Beyond Computation: Calculators Can Enhance Problem Solving Process

Yeo Kai Kow Joseph
kaikow.yeo@nie edu.sg
National Institute of Education
Nanyang Technological University
Singapore

Abstract: *Although the role that skill acquisition plays is being deliberated in the development of conceptual understanding, the key role of the calculator as a tool for exploring mathematics and solving problems at the primary level is unquestioned. In this paper, we show how pupils can use scientific calculators as a supporting medium to improve the quality of solving word problems. To this end a flexible six-stage process, acronymically named RESCUE, is presented. The process is illustrated using a non-routine item that possesses the quality of complexity. In addition, the six stage process also deals with ways to use calculators to enhance multiple problem-solving strategies, encourage discourse and facilitate mathematical thinking through solving word problems at the primary level. The process of solving the problems allows pupils to become familiar with multiple ways of using a calculator to obtain an answer, learn to interpret the calculator display and identify the limitations of the calculator.*

1. Introduction

Calculators used at the primary level are basically a suite of ways of handling fractions as well as decimals; so they allow smooth transition between fractions, decimals and percentages and facilitate both arithmetic operations with whole numbers, fractions and simplification of fractions. Calculators are now completely accepted as the normal way of performing tedious computations at the primary level. Their efficiency, accuracy and speed in carrying out lengthy calculations have provided pupils and teachers with new opportunities to refocus mathematics lesson time and attention. Although the role that skill acquisition plays is being deliberated in the development of conceptual understanding, the key role of the calculator as a tool for exploring mathematics and solving problems at the primary level is unquestioned. This was supported by the recent Singapore's Ministry of Education press release statement on the introduction of calculators in Primary 5 - 6 Mathematics which stated that:

> Calculators facilitate the use of more exploratory approaches in learning mathematical concepts, some of which may require repeated computations, or computations with large numbers or decimals. With a calculator, pupils can perform these tasks and better focus on discovering patterns and making generalisations without worrying about computational accuracy. Second, the use of calculators also enables teachers to use resources from everyday life, such as supermarket advertisements, to set real-life problems with real-life numbers that may be difficult for pupils to work with without a calculator. Pupils would hence be better able to see the connection between mathematics and the world around them. [10]

However, many teachers continue to fear computational skills will not be learned if calculators are used ([1], [7]), that students will become over-reliant on them [3] and that access to calculators gives students an unfair advantage and is a form of cheating [12].

At the primary level, solving word problem is a regular feature in the Singapore primary mathematics classroom. While judiciously chosen word problems is a crucial part of mathematics instruction at the primary level, teachers need to develop pupils' abilities to think and reason when solving word problems using a calculator. Apart from the particular tool or strategy used for calculations in the word problems, mathematics teachers should also value looking back at the

computed answers. Pupils should be asking questions such as: Does the answer make sense given the context of the problem? Do we expect a whole number for the final answer? Whether a problem is solved by paper and pencil methods or by a calculator, these questions are critical aspects of the problem solving process and should be part of the inherent reflection process of the pupil. Experienced calculator users are aware that these tools do not always give correct information either due to input errors or incorrect reasoning. Are pupils mindful and reflective concerning calculator answers? Do they have both the confidence, assurance and reasoning skills to evaluate unreasonable results?

Calculators are an important part of the primary school mathematics classroom but a current topic of concern among teachers is the effect of prevalent calculator use on students' computation abilities. Since the 1990s great attention has been focused exploring pupil ability and thinking related to computational estimation and ways to support the teaching and learning of estimation ([2], [13]). Mathematics educators have began to view estimation within the broader context of number sense ([9], [15]) so that the focus is placed on assisting pupils to understand and make sense of numerical computations and plan meaningful ways to solve problems and interpret results. Research studies have indicated that children who see themselves as making sense of numerical computations do indeed use their conceptual understanding to estimate and look back on their answers [8].

The problem solving approach to mathematics instruction was introduced in Singapore in 1992. Concerns about this teaching approach were raised shortly afterwards [6], when a group of teachers articulated their concern that they lacked skills to teach of mathematical problem solving. For instance, teachers felt inadequate about their own teaching approaches to problem solving, especially with non-routine problems. In a two-year research, Chang et al studied the pedagogical practices within primary mathematics classrooms in four Singapore schools [4]. In all a traditional teaching approach predominated amongst the primary teachers — expository teaching, followed by the students practicing routine exercises to consolidate the concepts, knowledge and skills. Anecdotal evidence further suggested that such an approach to teaching was common in most primary schools. Somehow the focus on problem solving did not quite 'sieve through', as it were, to classroom teaching practice.

