Study on Instruction of elementary functions through geometric construction activity with dynamic geometry environment

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Abstract: *Providing middle school students with an opportunity to construct function with dynamic geometry based on the proportion between lengths of triangle, this study aims to activate students' intuitive relationship between the elementary algebraic functions and their geometric properties. In addition, emphasizing the process of justification about the choice of students' construction method, this study aims to improve students' deductive reasoning ability. This study will investigate the following research questions: (1) What are the characteristics of the students' thinking process when students construct elementary functions with the Geometer's Sketchpad? (2) What are the roles of the teacher in the process? This study uses qualitative case study as its purpose to observe and analyze interaction between students and between students and teacher.*

1. Introduction and Theoretical framework

The most general process of teaching the elementary algebraic functions in Korean middle schools is as follows. First, when relation expression about a specific function is presented, students try to get the values of the function through algebraic calculation. And students draw a graph on the coordinate plane one by one. However, it's not easy to draw a graph of functions such as $y = \frac{1}{x}$ and $y = \sqrt{x}$ exactly. Besides, students have missed an opportunity to recognize dynamic, geometric and proportional properties of the function while emphasizing correspondence activity which is centered on algebraic calculations.

This study provides middle school students with many opportunities to construct functions such as $y = \frac{1}{x}$, $y = \sqrt{x}$ and $y = x^2$ with dynamic geometry like Geometer's Sketchpad based on the proportion between lengths of similar triangles and the analysis method formalized by Greek mathematician Pappus.

This study will investigate the following research questions.

1) What are the characteristics of the students' thinking process when they construct elementary functions with the Geometer's Sketchpad?

2) What are the roles of the teacher in the process?

We expect this study to activate students' intuitive relationship between the elementary algebraic functions and their geometric properties. And we also expect them to improve their deductive reasoning ability through the process of justifying the choice of construction method.

The characteristics of dynamical geometry environments are continuous movement, direct operation, and immersive environment. "Continuous movement" means that mathematical targets of the computer screen move maintaining logical interrelations and the whole shape while dragging, and students can observe the entire process in which targets are changing as well (see [7]).

It’s hard to dynamically connect functions with table, graph and algebraic equation in paper and pen environment and thereby students have difficulty in figuring out their relations effectively. The use of dynamic software can overcome these limits and help mathematical expressions get interconnected meaningfully (see [10]).

Analysis method is one of the oldest mathematical heuristics and was arranged systematically by Pappus around 300 BC. Analysis method begins with the supposition that the solution is already given or the theorem is already proven and it tries to find out the previous knowledge or proposition. This process would be done repeatedly until it reaches self-evident assumption (see [7]).

The use of analysis in dynamic geometry environments made it easier to construct in more advanced level problems (see [2]).

High school mathematics teachers teaching senior classes well-equipped with technological resources are found to have various responsibilities, challenges, roles. The roles of the teacher in proof learning are counselor, technical assistant and collaborator (see [2] and [3]).

2. Methodology

Class experiments were conducted with 2 ninth grade students who had no experience in using GSP before. In terms of level of academic achievement, both of them were in the upper group of their class and they had enjoyed solving mathematics problems. They have already learned about constructions such as bisector of the angle, perpendicular bisector, and angular movement etc in the first year of middle school. They thought that there was no connection between construction and other mathematical themes like algebra or function, but it is worthwhile to learn construction in itself.

Experimental classes were conducted 4 times and each class took about 1 hour. Two students were provided with one computer and required to write down the process of solving the problems, particular difficulties and feelings of using GSP. The Geometer's sketchpad 4.03 was used for the experiment and each class was recorded with video and audio files. The analysis was conducted using triangulation based on transcripts of the classes, the video images, worksheets and interview data.

3. Result and Analysis

1. What are the characteristics of the students' thinking process when students construct elementary functions such as $y=\frac{1}{x}$, $y=\sqrt{x}$ and $y=x^2$ with the Geometer's Sketchpad?

1) Students observed the continuous change of the functions in dynamical geometry environments so they came to understand the continuous characteristics of the functions.

