Learning mathematics with Computer Algebra Systems (CAS):

Middle and senior secondary students' achievement, CAS experience and gender differences

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The Mathematics and Technology Attitudes Scale (MTAS) is a scale for secondary students which may be used to monitor five affective variables relevant to learning mathematics with technology. The subscales measure mathematics confidence, confidence with technology, attitude to learning mathematics with technology and two aspects of engagement in learning mathematics. The paper reports the responses of 835 students from 9 independent Victorian schools. The supplementary variables were the following: gender, mathematics grade, year level and learning setting. Principal Components Analysis (PCA), t-tests, correlations and MANOVA have been used for the analysis of students' responses. Statistically significant differences were found between gender, Mathematics Confidence and Technology Confidence, between achievement and Mathematics Confidence, Behavioural Engagement, and Affective engagement, between year level and Behavioural Engagement and between three years of CAS experience and one year of CAS experience.

The aim of the study was to investigate the interrelationship between secondary students' mathematics confidence, confidence with computers, attitude to learning mathematics with computers, affective engagement and behavioural engagement, achievement, gender, achievement, year level and years of CAS Use. The *Mathematics and Technology Attitudes Scale* (MTAS) was used to examine the role of the affective domain in learning mathematics with technology, and it reports some of the results from the use of MTAS in Years 9-12 classrooms in Victoria, Australia.

Reports of a number of teaching innovations over the last forty years include data on students' attitudes to the innovation as well as their mathematical achievement. McLeod (1992) put forward a strong position that affective issues play a central role in mathematics learning. McLeod's (1992) definition of beliefs, attitudes and emotions has been considered adequate for this study.

The importance of intrinsic motivation for achievement and participation in advanced mathematics courses, and the apparent differences between boys' and girls' views has been demonstrated by Watt's (cited in Vale and Bartholomew, in print) argument that:

> Boys maintained higher intrinsic value for maths and higher maths related self-perceptions than girls throughout adolescence…We need to understand how it is that boys come to be more interested and like maths more than girls; and also why girls perceive themselves as having less talent, even when they perform similarly.

The authors also cited a finding from the Program of International Student Assessment (PISA) 2003 study relating to girls' confidence in mathematics: "females appear to be less engaged, more anxious and less confident in mathematics than males. It is our contention however, that computer (and technology) confidence is a very different construct to that of mathematical confidence. Mathematical confidence is an affective dimension closely associated with mathematics achievement.

Student engagement with the intellectual work of learning is, according to Marks (2000), an important goal for education, leads to achievement and contributes to students' social and cognitive development, and as far as research examining the effect(s) of engagement on achievement it is: "comparatively sparse, existing studies consistently demonstrate a strong positive relationship between engagement and performance across diverse populations" (p. 155). Fredricks, Blumfield & Paris (2004) assumed school engagement as a concept that is malleable, responsive to contextual features and amenable to environmental change. They proposed the following three dimensions: *Behavioural engagement*, which draws on student participation, *Emotional engagement*, encompassing both positive and negative reactions to staff and the school in general, and *Cognitive engagement*, which draws on the principle of students making an investment in learning. Two of the dimensions of this framework, i.e. behavioural engagement and emotional engagement form part of the MTAS instrument.

Weglinsky (1998) evaluated the educational technology and student achievement in mathematics with a USA national sample of 7,146 Year 8 (second year junior high school) students. He reported that "high-achieving students are more likely to use technology in certain ways rather than these uses of technology promote high levels of academic achievement" (p.4).

The use of CAS in mathematics classes and the effects of attitudes and behaviours on learning mathematics with computer tools have attracted a broad range of attention from researchers in many parts of the world in the past decade (Pierce & Stacey, 2004; Artigue, 2002; Drijvers, 2000; Guin & Truche, 1999; Reed, Drijvers & Kirtschner, 2009). Referring to a number of recent studies, Reed, Drijvers & Kirtschner (2009), lamented the fact that:

> Although the use of computer tools in schools is widespread, actual outcomes of employing such tools have been disappointing. While computer tools are purported to enhance the learning experience and to bring learners to higher levels of understanding, motivation, engagement and self-esteem, they are often marginalised within existing classroom practices, or used only for repetitive, delimited activities, rather than to promote complex learning. In mathematics education also, research has shown that the potential benefits of employing mathematical computer tools are not always realised (p. 1).