In this paper, we show how pupils can use scientific calculators as a supporting medium to improve the quality of solving word problems. To this end a flexible six-stage instructional process, acronymically named RESCUE, is presented. In most non-routine word problems, the six-stage instructional process leads pupils through initial exploration to multi-stranded exploration of the word problem to be solved. The process is illustrated using one non-routine word problem that possess the quality of complexity. In addition, the six stage process also deals with ways to use calculators to enhance multiple problem-solving strategies, encourage discourse and facilitate mathematical thinking through solving word problems at the primary level. The process of solving the problem allows pupils to become familiar with multiple ways of using a calculator to obtain an answer, learn to interpret the calculator display and identify the limitations of the calculator.

2. Six-Stage Process of Teaching Mathematical Problem Solving

The **RESCUE** (**R**ead, **E**stimate, **S**elect, **C**arry out, **U**se, **E**xplore) plan's instructional approach is adapted from Polya's original problem solving strategy [11] but has also incorporate scientific calculator into the process. This instructional approach can be employed across the primary levels. Teachers could adopt this **RESCUE** plan in the teaching of problem solving especially related to solving non-routine problems. After pupils individually estimate the answer, the following

problem solving approach can be done individually or in small groups before discussing with the whole class.

1. Ask the pupils to **R**EAD the word problem before making any attempt. After reading, teacher will ask guiding questions to help the pupils to look for the information and data in the word problem. Pupils could organize the information and data using diagrams and tabulation.
2. Require each pupil to **E**STIMATE and write the answer and give reasons for their estimation. Next, the teacher should request the pupils to share and discuss estimates and rationales. They should do this first in pairs before whole-class discussion. Pupils comment on the extent to which they are convinced of the accuracy of their estimation. The teacher remains neutral in word and body language to all contributions.
3. Teach the pupils how to **Select** appropriate heuristics to solve the word problem. In order to achieve this aim, the teacher needs information about which heuristics pupils are likely to be able to use to successfully solve various types of problems.
4. Have pupils **C**arry out the computations with a calculator, keeping track of the key sequences they use so that they can share their work with their classmates.
5. Ask the pupils to **U**se the obtained answer to check whether their answer satisfies the problem requirements, and whether their answer makes sense, particularly in a real world context.
6. **E**xplore the word problem together with pupils to interpret the mathematical solution in terms of the problem and explore further such as what happen if some of the original details are changed in the problem.

The six-stage **RESCUE** plan (**R**ead, **E**stimate, **S**elect, **C**arry out, **U**se, **E**xplore) can be a practical and effective instructional approach that utilize a calculator to assist the teachers in the teaching of mathematical problem solving in the primary mathematics classroom.

3. An Illustrative Example Using RESCUE Plan

Although the six points of the plan are all critical, they often overlap and interact in real classroom teaching which can make specific identification of the six stages artificial. The philosophy and the variability of the procedure, in the following non-routine item will be illustrated.

Decimal Problem :(Primary 5)

At a sale, all pens are sold at the same price. If 13 pens cost more than $20 and 11 pens cost less than $17, find the cost of one pen to the nearest cent.

Read

At the first glance, this non-routine problem demands more than computation procedure. After reading, the following guiding questions could be asked by the teacher to help the pupils to visualize the problem situation.

- Do all the pens sell at the same price?
- What is the cost of 13 pens?
- What is the cost of 11 pens?
- What are you supposed to find in this problem?

Pupils after reading should be able to organize the information and data.

Estimate

When pupils are ready to begin solving the problem, they should always have an initial "guess" or estimate as to what they might expect the answer to be. Every pupil should have the worksheet (see Appendix A), pen and a calculator. Each pupil is told to read the problem quietly and write the estimate of the answer together with the reasons. Using a pen to write the estimate of the answer would reduce the temptation to change an estimate and reasons. The discussion of these estimates should be conducted by showing the procedures on the board while listening to the pupils' reasons. The teacher need not comment on the accuracy of any of the estimates or reasons but should seek other pupils' views about their classmates' contributions. Some pupils might estimate a solution using their knowledge of dividing a number by 10 and 15, resulting in an estimate of $1.70 or $1.3. Others may use the fact that the total cost of the pens is $37 and total number of pens is 24 and round off to the nearest 10 to get the cost of one pen to be $20. Some pupils may make a guess that the cost of one pen is more than $1.50. It would not be surprising if a pupil ignores the mathematics and finds an estimate based on the cost of a pen that they purchase from the bookshop. The teacher writes each of these answers so that the initial suggestions remain public throughout the time required to solve the problem. A discussion of these estimates as well as their reasons will encourage discourse, assist pupils to develop the ability to take risks and trust their own judgment and expose pupils to multiple ways of thinking about a problem. This will motivate pupils to complete the problem with working clearly written.