The most general process of graphing the elementary algebraic functions in traditional instruction is as follows. First, relation expression about a specific function is presented and students try to get the values of the function through algebraic calculation. And students draw a graph on the coordinate plane one by one. As the domain is expanded, students have to guess how the graph will turn out in the case of $y=\frac{6}{x}$ (see Figure 3.1).

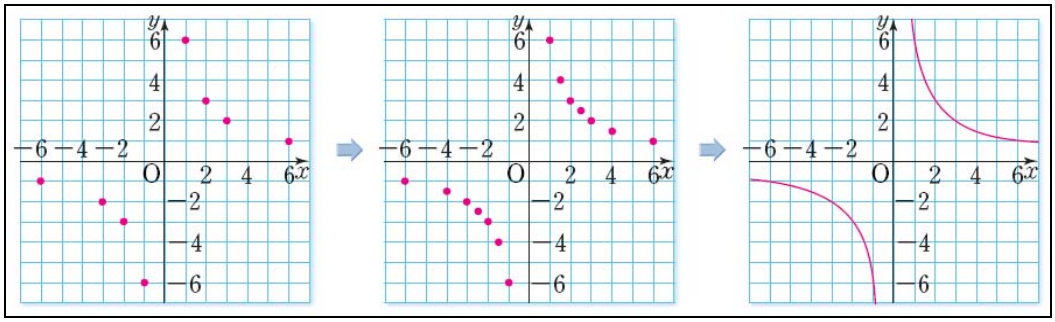

Figure 3.1 [Chunjae Moonwha, Park young hoon] 7th grade textbook p.148

What follow is the process that students draw a graph of the function, $y=\frac{1}{x}$ in paper and pen environment and dynamical geometry environment. First, students tried to transform the equation $y=\frac{1}{x}$ into other forms such as $xy=1, x=\frac{1}{y}$ and $x:1=1:y$ etc. Second, they thought of the proportion between lengths of triangle and drew two similar right triangles on the worksheet (see Figure 3.2). They came to recognize x, y as sides of a triangle in this process.

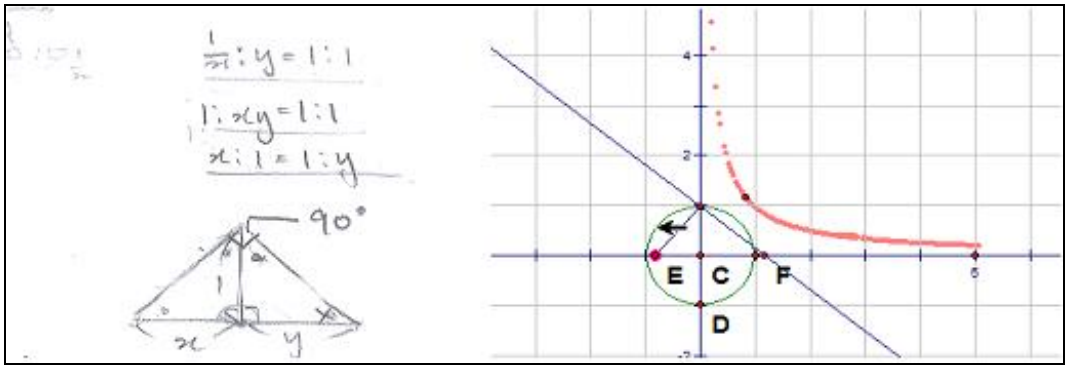

Figure 3.2 worksheet and GSP activity for $y = \frac{1}{x}$

Third, they constructed this figure with GSP and observed the continuous process of drawing a graph, $y = \frac{1}{x}$ by moving point E (see Figure 3.2). Through this process, students could observe the continuous change of the graph and had an opportunity to recognize the continuous property of the function. We can say that mathematical inquiry could be possible in harmony with paper and pen environment and dynamical geometry environment.

2) Students could distinguish independent variable x from dependent variable y through construction activities.

Students transformed the relation expression, $y = \sqrt{x}$ into $y^2 = x$, $x : y = y : 1$ etc. They could draw similar right triangles which corresponded with $x : y = y : 1$ (see Figure3.3) but they couldn't precisely distinguish independent variable from the dependent variable earlier in the class. At first, they constructed $\overline{BD}$ whose length is 1 on the x axis and point C on the y axis and then constructed the intersection point A of perpendicular line of $\overline{CD}$ and x axis (see Figure3.3): They used the origin point (0,0) and the unit point (1,0) in order to construct $\overline{BD}$ whose length is 1.

