Graphing Calculator and Black Box Phenomenon and Strategies of Consolidation into Math Courses in China

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Abstract: *The Graphing Calculator (GC), which is regarded as information technology that probes into math courses* [1] *, plays an important role in combining information technology with the math course. At present, many researches and practices are undertaken about the relationship between GC and new math course in many places around the country. Meanwhile many excellent teaching cases have been developed. The Standard of Math Course in High School says that the basic principle of combination of using information technology in math course is beneficial for the students to learn the essence of mathematics. But some of cases are called "black box" phenomena which lay emphasis on implication of technology, and focus on the math's conclusions or results through GC, while the math's concept even principles of math's knowledge are laid in second place or ignored.*

1. The "black box" Scenario

1.1 Focusing on operating results but not the mathematical principles.

The idea that "Focusing on operating results but not the mathematical principles" refers to focusing on the issue directly by using GC results, and ignoring the mathematical theory behind them. Using GC to teach, in this way, the math class is turned into an Information Technology Operations class.

Case 1 Seek the Approximate Solution with the Help of Information Technology [2], The teaching objectives are: 1. Show Algebraic function of GC; 2. Show image processing functions of GC. The general direction of orientation was in the technical operations, reflecting no mathematical knowledge learning objectives, not to mention the high level goals for mathematical thinking. In particular the teaching process was to seek the approximate solution of $2^x + 3x = 7$. The students conducted 4 methods of approach. The first three kinds of solution used the GC: equations for functions, algebra function. The approximate solution will be displayed immediately after putting in the equation, or drawing the images of the two functions $y_1 = 2^x + 3x, y_2 = 7$, tracking the intersection coordinates and reading out direct the approximation.

Clearly, to determine the approximate solution of the equation, the mathematical theory is the existence theorem of zeros. Mathematics is the dichotomy, mathematical thought is thought of the algorithm, these elements are reflected in solution 4 (using the dichotomy programming). While the first 3 approaches are not reflected, we believe that as an independent method for solving, method four could be ignored. The math could barely be seen in the other three kinds of mathematical methods (Solution Combination of Number 3 reflects the ideas, but this is clearly not the core of this issue), in particular, just using " function solution equation, algebra function to “press "the equation, it is a" black box “for the students. They cannot not see any mathematical component equation, but only get a result, not to need for thinking. The students know this but do not know why it is the solution. “Without thinking it is impossible to build critical thinking, without thinking it is impossible for teaching to build thinking.” In other words, as opposed to the basic goal of mathematics education: "learning mathematics, fostering mathematical thinking", the first 3 methods almost have no educational meaning. What students learn is the operation method of GC. The results they can use the GC without knowledge and thinking skills being improved. Technical operations make students overlook the learning of mathematical theory, the training of the mathematical thinking and the improvement of the basic technical skills.

1.2 Intuitive conclusion but no logical process

The so-called "just intuitive conclusion but no logical process" refers to the fact that when doing Inquiry-based learning, students use the functions of GC to get the results directly, without the necessary logical analysis and even necessary deductive proof.

Case 2 [4], the first term of the number series $\{a_n\}$ is a_1=2, and $a_{n+1} = \frac{2a_n + 1}{3} (n \in Z^+)$, seeking to set up the smallest positive integer inequality $|a_{n+1} - a_n| < 10^{-9}$.

As traditional problems, from the level of knowledge, the theme of mathematical significance is, through a specific sequence to show that in certain conditions (existence limit), when the number of the column is large enough, the neighboring columns adjacent to the value is infinitely close. From the Capacity level, the theme is to build mathematical reasoning for the students, computing power in the process of problem solving. The determination of n is based on solving the inequality $(\frac{2}{3})^{n-1} < 3\times10^{-9}$. In order to solve the inequality, computational tools must be rely on or the transcendental operation can not be completed by the manual operation. The basis of 1-5 Solution of the case is helped with the use of GC for the computing. Method 9 is using a loop to write 4 procedures to run the four programs on GC. These can meet the conditions to determine the smallest positive integer. It also better reflects the new curriculum ideas using the algorithm. The problem is that solution to 6-8 makes use of the recursive computing functionality the GC and puts the first and the recurrence formula in GC. The GC automatically figures out each progressive display, until the conditions are met to satisfy the inequality (there are nine zeros behind the decimal in the two neighboring rows of value). (See Figures as follow).