The authors found that promoting learning with mathematical tools must take several factors into account simultaneously, including the improvement of student attitudes, learning behaviours and providing ample opportunity for the construction of new mathematical knowledge from acquired tool mastery. Embedding tool use in meaningful mathematical discourse in which ideas are discussed and reflected upon, was considered the most important aspect in this process. The study also found students' attitudes and behaviours to be influenced by school and classroom factors, in agreement with the outcomes of the present study.

Aims of the study

The aims of the study were to investigate:

The factorial structure of the MTAS, and

The existence of gender, achievement and CAS experience differences in each of the five MTAS subscales

Research Methods

Sample

The participants were 835 Year 9-12 students, from nine Independent schools, in Victoria, Australia. From the 45 randomly selected schools, eleven agreed to participate in the study. The Head of Mathematics in each school selected students from Year 9-12 classrooms that had been using CAS calculators for one or more years (3 schools commenced using CAS calculators in Year 9, 2 schools in Year 10, 2 schools in Year 11 and 1 school in Year 12). The study took place in 2009 (Term 4). The grade categories used in the study were the following: A (80-100%), B (70-79%), C (60-69%), D (50-59%) and E/F (<50%).

Instrument

For our research we used the Mathematics and Technology Attitudes Scale (MTAS) developed by Pierce, Stacey & Barkatsas (2007). The instrument consists of 20 items. A Likert-type scoring format is used for each of the subscales: *Mathematics Confidence [MC], Confidence with Technology [TC], Attitude to learning Mathematics with Technology (whether computers, graphics calculators or computer algebra systems in the original scale – CAS in this study) [MT], Affective Engagement [AE].* Students are asked to indicate the extent of their agreement with each statement, on a five point scale from strongly agree to strongly disagree (scored from 5 to 1).

A different but similar response set is used for the *Behavioural Engagement [BE]* subscale. Students are asked to indicate the frequency of occurrence of different behaviours. A five-point system is again used – Nearly Always, Usually, About Half of the Time, Occasionally, Hardly Ever (scored again from 5 to 1). The rationale for the selection of the items and the naming of the subscales, as well as the psychometric properties of the scale, may be found in Pierce, Stacey & Barkatsas (2007) and Barkatsas (2005). The data analysis is presented in the next section.

Data Analysis and Discussion

Principal Components Analysis (PCA)

The questionnaire items were initially subjected to a Principal Components Analysis (PCA, extraction method: maximum likelihood). The PCA analysis using data from 1088 students' responses to the twenty items forming the MTAS (Table 1) indicates that the data satisfy the underlying assumptions of the PCA and that together five components (each with eigenvalue greater than 1) explain 69.75% of the variance, with 29.15% of the initial eigenvalues (extraction sums of squared loadings) and 16.3% of the rotated sums of squared loadings attributed to the first factor - *Mathematics Confidence* (MC), and a further 16.87% of the initial eigenvalues and 14.90% of the rotated sums of squared loadings attributed to the second factor - *Attitude towards use of CAS for learning mathematics* (MT). The communalities for nineteen of the items were greater than .6 and the lowest communality for only one BE item was .502.

Table 1
Rotated component matrix

	Component				
	1	2	3	4	5
(MC) I can get good results in mathematics	.855				
(MC) I know I can handle difficulties in maths	.841				
(MC) I am confident with mathematics	.805				
(MC) I have a mathematical mind	.766				
(MT) Mathematics is more interesting when using CAS calculators		.871			
(MT) CAS calculators help me learn mathematics better		.850			
(MT) Using CAS calculators in mathematics is worth the extra effort		.845			
(MT) I like using Computer Algebra Systems (CAS) calculators for mathematics		.817			
(TC) I am good at using computers			.871		
(TC) I can fix a lot of computer problems			.850		
(TC) I am quick to learn new computer software needed for school			.827		
(TC) I am good at using things like VCR's, DVD's, MP3's, iPods, Wii's and mobile phones.			.790		
(BE) If I make mistakes, I work until I have corrected them				.773	
(BE) I concentrate hard in maths				.727	
(BE) If I can't do a problem, I keep trying different ideas				.704	
(BE) I try to answer questions the teacher asks				.685	
(AE) In mathematics you get rewards for your effort					.742
(AE) I get a sense of satisfaction when I solve mathematics problems					.726
(AE) Learning mathematics is enjoyable					.723
(AE) I am interested to learn new things in mathematics					.660

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.
Rotation converged in 6 iterations.