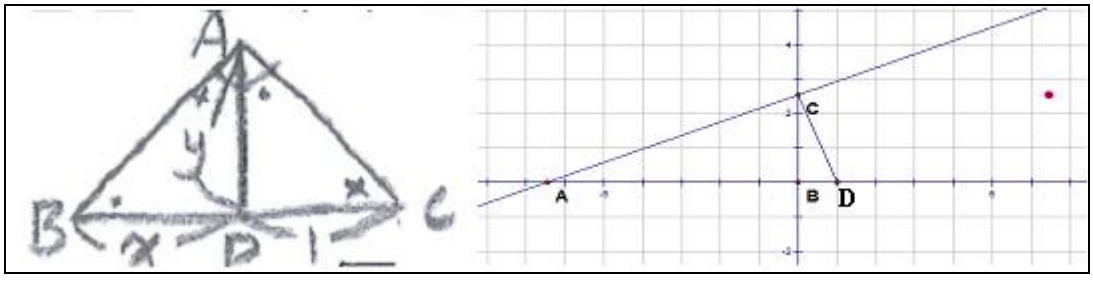

Figure 3.3 worksheet and GSP activity for $y = \sqrt{x}$

They plotted the point whose x coordinate is $\overline{AB}$ and y coordinate is $\overline{BC}$ and observed how the plotted point moves dragging point A. But they couldn't observe anything and found out that they had to drag point C in order to observe the graph (see Figure 3.4).

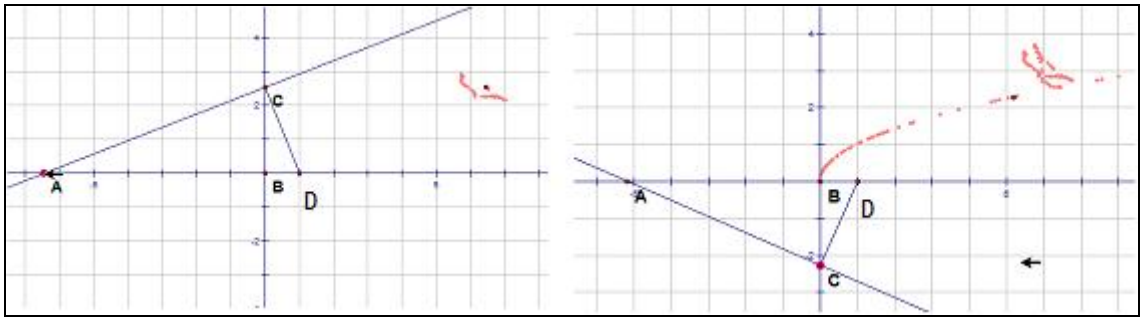

Figure 3.4 GSP activity for $y = \sqrt{x}$

< Transcript A>

1. Jin yong: I guess we changed x and y

2. Teacher: Here, $\overline{BC}$ must change according to $\overline{AB}$.

3. Jin yong: Yes, but it doesn't. I don't know what to do.

4. Teacher: What are the changing properties and unchanging properties?

5. Sue jin: $\angle ACD$ and $\angle ABC$ are $90°$. And the fact that $\overline{BD}$ is 1 must not change.

6. Teacher: The right angle has to be maintained while the length of $\overline{AB}$ is changing. Then what will be the trace of the point whose angle is always right no matter where it is?

(The students thought for a while and remembered that the angle of the circumference about diameter was always $90°$.)

7. Jin yong: circle!

They constructed the circle whose diameter is $\overline{LK}+1$ and plotted the point whose x coordinate is $\overline{LK}$ and y coordinate is $\overline{MK}$ and observed how the plotted point moves dragging point L(see Figure 3.5).

