Tasks, Technologies and Aesthetics: Aspects of One Approach to the 'Reconceptualization' of the Teaching and Learning of Mathematics

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Abstract: *In December of 2006 participants at the 17th ICMI Study Conference: "Digital Technologies in Mathematics Education - Rethinking the Terrain" heard one of the keynote speakers argue that: "Although the computer hardware and software options have been present for decades, we have still not seen a major shift in pedagogy within our education systems such as was widely predicted... We need to dedicate perhaps 10% of our individual energy and working lives to the exploration of new ways of teaching—of reconceptualizing how it is that we teach and students learn mathematics at all levels". This paper is a response to that statement. Following a brief consideration of the claim, an extended example from a 'reconceptualized' approach is developed. Three central characteristics of this 'constructive aesthetic' approach are rich tasks, conceptual frameworks and powerful ideas.*

1. Introduction

The question of how to make effective use of the various devices of Information and Communication Technology has been of long-standing interest to the international mathematics education community. The first ICMI (International Commission on Mathematical Instruction) Study, held in Strasbourg, France in March of 1985, was on the theme of "The Influence of Computers and Informatics on Mathematics and its Teaching". Of the fourteen succeeding Studies sponsored by this group in the next two decades, this was the only one to have not just a substantial set of meeting proceedings published (Churchill et al, 1986), but also to have substantially revised 'second edition' appear some six years later (Cornu and Ralston, 1992). This theme is also the first in this series to have been revisited, with the 17th Study Conference held in Hanoi, Vietnam ("Digital Technologies and Mathematics Teaching and Learning: Rethinking the Terrain") in December of 2006.

Given the prominence of these concerns and the massive investment in hardware, software and related curriculum materials over the period in question some might see it as surprising that there is a widespread and strong feeling that little authentic progress has been achieved. In the opening keynote address of the Hanoi meeting, for instance, participants were told that: "Although the computer hardware and software options have been present for decades, we have still not seen a major shift in pedagogy within our education systems such as was widely predicted... We need to dedicate perhaps 10% of our individual energy and working lives to the exploration of new ways of teaching—of reconceptualizing how it is that we teach and students learn mathematics at all levels" (1). Given that the speaker, Seymour Papert (2), would be seen by many as one who contributed substantially to the prediction of a 'major shift' (1980), some might feel that his views where less than objective. Space does not permit a detailed examination of the claims of underachievement. We note just two sources of supportive data and argument. First, for a broad historical consideration of the claims and realities of new technologies in education, the work of Cuban (1986, 2001) is pertinent.

On the mathematical side an important piece of research on the perceptions and attitudes of mathematics learners carried out by Nardi and Steward with a set of three, Year Three (13/14 year olds), classes in the United Kingdom and reported in the British Educational Research Journal in 2003. The subtitle of their paper, "A Profile of Quiet Disaffection in the Secondary Mathematics Classroom", captures the general thrust of their findings. In particular, the students claimed that much of their experience was captured by the mnemonic, T.I.R.E.D, the elements of which are: Tedium, Isolation, Rote Learning, Elitism and Depersonalisation.

2. 'Fatigue-Inducing' Mathematics Teaching and Learning: Two Examples

The types of student experience in mathematics classrooms of the sort revealed by Nardi and Steward are not, in fact, new. Nor are they geographically restricted. Findings from related studies in other parts of the world report similar results as do several of the international studies of mathematics achievement conducted over the last half-century. If one wishes to gain better insight into why these feelings emerge and what might be done to ameliorate them it might be helpful to work through some examples of one dimension of the experience, namely the types of question that students are asked to reflect on in their efforts to increase their comprehension of chosen mathematical concepts. We begin with a consideration of two examples of 'problems' that might be representative of ones presented to secondary-level mathematics students over a period of more than a century (3). The first, conceivably, could have been addressed to students tackling Euclidean Geometry more than a century ago.

> *Consider square ABCD where E is the midpoint of AB and F the point of intersection of AC and DE. What fraction of the area of triangle CFD is the area of the triangle AEF?*

The reader is encouraged to take a moment or two to reflect on this question, perhaps (as would likely have been the case for its original intended audience) with the aid of a pencil and paper. For a more recent example, think of a class of students from the last decade of the twentieth century facing this instruction:

> *Use your geometry software to construct square PQRS. On this figure locate and label T, the midpoint of PQ and join it to vertex S. Construct diagonal PR and label the point of intersection of ST and PR as U. Using the tools on the 'Measure' menu find what fraction the area of the triangle PTU is of the area of triangle SUR.*

To the extent that they are, in fact, moderately accurate representations of the sorts of 'prompts' given to math students for a very long time, these two scenarios have a number of features in common. At a basic level they are mathematically identical. The first (which in its day might have been called a 'rider', ie, a question arising either directly or indirectly out of the propositions to which they are appended) is a simple application of the Euclidean proposition which states that the areas of similar polygons are proportional to the squares of the ratios of corresponding sides. In the system of the time this would have been an "A exercise" (or 'easy rider'?) as opposed to the more challenging "C exercises" since it only requires the student to sketch something like Figure 2.1 below, and to realize that triangles AEF and CDF are similar with corresponding sides AE and CD being in the ratio of 1:2. Hence, applying the recently examined 'proposition', the ratio of the smaller triangle to the larger one must be 1:4.