Further, according to Coakes & Steed (1999), if the Kaiser-Meyer-Olkin (KMO) Measure of Sampling Adequacy is greater than 0.6 and the Bartlett's Test of Sphericity (BTS) is significant then factorability of the correlation matrix is assumed. For this part of the study, the KMO=.874 and BTS<.001, so factorability of the correlation matrix is assumed. Reliability analysis yield satisfactory Cronbach’s alpha values for each subscale: MC, 0.93; MT, 0.90; TC, 0.88; BE, 0.78 and AE, 0.70. This indicates a strong or acceptable degree of internal consistency in each subscale.

Further Statistical Analyses

In order to investigate gender, achievement, year level and Number of CAS years use statistically significant differences in the MTAS subscales, t-tests, one-way ANOVAs and Multiple Analysis of Variance (MANOVA) tests were conducted. Some of these findings are discussed in the following sections.

Gender Differences

Gender differences in attitudes to mathematics have long been of interest (Fennema and Sherman, 1976; Forgasz, H., Leder, G.C. & Barkatsas, A. (1998); Forgasz, Leder & Barkatsas, 1999; Barkatsas, Forgasz & Leder, 2001; Pierce, Stacey and Barkatsas, 2007; Forgasz, Barkatsas, Bishop, Clarke, Keast, Seah & Sullivan, 2008; Barkatsas, Gialamas & Kasimatis, 2009) and a question of current interest is whether using technology to learn mathematics exacerbates differences. This section reports results on the five subscales by gender.

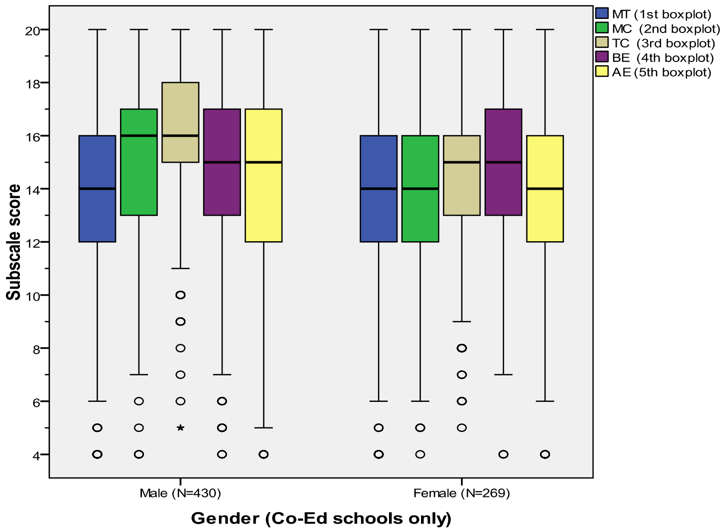

Figure 1. MTAS subscale scores by gender.

The breakdown of these scores by gender, illustrated in Figure 1, reveals that boys have higher scores than girls for each subscale except *MT* and *BE*, and statistically significantly higher scores for *MC* [t(720) = 6.673, p < .001 (two-tailed, equal variances assumed)] and *TC* [t(727) = 8.438, p < .001 (two-tailed, equal variances assumed)]. While 50% of boys score 16+ on MC, this was true for only 25% of girls. TC scores are even more strongly higher for boys, with approximately 75% of boys scoring 15+ and only 50% of the girls. These results reflect the common finding that boys express greater confidence than girls on both mathematics and technology. It is important to note that the BE scores and the MT scores are identical for both boys and girls, in contrast to the Pierce, Stacey and Barkatsas (2007) study, which found a statistically significant difference between boys' and girls' mean scores for MT and reported that: "The distributions of MT have a long tail for both boys and girls and high inter-quartile range" (p. 296).