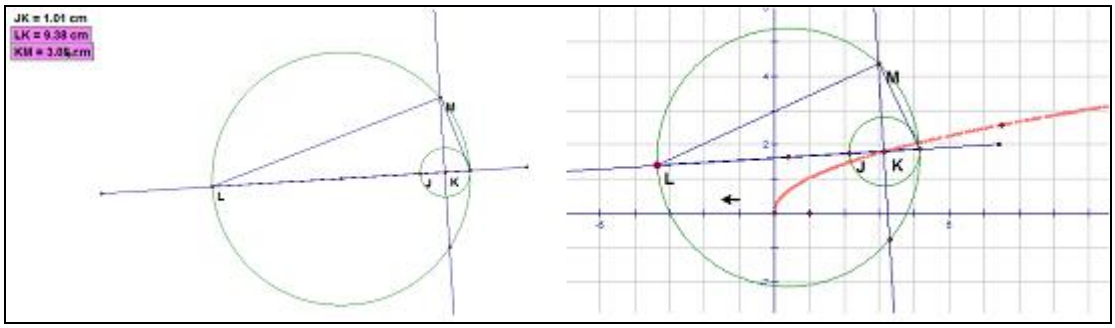

Figure 3.5 GSP activity for $y = \sqrt{x}$

They couldn't distinguish the independent variable from the dependent variable precisely at first. After students were guided by the teacher's question about invariability and variability of the figures and thought out the characteristics of the circle. Since then, construction activity went on smoothly. Through this process, students could experience the dependent property, one of the most important characteristics of the function in dynamical geometry environment.

3) Students could perceive the mathematical connection through constructing functions in dynamical geometry environment.

Students learned about construction in the first year of middle school, proportion between lengths of similar triangles in the second, and learned about the property of the angle of circumference in the third. Now, they experienced that these concepts are connected with functions. Therefore they came to perceive that mathematical themes are connected between different grades and domains. Also, they realized that mathematics is not just a set of arbitrary rules. The following describes some of contents that students wrote down on the worksheets after each activity.

- I'm very glad to learn $y = \frac{1}{x}$ by the new method using proportion and construction. Through this activity, I have understood functions from algebraic and geometric aspects. From now on, whenever I draw a graph of a function, I'll try various methods such as algebraic calculation and construction with GSP.
- I realized that the property of proportion can be used in drawing a graph and this fact was new to me. I think using computer makes learning clear and convenient...
- This time was different from last time, so I failed again and again. Since I started to think of the angle of circumference, it has been easier. This function is more complex than the last one.
- My ability seems to gradually improve by constructing a new function. Now I can draw a graph of any function.

4) Students experienced 'analysis method' in the process of exploring the plan to construct.

When students constructed functions such as $y = x^2$, they supposed that they already got the value of y and transformed the relation expression freely. While transforming the relation expression, they paid attention to proportion $x : y = 1 : x$. And then they could draw similar right triangles satisfying this proportional expression and construct a graph of $y = x^2$. In this process, the analysis method provided a critical idea to construct a graph of $y = x^2$ (see Figure 3.6).