Select

With their own estimation of the answer, pupils will be encouraged to think of an appropriate heuristic to solve the problem. For example, in this problem, pupils could use systematic listing, guess and check, tabulation and logical reasoning to solve the word problem.

Carry Out

Each pupil will use their calculator to solve the problem and write each key sequence on their worksheet. Some pupils may key in 20, ÷, 13, = and key in 17, ÷, 11, =. Others might key in 37, ÷, 24, =. Someone might just use guess and check with the aid of a calculator to find the answer. Pupils could also begin by guessing that the cost of one pen is $1.53, $1.54 or $1.55 and from there compute and check the conditions for the cost of 11 pens and cost of 13 pens. Some pupils might key in the wrong sequence that yield an incorrect response. Questions to stimulate discussion might include these: Did every pupil obtain the same answer? Explain clearly. Which approach, if any, minimizes the key sequence and could be considered more efficient?

Use

Teacher could use the obtained answer to discuss the different approaches of getting answers from the calculator. It is also necessary for the pupils to explain why the final answer displayed by the calculator is correct. As a whole class, teacher should also alert the pupils if the answer obtained is reasonable. For example in this problem, it should obviously be a decimal because we are referring to dollars and cents. Having the cost of one pen to be $2 is unlikely to be an acceptable answer. To give an indication to the pupils if they have made a good estimation to the answer, they will need to find the difference between their initial estimate and the final solution. By discussing different approaches and analyzing why the answers make sense, helps the pupils to focus on the underlying mathematics concepts including decimals, the meaning of division and rounding off.

Explore

Explore the word problem together with pupils to interpret the mathematical solution in terms of the problem. Thus, a discussion may now focus on interpreting this value. Should the correct answer be $1.53, $1.54 or $1.55? Why choose one answer over the other? In addition, the original

form of the problem could be modified with different cost for different number of pens. Pupils should also be mindful that to modify a problem, the context of the word problem must be meaningful. Thus, all these discussions will create an opportunity for the pupils to relate their personal experiences and opinions and may assist them develop analytical and critical thinking skills.

Therefore in performing this word problem with authentic data, it is necessary to perform numerous computations which can be tedious if a calculator is not used. When a calculator is used, the ability to solve this word problem does not depend so much on the ability to perform the division algorithm, which is done by the calculator. Rather, the pupils must have the ability to familiar with multiple ways of using the calculator to obtain answers, learn to interpret calculator display and identify limitation of the calculator during the guess-and-check and reasoning process, which requires good number sense. Extraneous information may be present in authentic sources and pupils have to choose the relevant information to use.

4. Conclusions

In this paper it is demonstrated how a calculator can be integrated into a pupil's problem solving process so as to enhance the quality of the pupil's written solutions. The **RESCUE** plan would ensure that the problem solver becomes aware of the stages of problem solving. It also assists the solver to organise their thoughts and approach the problem systematically as well as reminds solver to evaluate own success in solving the problem. One might ask, is it necessary to use a calculator if the high ability pupil is able to work out the required solution without such assistance? The author response would be that in this **RESCUE** plan, the calculator is able to give the pupil the confidence and competence in doing mathematics and the ability to persevere even when the problem is challenging. The **RESCUE** plan also provides a broad, early attempt at synthesizing how problem solving could be integrated with the use of calculator in the primary years. In addition, it is needful to dispel the notion that problem-solving instruction typically involves only problems that can be solved in "five minutes or less" (e.g., [5], [14]). This type of rapid and artificial problem solving is most apparent when primary pupils are taught to search for key words (e.g., "more" or "gave away") or use a single strategy (e.g., "make a table") to solve a set of predictable or highly structured problems. Such an approach may enable pupils to complete what has been called "end of the chapter problems." However, critics argued that these methods do little to foster a deeper sense of problem solving. In fact, they may lead many pupils to give up when faced with more non-routine problems that require extended effort or several incorrect attempts before a correct solution is achieved. Yet, non-routine problems reflect the kind of effort required in authentic problem solving and more sophisticated mathematics.

Another challenge plaguing primary pupils is not that they lack of confidence in estimation skills but rather that they may be reluctant to question or reflect on their own mathematical thinking. It should be obvious to pupils that finding the answer is not the only focus in mathematical problem solving. Reflection on process should become second nature to pupils so that when they experience a situation where a calculator does not provide the correct or needed information, confidence in their own mathematical reasoning will guide them.

