Instructional Design of Inquiry-Oriented Differential Equations

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Abstract. *To improve undergraduate mathematics learning teachers need to recognize and value characteristics of classroom learning environments that contribute to powerful student learning. The broad goal of this special issue are to share such characteristics and the theoretical and empirical grounding for an innovative approach called the inquiry oriented differential equations (IO-DE) project. We use the IO-DE project as a case example of how undergraduate mathematics can draw on theoretical and instructional advances initiated at the K-12 level to create and sustain learning environments for powerful student learning. In addition to providing an overview of the five articles in this special issue, in this introductory article we highlight the theoretical background for the IO-DE project and provide a summary of two quantitative studies done to assess the effectiveness of the IO-DE project on student learning.*

University mathematics departments, faced with the combination of an increasingly diverse student body, a declining number of mathematic majors, and looming accountability concerns are more aware than ever of their need to develop effective and innovative curricula and instructional practices (see [2], [5] and [17]). These innovations need to be capable of supporting students in developing productive dispositions as well as deep conceptual understandings of important mathematical ideas. How to accomplish this daunting task is an open question that offers an opportunity for mathematicians and mathematics educators to work together on problems of teaching and learning.

The Inquiry Oriented Differential Equations (IO-DE) project is an example of such a collaborative effort that seeks to explore the prospects and possibilities for improving undergraduate mathematics education, using differential equations as a case example. In addition to providing an overview of the articles in this issue, the goals of this article are to highlight the theoretical background for the IO-DE project and to provide a summary of two quantitative studies done to assess the effectiveness of the IO-DE project on student learning.

IO-DE Background Theory

Perhaps not surprisingly, different research communities characterize inquiry in different ways. For example, in science education the National Research Council [10] states that inquiry includes identification of assumptions, use of critical and logical thinking, and consideration of alternative explanations. In mathematics education, Richards [16], characterizes inquiry as learning to speak and act mathematically by participating in mathematical discussions, posing conjectures, and solving new or unfamiliar problems. Both of these characterizations highlight important aspects of student activity. While characterizations of student activity are useful, it only addresses part of the process of inquiry. In order to more fully understand the complexity of classroom learning, our definition of inquiry also encompasses *teacher* activity. In particular, IO-DE teachers routinely *inquire* into their students' mathematical thinking and reasoning. Teacher inquiry into student thinking serves three functions. First, it enables teachers to build models for how their students interpret and generate mathematical ideas. Second, it provides an opportunity for teachers to learn something new about particular mathematical ideas in light of student thinking. Third, it better positions teachers to follow up on students' thinking by posing new questions and tasks. Students, on the other hand, are learning new mathematics by *inquiry*, which involves engaging in mathematical discussions, posing and following up on conjectures, explaining and justifying one's thinking, and solving novel problems. Thus, the first function that student inquiry serves is to learn new mathematics by engaging in genuine argumentation. The second function that student inquiry serves is to empower learners to see themselves as capable of reinventing mathematics and to see mathematics itself as a human activity.

Mathematically, the IO-DE project drew its initial curricular inspiration from contemporary, dynamical systems approaches, such as that developed by [1], [6], and others. These approaches represent a significant departure from conventional treatments that emphasize a host of analytic techniques for solving special classes of well-posed problems. Departing from this conventional approach, Blanchard, Devaney, and Hall, for example, develop analytic, graphical, and numerical approaches as three different techniques. They also incorporate application problems that illustrate the usefulness of differential equations in real world problems.

As our extended research team[1] systematically investigated the learning and teaching in such approaches, we developed three goals that extended contemporary dynamical systems approaches that related to our definition of inquiry.

- First, we wanted students to essential reinvent many of the key mathematical ideas and methods for solving differential equations. Thus, rather than being introduced to analytic, graphical, and numerical methods as pre-existing techniques for solving differential equations, IO-DE students are invited to engage in challenging problems that provide an opportunity for them to create their own analytical, graphical, and numerical methods. Teachers, for their part, facilitate and support the growth of students' self-generated mathematical ideas and representations toward more conventional ones.
- Second, and related to the first goal, we wanted challenging tasks, often situated in realistic situations, to serve as the starting point for students' mathematical work. In other words, experientially real[2] situations drive the need for and creation of key mathematical ideas and various methods of solving differential equations. This stands in contrast to application type problems that typically appear at the end of a section.
- Third, we wanted to maintain a balanced treatment of analytic, numerical, and graphical approaches, but we wanted these various approaches to emerge more or less simultaneously for learners. The desire for co-emergence of these methods stemmed in part from earlier

[1] The extended research team includes Mark Burtch, Mi-Kyung Ju, Karen Allen Keene, Michael Keynes, Karen King, Oh Nam Kwon, Karen Marrongelle, Chris Rasmussen, Bernd Rossa, Wei Ruan, Kyunghee Shin, Michelle Stephan, Joe Wagner, and Erna Yackel.

[2] The term experientially real refers to problem situations for which learners can engage their existing ways of reasoning to make progress on the problem. Such experientially real situations may be grounded in real world settings, but depending on the background and experience of learners, may also be grounded in more symbolic, mathematically oriented settings.

research in a reform-oriented differential equations course in which students had compartmentalized understandings of analytic, numerical, and graphical methods (see [11] and [12]). Teacher assistance in identifying analytical, numerical, and graphical methods as three different techniques comes *after* students have made progress in reinventing such methods.

Accomplishing these three goals was facilitated by conducting research in three related strands: (1) Adaptation of an innovative instructional design approach to the undergraduate level, (2) Systematic study of student thinking as they build ideas and teacher knowledge to support students' reinvention, and (3) Careful attention to the social production of meaning and student identity. These three strands do not represent a linear progression in our research. We conduct research in these three strands concurrently and view the strands as complementary. We next briefly turn to these three research strands of the IO-DE project, and situate the articles in this special issue in relation to these three areas of research.