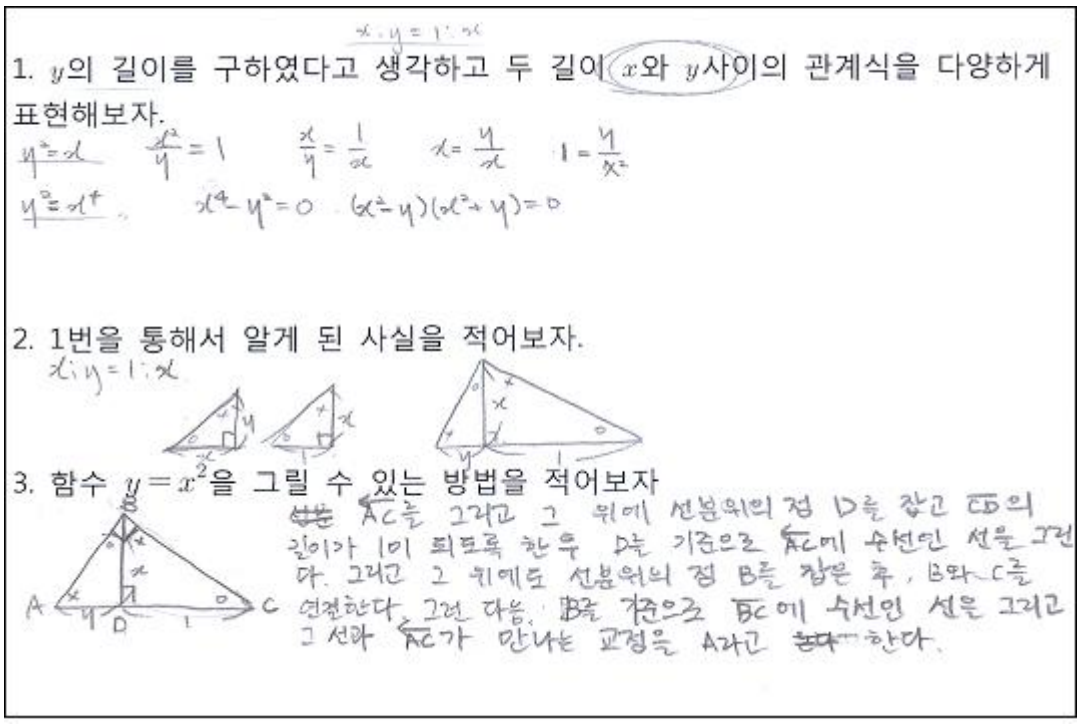

Figure 3.6 worksheet and GSP activity for $y = x^2$

5) Students came to perceive $\sqrt{x}$ in geometrical as well as in algebraic aspect through constructing $y = \sqrt{x}$.

Students already had a geometrical image of irrational number such as $\sqrt{2}$ from the early stage of this study, but it was a good chance to experience the geometrical meaning of $\sqrt{2}+\sqrt{3}$ considering the fact that the educational content of irrational number is mainly about algebraic calculation.

2. What are the roles of the teacher in the process?

1) The teacher has the role of counselor who points out mistakes made by students and helps them seek the solution.

Students constructed the figure on the GSP based on the worksheet activity to draw a graph of $y = \sqrt{x}$ and were confused by the result (see Figure 3.7).

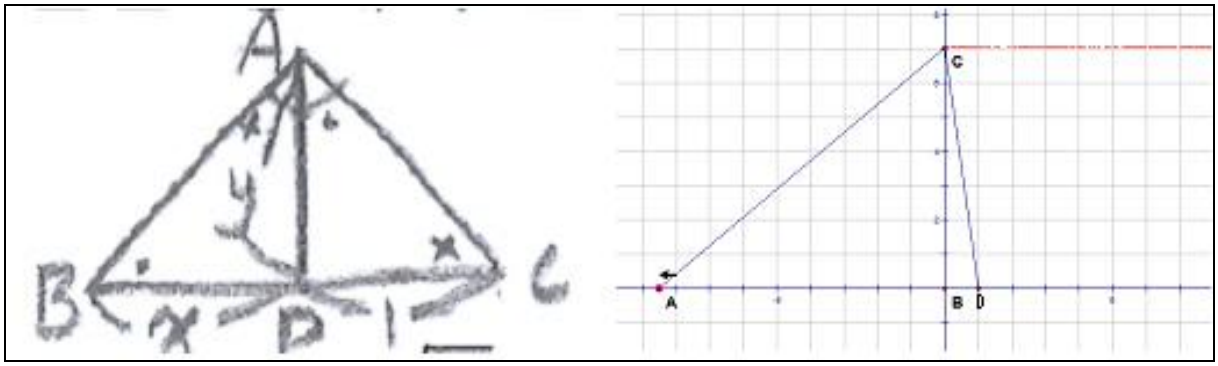

Figure 3.7 worksheet and GSP activity for $y = \sqrt{x}$

< Transcript B>

1. Teacher: Here, $\overline{AB}$ is changing. How about the length of $\overline{BC}$?

2. Jin yong: It doesn't.

3. Teacher: That means it's constant. Why does this happen?

4. Jin yong: It will go well if both of them change together...

5. Teacher: **If those figures are similar, $\overline{BC}$ will change according to $\overline{AB}$. Why does this happen? Let's reflect our work and check whether your works satisfy all the necessary conditions.**

6. Sue jin: Wait a minute! We forgot to maintain the right angle condition. We are constructing similar right triangles.

7. Jin yong: Aha, We didn't draw a perpendicular line to $\overline{CD}$.

The teacher recommended that students should reflect their work so that they could discover the cause of error (5). Through this process, students could proceed to the next stage of construction.