Achievement differences

Barkatsas, Kasimatis & Gialamas (2009) investigated the complex relationship between students' mathematics confidence, confidence with technology, attitude to learning mathematics with technology, affective engagement and behavioural engagement, achievement, gender and year level. They reported that high achievement in mathematics was associated with high levels of mathematics confidence, strongly positive levels of affective engagement and behavioural engagement, high confidence in using technology and a strongly positive attitude to learning mathematics with technology.

In this study, statistically significant differences were found between *achievement and MC* [F(4, 37) = 13.429, $p < 0.001$, $\eta^2 = .109$], *achievement and BE* [F(4, 37) = 3.470, $p < 0.05$, $\eta^2 = .031$] and *achievement and AE* [F(4, 37) = 3.296, $p < 0.05$, $\eta^2 = .029$].

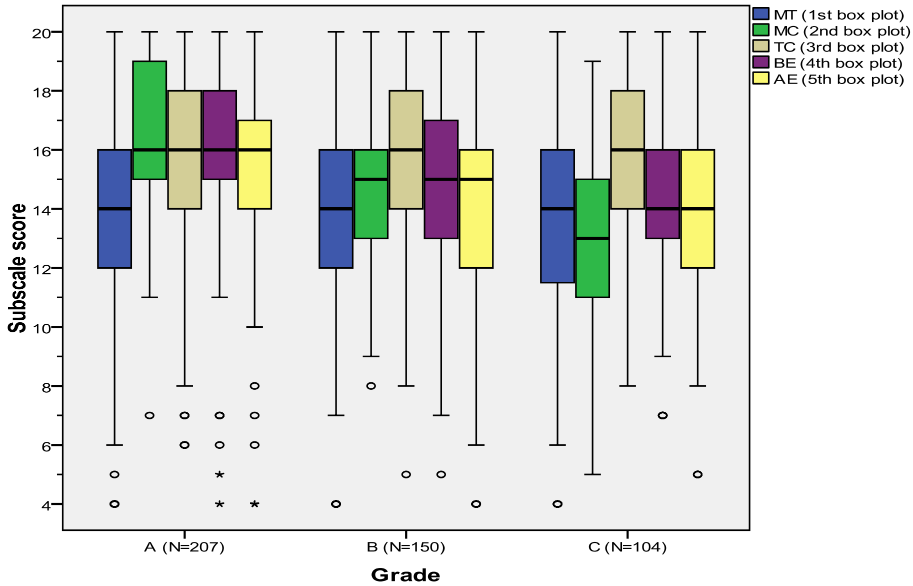

Figure 2. MTAS subscales by grade. (The Grades categories used in the study were: A (80-100%), B (70-79%), C (60-69%), D (50-59%) and E (<50%). Grades D+E have been removed due to the small number of students in these two categories).

The breakdown of these scores by grades, illustrated in Figure 2, reveals that students with excellent achievement in mathematics (students with grade A, which incorporates the grade A^+ in this study) demonstrated higher levels of mathematics confidence than all the other students in the sample and that the higher the students' grade the more positive their mathematics confidence. A similar outcome is evident for both behavioural engagement and affective engagement. The *MT* inter-quartile ranges are identical for students with grades A and B, with a slightly smaller value for the first quartile (25%) for students with a C. The median value is the same for all grades (14) and the scores for each grade are less than the corresponding *TC* scores, respectively.

Differences between MT and TC, and CAS experience

Statistically significant differences were found between the variable *three years of CAS experience* and the variable *one year of CAS experience* [t(626) = -1.806, $p < .05$ (two-tailed, equal variances assumed)] but no statistically significant differences were found between the variables two years of CAS experience and one year of CAS experience, and the variables two years of CAS experience and three years of CAS experience. It could be argued that the more time students get accustomed to the CAS calculators' functionality the more positive their attitudes towards their use in mathematics learning are.

