Integration of Graphing Calculators in Mathematics Teaching in China

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Abstract: *This study investigated the benefits of using the graphic calculators in mathematics teaching and examined the effects of the graphing calculators in secondary mathematics classroom, such as: (1)The use of graphing calculators in middle school mathematics teaching does not affect mathematics learning, but helps in the understanding of mathematics concepts, qualities etc., and promotes changes from the passive, old learning approach to diverse learning thru self-learning, cooperation, and exploration. (2) The use of graphing calculators in secondary mathematics teaching is conductive to the reform of evaluation systems, especially in the way exams are constructed. (3) The use of graphing calculators promotes the modernization of mathematics textbooks, and is conductive to the information, visualization, diversification, and popularization of the contents of teaching materials. In addition, this study discussed the advantages of using the graphic calculators in mathematics education programs in higher education instructions and in professional development.*

1. Introduction

According to the National Council of Teachers of Mathematics (NCTM, 2000), technology is an essential component in teaching mathematics; it influences the way that mathematics is taught and learned. However, some educators are uncertain and anxious about the functions of technology on their student learning. This is also true in Chinese mathematics classrooms in terms of using graphing calculators. Although integrating graphing calculators in mathematics classrooms has been developed and used in the U.S. since the 1980s (National Research Council, 2000), Chinese mathematics educators have struggled with whether to use graphing calculators in mathematics classrooms and wondered what the vital roles of graphing calculators are in student learning and the teaching of mathematics.

2. Conceptual Framework

2.1 Optimizing Mathematical Problem-solving Processes

The traditional mathematical concept is often considered to be the memorizing formulas, substituting numbers in equations, repeated practice and long and monotonous calculations. Many students have become tired of mathematics, and have strayed from mathematics because it is boring and has complicated calculation. However, the use of graphing calculators to handle complex calculations can reduce the burden on students, so that students spend more me on understanding, reasoning, and the applications of mathematics, which can stimulate their enthusiasm for learning (Streun, Harskamp, & Suhre, 2000).

The graphing calculator has a powerful algebraic function; the use of a variety of its built in programs can carry out different kinds of calculators and transformations of polynomials, matrices, determinants, factorizations, equation solving, the seeking of limits and trigonometric functions, etc. Such functions have not only provided strong support for the teaching of mathematics, especially beginning calculus and other higher mathematics content at the secondary school level, but also have become good tools for independent exploration and experiments (Shore & Shore, 2002).

2.2 The Development of a Real Context and Optimization of Process Education

With the development of new curriculum ideas, mathematics teaching pays more attention to process education, which focuses more on the process in which students can experience the development of knowledge so as to cultivate their abilities of exploration, abstract reasoning and reflection.

In this learning process, teachers should guide students to learn mathematics; then students experience and obtain information through their personal experience. However, in the traditional context of mathematics teaching, teachers would not use a lot of real situations, so students could not relate to any experiences in the real situations. In contrast, graphing calculators are able to provide multiple visual representations, allowing students' hands-operation to enable them to experience different forms of relationships through real-world examples (Laughbaum, 2002). It really provides a platform of "doing math" to students. Taking inequality as an example, every inequality can have an important mathematical representation, in particular the geometric representation. If the teachers can bring some geometry contexts of inequalities correctly in teaching, the students are in the visual process of observation and can grasp such inequalities well. This is far more meaningful than demanding students to solve inequalities by calculation. At the same time, teachers can train students' ability for logical thinking by "process education" and allow them to feel that many mathematical principles can also be proven by both intuitive and visual approaches.

2.3 Research Questions

The research questions asked in this study were:

1. What are the benefits of using the graphic calculators in mathematics classrooms?

2. What are the effects of the graphing calculators in mathematics teaching in secondary mathematics classrooms?

3. What are the advantages of using the graphic calculators in mathematics education programs in higher education instructions and in professional development?

3.Methods

3.1 Subjects

In April 2000, Hangzhou Normal University (HNU) formed a research team to study the effects of the graphing calculators in mathematics teaching. Three secondary schools in Zhejiang Province participated in this study. Eight mathematics classrooms were involved, including two college junior classes, three freshmen classes and three sophomore classes at the high school level. A total of 400 students participated in the study.