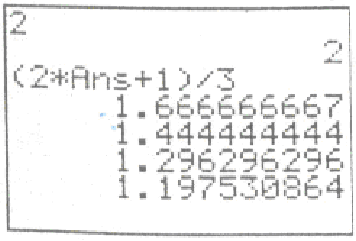

图 6-11

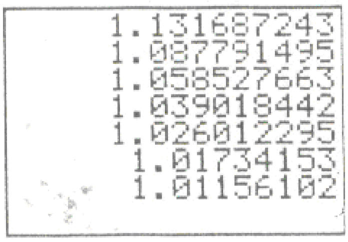

图 6-12

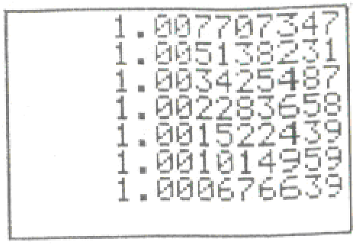

图 6-13

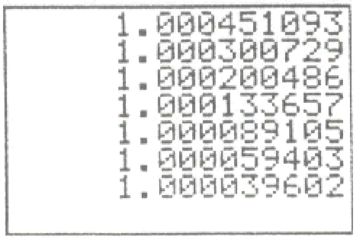

图 6-14

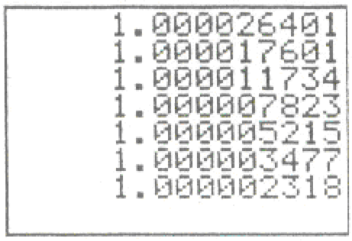

图 6-15

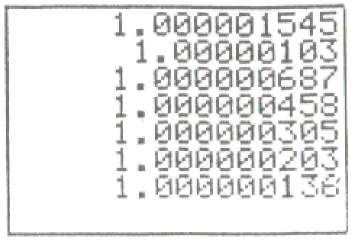

图 6-16

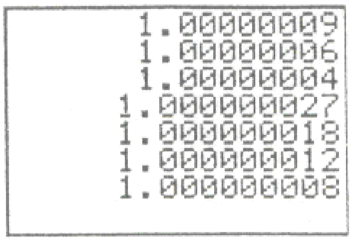

图 6-17

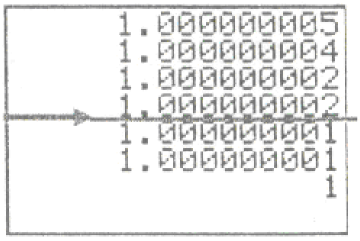

图 6-18

If you insist on finding the mathematical sense in the solution of 6-8, we can only say that it lets students know (and see) a mathematical fact: a series, if you know it's first and the recurrence relations , then you can identify all the items in the series. But this is clearly not the core of the problem.

Case 3: There are two exercises in Mathematics Compulsory Book I for Jiangsu high school students:

Page 30 Example 2: Draw the image of the following function: $f(x)=|x|$ and obtain its value.

Page 31 Exercise 2: Draw the image of the following function: $f(x)=|x+3|$ (Textbook marginal note: What is the connection among the images of Question 2 and Example 2?)

A teacher replaced the two above issues with the following two questions when doing a GC teaching demonstration classed in a Discussion group in Suqian experimental zone.

① Draw the image of the functions: $y = 2x +1, y = 2 | x | +1, y = | 2x +1|$ respectively and observe the contact between them.

②Draw the image of the functions: $y=x^2-2x-3, y=x^2-2|x|-3$, and then draw another function image: $y=|x^2-2x-3|$, observe the connection between them.

The teacher asked the students to explore the functions using the GC independently. By contrast, they summed up the turnover transform conclusions quickly after drawing the two function images using the GC.

It was the first time they tried to use the GC in teaching, and the teacher was pleased when they solved the problem successfully. The procedure was not smooth in the non-experimental classes which were not equipped with the GC. Teachers of non-experimental classes who participated in the demonstration lesson recognized the power of information technology because of their difficulties in teaching this content.