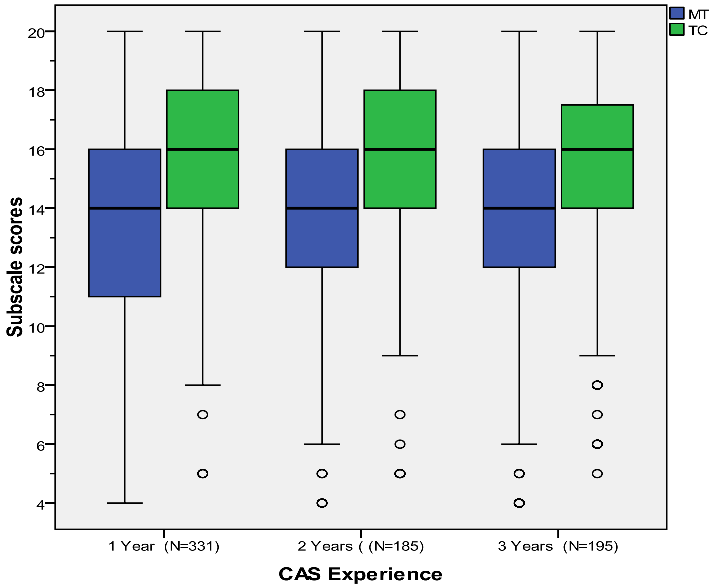

Figure 3. Attitude to learning mathematics with CAS [MT] and Confidence with Technology [TC] subscale scores by CAS experience.

In Figure 3 the *MT* and *TC* subscale scores by the number of years students have been using CAS calculators, are shown. The rationale for the inclusion of this figure in this paper is that the TC scores are consistently higher than then *MT* scores for all the analyses carried out for this study. Students demonstrate higher levels of *TC* irrespective of CAS experience, compared to *MT* scores. It could be argued - with caution - that it may take at least two to three years for students to get accustomed to the complex functionality of CAS calculators (or other CAS software), for their confidence with CAS to reach the levels of their confidence with other technological devices, tools and software they have used and been accustomed to, in the past. It may also be the case that students demonstrate more confidence with other forms of technology because they do not associate them with school and/or mathematics work.

Conclusions

In this paper we investigated the complex relationship between secondary mathematics students' mathematics confidence, confidence with technology, attitude to learning mathematics with technology, affective engagement and behavioural engagement, achievement, gender and year level. The following is a summary of the study's findings. Statistically significant differences were found between:

Gender and the following subscales:

- *Mathematics Confidence [MC]:* $t(720) = 6.673$, $p < .001$ (two-tailed, equal variances assumed)] and
- *Confidence with Technology [TC]*: $t(727) = 8.438$, $p < .001$ (two-tailed, equal variances assumed)].

A*chievement* and the following subscales:

- *Mathematics Confidence [MC]: F(4, 37) = 13.429, p < 0.001, η2 = .109,*
- *Behavioural Engagement [BE]: F(4, 37) = 3.470, p < 0.05, η2 = .031 and*
- *Affective Engagement [AE]: F(4, 37) = 3.296, p < 0.05, η2 = .029*
- *Year level and Behavioural Engagement [BE]*: F (3, 37) = 3.977, $p < 0.05$, $\eta^2 = .026$.

The variable three years of CAS experience and the variable *one year of CAS experience*: t(626) = -1.806, p < .05 (two-tailed, equal variances assumed).

Overall, students demonstrated higher levels of *TC* - irrespective of CAS experience - compared to *MT* scores. It could be conjectured that, for the majority of students, it may take at least two to three years to get accustomed to the complex functionality of CAS calculators (or other software) and for their confidence (to learn mathematics) with CAS, to reach the levels of their confidence with other technological devices, tools and software that they have used and have been accustomed to. It may also be the case that students demonstrate more confidence with other forms of technology because they do not associate them with school and/or mathematics work.

Drijvers (2000) put forward five obstacles that students may encounter when using CAS. All of them however may be characterised as cognitive obstacles. It is my conviction that the time is ripe for the mathematics education community to engage in longitudinal studies which will focus on the identification of affective obstacles students encounter when using CAS in learning mathematics. It is important that we invest in teaching innovations that will boost low and average-achieving students' confidence in learning mathematics and that teachers require extensive training in identifying and implementing appropriate teaching and learning approaches that will enable them to avoid reinforcing female students' learned helplessness in mathematics in order to close the gender gap in mathematics achievement and in using technology for mathematics learning.

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