3.2 Procedure

The research was divided into three parts. The first mainly studied a series of applications of graphing calculators in mathematics teaching by HNU. The second one carried out the applications of the graphing calculators in the practice of teaching, which the three secondary schools participated in applications. This study was based on the teaching and research experiences in 2002 and 2004. During this period, graphing calculators were used in mathematics teaching in courses for the undergraduates, graduate students, and in-service teachers. The third part consisted of two stages: familiarity with using the graphing calculators, and modern educational technology by the mathematical teachers in Zhejiang Province in China.

3.3 Measurement

Effectiveness of problem solving was used as a measure of effect of the graphing calculators on mathematics teaching in this study in the first and second parts. In the third part, in 2005, in order to investigate knowledge of graphing calculators and modern educational technology by classroom teachers, 295 questionnaires were sent out. The questionnaire included twenty questions. In 2006, 295 questionnaires on teachers' knowledge of the application of modern educational technology were sent out, including teachers in 13 schools: five provincial first-degree key schools, two second-degree key schools, two ordinary schools and four junior middle schools. The questionnaire included twenty-five questions.

4. Results

The results show that graphing calculators have an important role in student learning and mathematics.

4．1 Benefits of Graphing Calculators on Mathematics Teaching

The benefits of using graphing calculators in mathematics teaching and student learning point to: (1) The use of graphing calculators in middle school mathematics teaching does not affect mathematics learning, but helps in the understanding of mathematics concepts, qualities etc., and promotes changes for the passive, old learning approach to diverse learning thru self-learning, cooperation, and exploration. (2) The use of graphing calculators in secondary mathematics teaching is conducive to the reform of evaluation systems, especially in the way exams are constructed. (3) The use of graphing calculators promotes the modernization of mathematics textbooks, and is conducive to the information, visualization, diversification, and popularization of the contents of teaching materials.

The following examples used in mathematics classrooms in this study provided evidence of effectiveness of using the graphic calculators in problem solving:

Example 1, (1997 national college entrance examination of science, section 18)

Seeking the value of $\dfrac{\sin 7^\circ + \cos 15^\circ \cdot \sin 8^\circ}{\cos 7^\circ + \sin 15^\circ \cdot \sin 8^\circ}$.

Solution: On the primary screen, input tExpand ($\sin(x+y)$), then enter $x = 7^\circ$ and $y = 8^\circ$, ans(1) $\to$ a， re-enter tExpand ($\cos(x+y)$)， $x = 7^\circ$ and $y = 8^\circ$, ans(1) $\to$ b 。

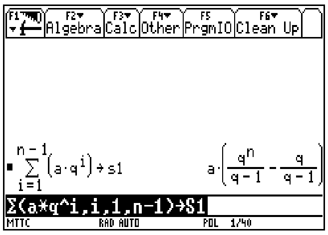
Then enter ： $(\sin 7^\circ + \mathrm{b} \cdot \sin 8^\circ) \big/ \left(\cos 7^\circ - \mathrm{a} \cdot \sin 8^\circ\right)$， and finally enter tCollect(ans(1)), get value $2 - \sqrt{3}$.

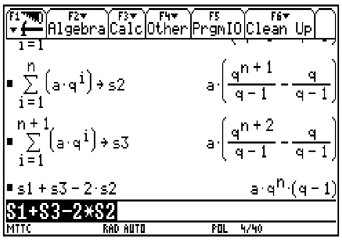
In this case, if we enter the formula directly into the calculator, we can not get results; only in accordance with the original problem-solving ideas step by step, we can get the right answer. Therefore, people who do not know the triangle formula and calculation methods can not solve the problem by a calculator. So we believe that the use of calculators will not affect the learning of mathematical knowledge and thinking.

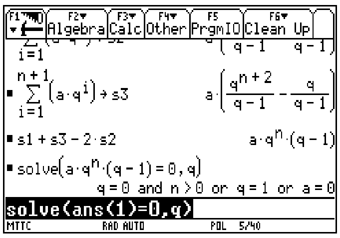
Example 2, (2001 national college entrance examination of science, section 15)

If $\{a_n\}$ is a geometric sequence in which q is its common ratio, S_n is the sum of its first n items. If $\{S_n\}$ is an arithmec sequence, then $q =$ _______.