Innovative Instructional Design

A cornerstone of the IO-DE project is adaptation of the instructional design theory of Realistic Mathematics Education (RME) to the undergraduate level. Central to RME is the design of instructional sequences that challenge learners to organize key subject matter at one level to produce new understanding at a higher level (see [3]). In this process, referred to as mathematizing, graphs, algorithms, and definitions become useful tools when students build them from the bottom up through a process of suitably guided reinvention (for illustrative examples and further theoretical development, see [7] and [15]).

The mathematization process is embodied in the core heuristics of guided reinvention and emergent models. Guided reinvention speaks to the need to locate instructional starting points that are experientially real to students and that take into account students' current mathematical ways of knowing. The search for such starting points is facilitated by examination of the history of mathematics, as well as students' informal solution strategies and interpretations. The heuristic of emergent models highlights the need for instructional sequences to be a connected, long-term series of problems in which students create and elaborate symbolic models of their informal mathematical activity (see [4]). The use of the term model is overarching idea that refers to student-generated ways of interpreting and organizing their mathematical activity, where activity refers to both mental activity and activity with graphs, equations, etc. From the perspective of RME, there is not just one model, but a series of models where students first develop *models-of* their mathematical activity leading to *models-for* reasoning about mathematical relationships.

Quantitative Assessments of IO-DE Student Learning

Rasmussen, Kwon, Allen, Marrongelle, & Burtch [13] conducted an evaluation study to compare students' routine skills and conceptual understandings of central ideas and analytic methods for solving differential equations between students in inquiry-oriented and traditionally taught classes at four undergraduate institutions in Korea and US. Whereas IO-DE project classes at all sites typically followed an inquiry-oriented format, comparison classes at all sites typically followed a lecture-style format.

The assessment consisted of routine skill problems and conceptual understanding problems. Routine skill problems focused on students' instrumental understanding such as an analytic and numerical nature of differential equations. On the other hand, conceptual understanding problems were aimed at evaluating students' relational understandings of important ideas and concepts. There was no significant difference between the two groups on routine problems. However, the IO-DE group did score significantly higher than the comparison group on conceptual problems.

Further, Kwon, Rasmussen, & Allen [9] investigated the follow-up study on the retention effect of conceptual and procedural knowledge one year after instruction for a subset of the students from the comparison study. Students' retention of knowledge was compared across a traditional and an IO-DE instructional approach. For the purpose of this analysis, procedurally oriented items were defined as those questions that were readily solved via analytic/symbolic techniques. Conceptually oriented items were defined into two categories, modeling tasks and qualitative/graphical tasks, each of which represent important and conceptually demanding thinking in mathematics, in general, and in differential equations, in particular. The two modeling tasks posed involved determining an appropriate differential equation to fit a given real-world situation. The qualitative/graphical tasks involved predicting and structuring the space of solutions. The analysis showed that there was no significant difference in retention between the two groups on the procedural oriented items. However, long-term retention of conceptual knowledge after students' participation in the IO-DE project as seen in student responses to modeling and qualitative/graphical problems was positive compared to retention by students in its traditional counterpart.

Conclusion

The implications of the IO-DE project are twofold. First, based on the results of the post-test and the delayed post-test (see [8], [9] and [13]), all IO-DE students from each of the four institutions, regardless of academic backgrounds and gender differences, outperformed traditionally taught comparison students on the post-test. Therefore, if the delayed post-test were given to all IO-DE students at the different sites, students may again similarly outperform comparison students. This result demonstrates that this instructional approach can be applicable to university mathematics regardless of academic preparations and gender differences. Secondly and more importantly, the instructional methods and curriculum design approach guided by RME are applicable to promoting student learning in all mathematics classrooms. Indeed, RME has its origins and broadest use in K-12 instructional settings. Yet, regardless of grade level or student differences, this inquiry-oriented instructional design could enhance long-term mathematics retention for all students. These findings support our conjecture that, when coupled with careful attention to developments within mathematics itself, theoretical advances that initially started at the elementary school classrooms (and which are beginning to spread to the rest of K-12) can be profitably leveraged and adapted to the university setting. As such, our work in differential equations may serve as a model for others interested in exploring the prospects and possibilities of improving undergraduate mathematics education in ways that connect with innovations at the K-12 level. Too often we hear teachers say that such and such an idea and approach to teaching and learning is fine at this or that level, but would not be appropriate for their level. We hope that this study calls into question such statements.

In our work we found that the heuristics from the instructional design theory of Realistic Mathematics Education and the constructs of social norms and sociomathematical norms constitute a useful collection of theoretical ideas that can inform and guide undergraduate mathematics education. Certainly these are not the only possibly useful theoretical ideas. Our position is, however, that innovations are more likely to result in positive outcomes if they are theoretically-driven rather than driven by the use of technology or collaborative learning, for example.

Additional significance of this work is evident in the extent to which the IO-DE research program is contributing to advancing the work of teachers and the professional development of mathematicians. For example, Rasmussen and Marrongelle [14] develop teaching strategies or "tools" that are teaching counterparts to the instructional design theory of RME. Using the IO-DE project as a case example, Wagner, Speer, and Rossa [18] detail the role of teachers' knowledge in the implementation of curricular reforms.

Thus, the IO-DE may be an example of an inquiry-oriented instruction at the undergraduate level which can help the type of conceptual understanding that can make mathematics meaningful to students and develop students' mathematical reasoning ability. The innovative approach to teaching differential equations suggested in this paper may give a possible arena for exploring the prospects and possibilities of improving undergraduate mathematics education.

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