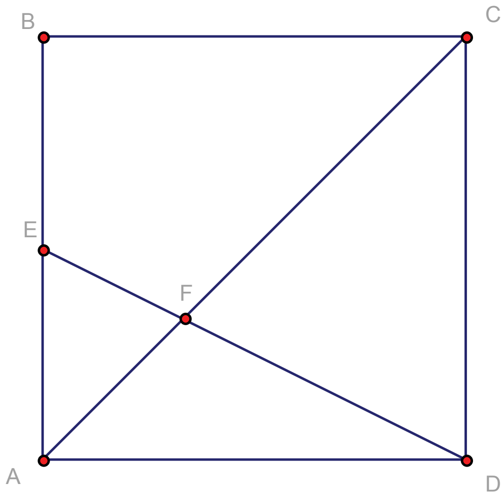

Figure 2.1 Square ABCD, midpoint E (of AB), and segments AC and DE intersecting at F.

Diligent learners in the second scenario will arrive at the isomorphic result but will likely do so without any appeal to classical geometric thinking. It would be likely that one of their teacher's higher priorities in including such an exercise in her or his lesson would be to give the students practice in using (and hence helping to learn) the often quite powerful digital tools at their disposal. Whatever the case may be, it certainly is not hard to see young minds exposed to a steady diet of these sorts of superficial and rather arbitrary exercises concluding that mathematics is tedious, mechanistic, disconnected from the world, and of significant interest to only a small-minority of people, many of whom would be seen as being socially inept. In short, the same type of views expressed by the subjects of the Nardi and Steward study. and a perspective consistent with the claims of Papert about underachievement.

3. Aspects of a 'Constructive Aesthetic' Reconceptualization: Conceptual Framework, Rich Tasks, and Powerful Ideas

If one aspires to avoid some of the pitfalls inherent in conventional approaches to mathematics instruction and to introduce curricular and pedagogic changes that might help learners to find mathematics classes both more effective and more enjoyable, how might one proceed and what roles might digital technologies play? This section begins with some general comments related to Papert's challenge to attempt reconceptualization and moves on to sketch briefly the foundations of one particular construction of this type, namely, a 'constructive aesthetic' one.

The field of mathematics education has paid considerable attention at a theoretical level to its scholarly underpinnings in terms of intellectual foundations. One thinks, for example, of the very extensive literature on the various types of "constructivism", Ernest's analysis (1991) of

Educational Ideologies, and Higginson's tetrahedral model of the field (1981), with Mathematical, Philosophical, Psychological and Sociological dimensions. There has been, however, very little reflection of this recognition of a plurality of possibilities in published curricula. The reasons for this are not entirely clear. In some contexts issues of politics or commerce may have played at least a partial role. Be that as it may, it would seem that one essential characteristic of any serious effort at reconceptualization would be a prominent portrayal of its academic assumptions and its intellectual heritage, namely the conceptual framework on which it is built. Its beliefs about the nature of the discipline, how it is constructed and learned, its role in human culture, the capabilities of learners and the purpose of the educational enterprise should not be buried. If this ambition were realized we might see the emergence of many different types of mathematical experience for learners somewhat akin to the broad range of musical styles. There are important similarities between and among the musical genres of folk, rock, classical and jazz, but there are significant differences too.

So, in a moment of high ambition, let me propose here a 'reconceptualization' that might aspire to being the mathematical equivalent of jazz. In other contexts (Sinclair et al, 2006) it has been called "maesthetics" to acknowledge its influence from both mathematics and aesthetics, but here we will use the related term, "constructive aesthetic" (first used in Upitis et al, 1997). This identifies an approach to mathematics that stresses its roots in fundamental human action and physicality, and in some species-specific predispositions to sensitivities to aesthetic characteristics of human experience such as symmetry, pattern and balance. It is a vision of mathematics and its teaching that gives primacy of place to play, connections, the physical and the artistic. It acknowledges its frequent origins in the spatial, the geometric, the visceral and the rhythmic, which often evolve into the numerical, the logical and the symbolic. Understanding a few fundamental ideas in depth is seen as being preferable to having a more superficial grasp of a longer list of "topics". It is profoundly anti-elitist in the sense that all human beings are seen as having considerable inherent potential in this approach. Powerful mathematical ideas are seen as being close to the surface of much 'natural' human experience. The identification, extraction, exploration and comprehension of these powerful ideas together with an understanding of their role in human societies and cultures are seen as the major aim of mathematics education. One particularly important method for attempting this is through the use of "rich learning tasks" (Flewelling & Higginson, 2003). These are situations that are latent with potential for the emergence of significant curricular concepts. Central to their development and use are teachers who are both highly knowledgeable from a academic perspective and skilled in working with both individuals and groups.