Solution: We can solve it by the function of summation of sequence on the calculator. Suppose the first term of series $\{a_n\}$ is S1.Input $\sum(a \cdot q\,\hat{}\,i, i, n-1) \to S1$ on the enter line of the primary screen. After pressing return, the sum of the first $n-1$ items are endowed variable S1, please see figure (1).







figure（1） **figure（2）** **figure（3）**

Then endow the sum of the first n items and the first $n+1$ items of $\{a_n\}$ to S2 and S3 respectively, please see figure 3. Because $\{S_n\}$ is the arithmec series, so $S1 + S3 - 2 \times S2 = 0$.

Enter menu F2, by 1: function of solving equation, solve $a \cdot q^n \cdot (q-1)=0$, then $q=0$ and $n>0$, or $q=1$, or $a=0$, please see figure (4). As we get $a_n \neq 0$, so $q=1$.

Example 3, (2003 national college entrance examination of science,(Beijing volumes, section 15)

If the general formula of the series $\{a_n\}$ is $a_n = \dfrac{3^{-n}+2^{-n}+(-1)^n(3^{-n}-2^{-n})}{2}$, $n=1,2,\cdots$, seeking the value of $\lim\limits_{n\to+\infty}(a_1+a_2+\cdots+a_n)$.

Solution: use its function of algebraic calculations, you simply need to enter $\lim(\sum(a_n,n,1,i),i,\infty)$ in the main screen, in which $a_n = \dfrac{3^{-n}+2^{-n}+(-1)^n(3^{-n}-2^{-n})}{2}$, then we can get the result $\dfrac{19}{24}$.

4．2 Application of the Graphing calculat1or in secondary school

Since 2000, some school campuses have achieved certain results by using graphing calculators in mathematics teaching. In this study, and in teaching practice, the use of modern handheld educational technology has constructed the basic model: experiment—guess—conclusion—theory verification by using calculators in mathematics teaching in secondary schools.

The Discovery pattern in the experiment process means that students participate independently in the experiment and find its process by using graphing calculators and combining teaching materials under the guidance of teachers. This teaching pattern applies mainly to the teaching process of the formation of knowledge such as concepts, rules, formulae, theorems and examples. It shows students' dominate positions in the process of finding teaching and pays attention to the cultivation of strategies and methods of the discovery of knowledge. The "experiment" includes measuring, mapping, calculating and so on. Through this kind of teaching, we strengthen and cultivate students' capacity and thinking of innovation. This highlights the "guess" in the overall structure, which are just the basic strategies and approaches of mathematics findings. In the above two processes, we combined the training of thinking in images, intuitive thinking and logical thinking, which reflects the dual nature of mathematics. According to the teaching content and conditions, this model can use a variety of instructional designs. It opens up a vast world for the full development of students' knowledge, abilities and personalities, and cultivation of the creative spirit and practical ability.

Example 4 During teaching "the nature of power function", two students shared a graphing calculator. Teachers mobilized students' thinking fully, combining theoretical thinking and experimentation , and achieved better teaching results.

The teaching process was as follows:

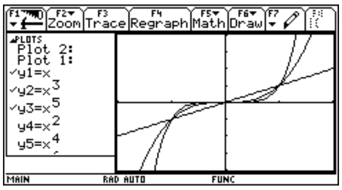
Comparison and classification——experimentation and observation——analysis and discussion——conclusion and synthesis

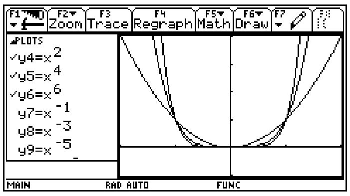
Comparison and classification: first, compare and classify various situations of the indexes of the power function.(abbreviated)

Experimentation and observation:

1.when n is a integer, choose n = 1,3,5 is as the experimental object when n is a positive odd number.

1) n = 2,4,6 is as the experimental object when n is a positive even number.