Teachers of experimental classes from different schools talked about this phenomenon, which stimulated further discussion in the after-school discussion group workshop. As there's no special studies on the images of function containing the absolute value. So a teacher asked these questions: what is the writers' intention when they composed the textbook? What should students learn through solving the problem? What kind of role should the GC play in teaching and learning? Many teachers believed that students were not good at dealing with problems using absolute value. Some students especially those who were grade one of senior high school from non-experimental classes were even poor at solving such questions. Therefore, the ability required to answer the textbook question from the perspective of knowledge was that students could draw the function image and had general understanding of translation transformation and turnover transformation. Students' capacity for classification, induction and generalization could be cultivated by discussing the value of x, positive and negative value of $f(x)$, as well as transforming some simple specific functions, such as $y = f(|x|)$, into sub-function. In non-experimental classes, students found it difficult drawing the images independently, so under the guidance of teachers they obtained the experience of learning classified discussion and induction. But in the experimental class, students can complete more complex function images by the aid of the GC, but the basic logical reasoning processes and mathematical thinking were overlooked because of the Perspicuity of the GC.

The phenomenon of overlooking logical process when using the GC often occurs in students in the experimental classes. Some students regard the results observed from the GC as formal conclusions without the consciousness of deduction and proof when they do some exploratory questions. A large number of students may use the GC to get an ideal image directly when they had too many assignments and they were too busy to complete them. The phenomenon is worthy of attention.

1.3 Many cases but without summary of a rule

The so-called "Full of many cases but without summary of rule" refers to the use of information technology in teaching to deal a concrete question neglecting these issues common nature to be summed. The students' cognitive level stays in the level of specific issues, not rising to a higher level of abstraction.

Case 4 [5] A teacher in teaching "the best value solution using the basic inequality problem (Senior

One / Compulsory Course)", $y = x + \frac{4}{x}(x > 0)$; $y = x + \frac{4}{x}(\frac{1}{2} \le x \le 3)$; $y = \sqrt{x^2+2} + \frac{1}{\sqrt{x^2+2}}$;

$f(x) = \frac{x^2+2x+a}{x}, x \in [1,+\infty)$; $f(x) = x + \frac{225}{x}, x \in (\frac{400}{29}, \frac{29}{2}]$ (a cost of pool application problems in the function), respectively, using the GC displays the image, observing the maximum (large) value.

Although the process of the lesson deal with a large number of the form such as $y = x + \frac{d}{x}(d > 0)$ by using the GC, they could not abstractly sum up the common nature of the whole function. Students can't understand to this kind of function.

Case 5[6] Known cases of any two non-zero vectors a, b, try to make $\overrightarrow{OA}$=a+b, $\overrightarrow{OB}$=a+2b, $\overrightarrow{OC}$=a+3b, can you determine the position of three-point A, B, C? Why?

By using Geometric mapping function, animated feature of the GC, we can make $\overrightarrow{OA}$=a+b, $\overrightarrow{OB}$=a+2b, $\overrightarrow{OC}$=a+3b in GC. Then drag any vector a, b, change its length, observe A, B, C's location, conjectured, and then prove it.

In this case, the GC helped students find the law; put forward a conjecture allowing the GC to put forward better cognitive functions. We believe that the problem will be resolved, and then lead students to think, for example, according to students' level of knowledge; you can ask students to answer the following questions:

According to this question, can you propose a more general context the problem and try to explore the result?

Or ask some of the more specific questions:

Can you find the three relations?

Can you find quantitative relationship between the coefficients?

Can you give A, B, C collinear three general terms?