Finally, it is important that teachers are prepared to convey to their pupils the importance of estimation and checking when using calculators. As calculators become more prevalent in the primary mathematics classroom and more integral to learning and solving mathematical problems, pupils need to perceive them as tools, not "dispenser of answers." Moreover, pupils who use calculators at the national examination should also need to be discriminatory as to when and how to use the calculator.

References

1. Burke, G. (2001). Computers and calculators in schools: A status report. Electronic Data Resources Service. Montreal, Canada: Mc Gill University ED 427 721.
2. Case , R., & Sowder, J. T. (1990). The development of computational estimation: A neo-Piagetian analysis. *Cognition and Instruction, 7,* 79-104.
3. Dresdeck, C. (1995). Promoting calculator use in elementary classrooms. Teaching Children Mathematics, 1(6), 300.
4. Chang, S.C., Kaur, B., Koay, P.L. & Lee, N.H. (2001). An exploratory analysis of current pedagogical practices in primary mathematics classrooms. The NIE Researcher, 1(2), 7-8.
5. Doyle, W. (1988). Work in mathematics classes: The context of students' thinking during instruction. Educational Psychologist, 23, 167–180.
6. Foong, P.Y., Yap, S.F. & Koay, P.L. (1996). Teachers' concerns about the revised mathematics.
7. Horton, S. V., Lovitt, T. C. & White, O. R. (1992). Teaching mathematics to adolescents classified as educable mentally handicapped: using calculators to remove the computational onus. Remedial and Special Education, 13(3), 36-60.
8. Kamii, C., & Lewis, B.A. (1993, January). The harmful effects of algorithm in primary arithmetic. *Teaching K-8, 23,* 36-38.
9. McIntosh, A., Reys, B. J., & Reys, R. E. (1992). A proposed framework for examining basic number sense. *For the Learning of Mathematics. 12*(3), 2-8, 44.
10. Ministry of Education (2007). Press Releases: Introduction of Calculators in Primary 5-6 Mathematics. Retrieved May 25, 2007, from http://www.moe.gov.sg/press/2007/pr20070427.htm.
11. Polya, G. (1988). How to solve it – a new aspect of mathematical model. (2nd Ed). Princeton, NJ: Princeton University Press.
12. Reys, B. J. & Arbaugh, F. (2001). Clearing up the confusion over calculator use in grades k-5. Teaching Children Mathematics. 8(2), 90-94.
13. Reys, T. E., Reys, B. J., Nohda, N., Ishida, J., Yoshikawa, S., & Shimizu, K. (199!). Computational estimation performance and strategies used by fifth and eighth-grade Japanese students. *Journal for Research in Mathematics Education. 22*(1), 39-58.
14. Schoenfeld, A. (1988). When good teaching leads to bad results: The disaster of "well-taught" mathematics courses. Educational Psychologist, 23(2), 145–166.
15. Sowder, J. T. (1992). Estimation and number sense. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 371-389). New York: Macmillan.

Appendix A

Decimal Problem Worksheet

The Problem — At a sale, all pens are sold at the same price. If 13 pens cost more than $20 and 11 pens cost less than $17, find the cost of one pen to the nearest cent.

Read

1. Do all the pens sell at the same price? _____
2. What is the cost of 13 pens? _____
3. What is the cost of 11 pens? _____
4. What are you suppose to find in this problem? _____

Estimate

5. Estimate the answer: _____ . Give reasons for your selection of this estimate.

_____

Select

6. Let's solve the problem by direct computation from calculator or guess and check.

Carry Out

7. Use the operation keys on your calculator to solve the problem. Write down all the key sequences and which values appear in the display each time

Key Sequences	Display

8. After our class discussion, fill in the blanks for the following statements:
   My approach to using the calculator was _____(correct/incorrect)
   If correct, I could have been more efficient using fewer key sequences: Yes / No
   Support your answer
   If incorrect, the key sequence I could have used were these:

   _____

Use the obtained answer

9. Explain the meaning of the answer that is displayed in the calculator.

_____

10. Does it match all the clues in the problem? _____
11. Is there an easier method? _____
12. Does the answer make sense? _____
13. The difference between my estimate and the final answer is _____

Explore

14. Explain why the cost of one pen is **not** equal to $1.53 or $1.55. Show your working clearly to support your reasoning.

15. What if the cost of one pen is increased by ten cents now? What would be the cost of 13 pens and the cost of 11 pens? Express your answer to the nearest cent.