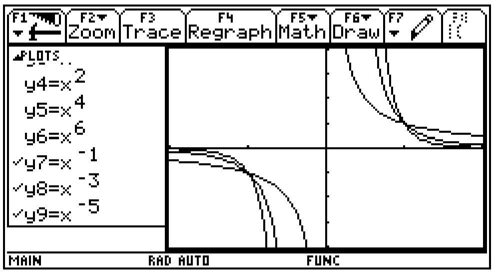


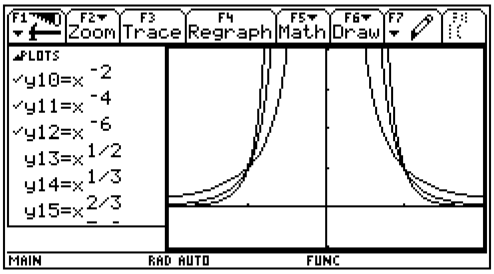


Figure(4) Figure(5)

2) n = -1,-3,-5 is as the experimental object when n is a negative odd number.

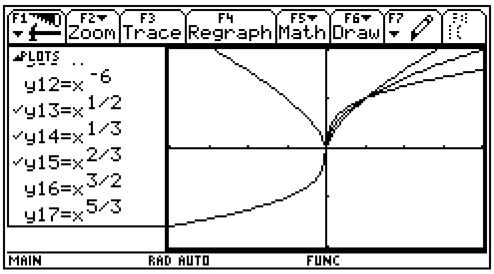
3) n = -2,-4,-6 is as the experimental object when n is a negative even number.





Figure(6) figure(7)

2 .when n is a fraction, choose n = $\frac{1}{2}$ 、 $\frac{1}{3}$ 、 $\frac{2}{3}$ is as the experimental object when n is a positive fraction which is greater than 0 and less than 1. n = $\frac{3}{2}$ 、 $\frac{5}{3}$ is as the experimental object when n is a positive fraction which is greater than 1.



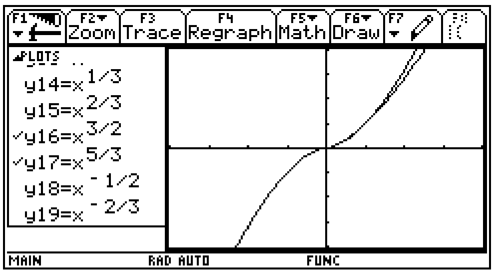


Figure (8) **Figure(9)**

3) $n = -\frac{1}{2}$ 、 $-\frac{2}{3}$ 、 $-\frac{3}{2}$ is as the experimental object when n is a negative fraction.

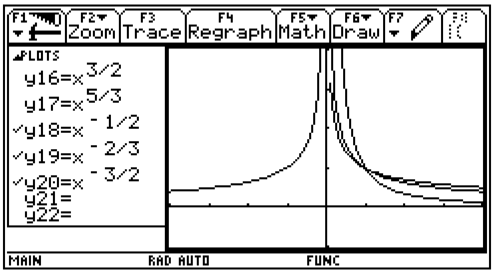


Figure (10)

Students observed and compared above experimental results repeatedly, finding the following three conditions of domain of definition of the power function.

(1) $x \in R$,such as n is a positive integer;

(2) $x \in (-\infty,0) \cup (0,+\infty)$, such as n is a negative integer;

(3) $x \in (0,+\infty)$ or $x \in [0,+\infty)$,if n = $\frac{q}{p}$, such as p is a even number, q is a odd number.

General conclusions: Teachers can guide students to synthesize as follows:

1. common point of the image;

2. shape of the curve, including positions of the progressive line;

3. Analysis of monotonicity

In the course of exploration and exchange, the students gained knowledge and a sense of accomplishment. It also stimulated their interest in learning and the desire to explore the unknown.

4.3 The Application and Research of Graphing Calculators in Higher Education Institutes

We have designed a second classroom program in undergraduate teaching which is to enable the students to understand graphing calculators and guide them to write theses on graphing calculators for graduation. For example, one faculty wrote “On graphing calculator and the teaching reform of higher mathematics”, which was published in Ningbo Institute of Journal Education in 2003. At the same time, by the use of graphing calculators, we have solved a lot of higher mathematics exercises and simplified the process of problem solving. We believe that the use of graphing calculators for higher mathematics teaching is conducive to the teaching of mathematics thinking.

Since 2003, we have opened “modern technology of mathematics education” for graduates in mathematics education mainly on how to use graphing calculators to solve problems in mathematics teaching which have achieved good results. Lin, 2003 graduate student in curriculum and teaching in mathematics education, published her paper entitled “Several applications of graphing calculator on the exploration of sequence of numbers” in the journal education of information and technology in 2006, and another entitled “On the exploration of the application of modern educational technology in mathematics education”, published by the journal Modern educational technology in 2006. Based on these publications, she wrote her master’s degree thesis entitled, “Modern education technology and mathematics teaching reform for middle school——taking graphing calculators as an example”.