2) The teacher has the role of technical assistant who provides clues to the construction of a graph.

The teacher assisted students in operating the dynamic geometry software so that they didn't have to worry about technical problems. Although students learned about basic functions of GSP, they couldn't master operating all functions in one class. In <transcript C>, we can observe that students were guided again by the teacher, when constructing a perpendicular line in GSP.

< transcript C>

1. Sue jin: Well, I think that we need to construct a perpendicular line at that point.

2. Teacher: Do you remember to construct a perpendicular line?

3. Students: **We can't.**

4. Teacher: **It's a technical problem...Select the point and segment. And choose Construct-Perpendicular Line.**

5. Students: Aha!

From that time they could construct a perpendicular line freely.

3) The teacher has the role of collaborator who links function concept to the idea of construction.

After all these activities, the students came to wonder if they could construct another kind of function in this way and tried to draw a graph of $y = x$ by using construction. The teacher supported and collaborated with their trial. In other words, teacher asked students what kind of relation expression was needed and they tried to transform $y = x$ into other expressions like former

cases. Then they got the relation equation, $x : y = 1 : 1$ and realized this was a congruent situation. They constructed congruent right triangles and confirmed the possibility of construction of $y = x$ in case of $x \geq 0$. In this process, the teacher had the role of collaborator who links function concept to the idea of construction by allowing students to extend their mathematical concerns (see Figure 3.8).

4. Conclusion

In this research, we investigated new possibilities in teaching elementary algebraic functions by providing middle school students with an opportunity to construct function in a dynamical geometry environment. This study has been concluded through observation and analysis.

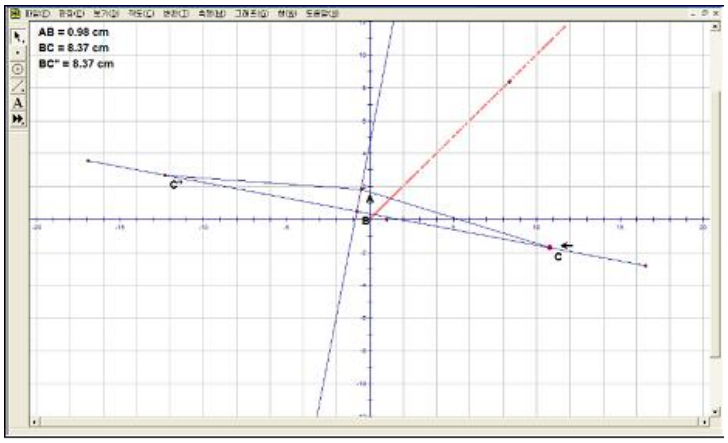

Figure 3.8 **GSP activity for** $y = x$

Students observed the continuous change of the functions in dynamical geometry environments so they came to understand the continuous characteristics of the functions and could distinguish independent variable x from dependent variable y through the construction activity. Students were also able to experience meaningful mathematical connections. Finally, Students learned the 'analysis method' in the process of exploring the plan to construct, so their deductive reasoning ability got improved by way of justifying the choice of construction method and came to perceive $\sqrt{x}$ in geometrical aspect like the length of side as well as in algebraic aspect through constructing functions.

The roles of the teacher in the process are as follows.

First, the teacher has the role of counselor who points out mistakes made by students and helps them seek the solution. Second, the teacher has the role of technical assistant who provides clues to the construction of a graph. Third, the teacher has the role of collaborator who links function concept to the idea of construction. With the findings from this study we offer the following suggestions that may be applied to mathematical education.

First, it is better that middle school mathematics teachers can utilize not only algebraic but also geometrical approaches in function instruction so that students can experience the various properties of the function and mathematical connection.

Second, many teacher training programs using dynamic geometry software have to be provided and further studies on developing teaching materials which can overcome the weakness of paper and

pen environment and maximize the strength of dynamic geometry environment are strongly called for. Finally, Students need to be able to learn in the dynamic geometry environment whenever they want.

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