The first issue was intended to develop student's awareness of problems and guide students to solve a specific problem. Then the problem will be extended to more general situations. This is a valuable mathematical result in mathematics teaching. Also, it is the focus of the training. The following three questions are to lead students to explore by raising the related issue. Because the student's capacity in levels is limited, probably students can not bring forward satisfactory methods of solving problems. Especially, since curriculum standards are not asking higher order questions. But this is good exploration material, which requires students to put forward conjecture, not give a proof. These methods can improve the students awareness and general ability, particularly in helping students aware the problems beyond individual level to form a more general pattern of " we could sum up the corresponding mode by understanding of the experience, enabling effective application is used to solve similar problems." [7] ,then experience the modeling features of Mathematical Sciences

2. Refractive conception root of "black box" scenario

Outwardly, black box phenomenon is caused by the inappropriate usage of the GC. It has a deep conception root. Many teachers are confused with the concept of combination information technology with math courses, and think that combination between Information and Math Courses means to usage of Information Technology in Math Teaching and takes for granted the class use of the GC, especially in an exclusive class, or where it is not regarded as a good class. Some math teachers still perceive math as static, and they think math is a kind of influx of a series of propositions, formulas, and rules, and learning math is to learn this influx; some math teachers regard learning math as figuring out questions. One of math teachers in a forum said that the GC can help students gain math conclusion quickly, and save much time to do the exercises. Looking at the proving of proposition and formula, some teachers and students think now that the result is worked out through the implication of information technology, which must be right, and the progress of implication of information technology is some kind of proof.

3. The effective way of avoiding the "black box" scenario

The study shows that using the GC appropriately can improve the level of math concept effectively, while using the GC inappropriately can influence the students' basic math abilities including calculating ability, understanding ability, judging ability and remembering ability. However, neither the positive effect nor negative influence is based on the GC itself. But is determined by how it is used, as it is vital for students to learn math.

3.1 Profound understanding of the basic objectives of mathematics education

Senior High School Mathematics Curriculum Standards points out that: "The extensive application of modern information technology are making a great impact on the mathematics content, mathematics teaching, mathematics learning and other aspects." However, " Senior high school education is basic education and senior high school mathematics curriculum works as the base of mathematics", "Improving students' math thinking, this is a basic goal of mathematics education." and this goal can be achieved by mastering the basic knowledge of mathematics, basic skills, the basic idea. Therefore, we should strengthen mathematics teaching and information technology integration on the premise of ensuring the written calculation training. Hence, the GC should be used to cultivate students' math thinking ability in mathematics teaching. Let the GC become a kind of tools to grasp mathematics' knowledge, thinking ability.

3.2 The basic connotation of combining information technology with mathematics curriculum

In general, simple using the GC cannot satisfy all the requirements of the mathematics curriculum. All activities must be carefully designed to achieve the integration of basic requirements. Professor Kekang-He indicated that the process of information technology combining with subject curriculum includes three basic attributes: creating a new teaching environment, achieving new ways of teaching and learning, changing the traditional teaching structures [9] . Looking at the GC also relies on the basic principles of mathematics teaching design. We should focus on three basic properties to design the use of the GC and also the basic characteristics (autonomy, inquiry, cooperation) [9] should be focused on under conditions of information technology in teaching and learning.

3.3 Adhering to the "math-based" in using the GC

Professor Shanliang-Li indicated that the GC works as a learning tool, so it's forbidden to produce function dissimilation, confining unnecessary needs of using the GC [10] . In mathematics teaching and learning, we should have a clear mathematical purpose in using the GC, requiring students to explore the change of patterns, study use the nature of the function, operate algorithm programs, deal with the statistical data, establish mathematical models and develop mathematical applications by using the GC. Then students can harvest mathematics knowledge, thinking ability, computing skills, logical reasoning, applications and other aspects. To get a simple result is not the ultimate goal in using the GC.

Written calculation should be used in any necessary place; ensuring basic written calculation is the premise of using the GC.

To enhance students' evidence awareness, and also there is great needs to give the necessary instructions and even a strict interpretation of the logical proof to the use of GC conclusion drawn by a special cases.

Overcome the phenomenon of sticking to one special question, and develop students' awareness of issues, development capabilities;

4. Conclusion

We believe that, in integrating GC and mathematics curriculum, we must not only reflect the “technology factor” to train students to use GC, but also embodies the “mathematical factors” to guide students to use the GC self-study, exploration and discovery. When student takes math study activities as their conscious behavior, not use the GC to replace necessary written calculation to obtain the results. Of course, while students' know the outcome of the questions, they still have the initiative to explore how to get this result by reasoning. For instance in the "isolated case "problem results in exploring that" type ", our mathematics teaching has changed from" teaching levels "up as" education levels "from the" impart knowledge "to" cultivate people ".

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