4.4 The Application of Graphing Calculators in Professional Development in the Promotion of the Qualities of Mathematics Teachers in Secondary Schools

In training of the professional teachers, using the graphing calculators was one approach This included the operation of graphing calculators in the essential algebra and geometry, which is very popular among teachers. Many of the teachers acquired the skills to apply graphing calculators in their teaching practices. In Huzhou Middle School, for example, He and Liu published their article “The practice and speculation on the application of graphing calculators in high school mathematics teaching” in the *Journal of Mathematical Information*, in 2005.

In 2005 , a study of knowledge of graphing calculators and modern educational technology with classroom teachers, 260 valid questionnaires were collected. The results of the questionnaires show that 17 teachers had some idea about graphing calculators, 6.54% in all. Moreover, only three of the teachers comparatively knew technology well, and 14 teachers knew just a little about it. From the investigation we found that 10 of the teachers think that the application of the graphing calculators can help students reinforce their creativity; 9 teachers think that graphing calculators can deepen students’ understanding of mathematical knowledge; nine teachers think that graphing calculators can help students to solve puzzles in mathematics. These findings show that in this study, the majority of the teachers think that graphing calculators can benefit students in their mathematics studies. However, we also found that the majority of teachers of mathematics in this study have never heard of graphing calculators, which shows that the promotion of using graphing calculators has not met teachers’ demand, so that the majority of the teachers in this study do not use this teaching facility.

The results of the March 2006 study of teachers’ knowledge of the application of this modern educational technology show that the teachers were not accepting of the application of graphing calculators in the teaching of mathematics. The majority of the teachers did not have the skills or did not use graphing calculators. In discussing the reasons, we think, there are several factors: firstly, the graphing calculator is new technology, which has been applied during a comparatively short me in mathematics teaching. The majority of teachers in this study had few chances to get to learn how to use graphing calculators; most of mathematics teachers just knew about them in their teacher training programs. Meanwhile, graphing calculators were a little expensive. The promotion was not sufficient, so that most of the teachers had no idea of graphing calculators. Secondly, the language of the software is English for graphing calculators, and the manual books and materials are all in the English language, and the mathematics teachers could not understand English very well; therefore, they can’t operate them properly and therefore not understand the benefits of the graphing calculators in mathematics teaching. Thirdly, they had not been used in the examinations, so they did not attract the teachers’ attention.

5. Discussion

This study shows the benefits of use of graphing calculators in secondary school mathematics teaching in understanding of mathematics concepts (Hopkins & Amelia, 1998; Laughbaum, 2002; Shore & Shore, 2003) and it encourages diverse learning such as self-learning, cooperation, and exploration (Shore & Shore, 2003).

In addition, it is also helpful to mathematics education programs in higher education institutions and professional development.

Although through investigation we achieve some good results, we found that it is necessary to do further studies in some of the problems such as, how can we deal with the relationship between students' independent thinking and cooperative study in applying modern educational technology. How can we make good use of the advantages of alternating teaching when we have modern technology in our hands? How can we associate graphing calculators with the CBL system and various sensors, and associate mathematics teaching with that of other subjects? Besides these difficulties, we have more problems in promotion of the use of graphing calculators in mathematics classrooms:

(1) graphing calculators are more expensive than what the countryside students can afford.

(2) graphing calculators just show the results of the solutions, not the procedures of the solutions. Since they are not allowed to use them in examinations, teachers and parents have little enthusiasm in using them.

(3) There remain some problems in dealing with the relationship of the advantages and disadvantages in using graphing calculators.

(4) There should be more establishments of the research teams to deal with the application of graphing calculators in China.

6. Conclusion

Graphing calculators have focused attention on the concern of mathematics teachers in their teaching in China, and much research work has been carried out; we have some achievements. We have come to know some of the advantages in using graphing calculators in mathematics teaching. With development of training, graphing calculators will gain more and more recognition. However, there is a long way to the popularization of graphing calculators in secondary schools, which points us to shift our teaching conceptions and ideas, keep up our research work, and maintain experiments in the schools.

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