduced after they were taught the technique to solve the problem using the calculator.

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