Significant intellectual influences include the American anthropologist Ellen Dissanayake with her concept of "Homo Aestheticus" (1995) and the Anglo-American philosopher/mathematician/ educator, Alfred North Whitehead, especially with his ideas on the pedagogic importance of the rhythmic cycle of: romance, precision and generalization (1967). [One very powerful criticism of the majority of contemporary mathematics classrooms, for instance, is that, from this Whiteheadian perspective they are almost totally bereft of "romance"]. Educational thinkers whose ideas have been influential in the development of this perspective include John Dewey, David Hawkins (1974), Seymour Papert (1980) and David Wheeler.

4. A 'Constructive-Aesthetic' Approach to a Rich Learning Task

In this section we attempt to outline the ways in which a constructive-aesthetic approach might play itself out with respect to a particular rich learning task. Partly in the interests of inviting reader participation (very much in the spirit of the approach), but also because of space constraints, the descriptions are brief, perhaps even verging on cryptic in places. We will not, for example, be able to provide diagrams for the variations suggested. Essential assumptions include the existence of a well-resourced classroom (in some ways, perhaps, more like an artist's atelier, or a scientist's laboratory) and a mathematically sophisticated and pedagogically skilled teacher who understands and believes in the suggested approach. The ages of the students could be quite diverse and it would be consistent with the approach to carry it out in a multi-age setting with the teacher as an active participant (in addition to her/his 'structuring' responsibilities). The extent to which some the potential of this task might be realized with any given group would be a function of many variables including, in particular, as noted above, the ability of the teacher, and the capabilities, experience and motivation of the learners.

The Task: Perhaps somewhat surprisingly, we will take as our 'exemplary' rich learning task the same mathematical structure we have already commented on at some length, i.e., the square with intersecting line segments. The 'trick' is, of course, that the pedagogic underpinnings and expectations are quite different. One reason for doing this is to underline the important point that mathematical richness is, to a great extent, in the eye of the beholder. There are some spiritual parallels here as well as some culinary ones. The enlightened can perceive beauty wherever they cast their eye and good chefs can generate succulent treats that generate only mush from lesser talents. There are distinct parallels with mathematics teaching. To the basic geometric description of the task (as above) we might then see, something of the order of (a 'rider' of sorts), the exhortation:

"Explore - Investigate - Extend".

What follows is a set of eight brief descriptions of different 'interpretations' or directions that groups of students might decide to follow in response to the very general instructions noted above. The initial emphasis in these descriptions is on the suprisingly extensive range of powerful mathematical ideas/curriculum concepts that can be generated by/connected to this deceptively simple initial structure. We conclude the section with some comments about the roles different types of technology might play in the evolution of this sort of activity.

Interpretation I: Tiling. One way to begin an elementary investigation of some of the mathematical properties inherent in this relatively simple structure is to create large-scale copies of the figure together with multiple copies of the smallest triangle (from an aesthetic perspective it might be desirable to have triangles of different colours). Then, in a parallel with jig-saw puzzling, the students could explore the ways in which copies of the smallest triangle might be placed in other places on the figure. One, especially 'elegant' result emerges, of course, from the fact that 4 of the small triangles, suitably manipulated, can exactly 'cover' the largest triangle. It might be noticed that midpoints play a particularly significant role in this solution. One of the smallest triangles can be nested, 'legally' in the mid-sized triangle leaving a second triangular region which is quite different in shape. Should students be able to make progess in these directions it would seem reasonable to claim that they have some of the background necessary to move toward some of the features of transformation geometry (especially notions of rotations, translations and reflections) and some aspects of classical geometry concerning figures with congruent bases and equal altitudes.

Interpretation II: Enumeration. There are some good classical 'measurement' possibilites here, but, perhaps especially for younger learners, the avenue of 'enumeration' is, perhaps, even more inviting. Geoboards or 'spotty paper' would be very useful in this regard, with sheets of 24 x 24 'spotty paper' being, for many, the tool of choice. In the 12 x 12 case, for instance, a 'subdivision' of the bottom-left quadrant into 3 x 3 subsquares opens up some arithmetic challenges (as well as insights that may be useful in some more advanced interpretations - see below). This experience would also be quite invaluable as a foundation for the development of ideas like "Pick's Theorem".

Interpretation III: The Total Square. A natural step from the consideration of aspects of the two 'similar' triangles is to explore the relative scales of the four polygonal areas created by the two intersecting line segments, ie the three triangular regions and the one quadrangular region. Working from the case where the students know that the ratio of the two similar triangles is 1:4 there are some very good opportunities to introduce concepts of elementary algebra here. If the area of the square as a whole is 'z' square units we can label the areas of the two similar triangles (in the appropriate square units) as 'x' and '4x' respectively. Let the area of the 'third' triangle be 'y' square units. Using fairly obvious features of the construction it can be seen that $4x + y = \frac{1}{2} z$ and that $x + y = \frac{1}{4} z$. Hence the ratio of the areas of the four polygons, from smallest to largest, is: 1, 2, 4, 5.

Interpretation IV: Special Points (Special Fractions). There is a lovely cognitive leap involved in seeing that our basic figure is, in an important mathematical sense, the second of a sequence of diagrams with similar, and very interesting mathematical properties. The first figure in this sequence is a square with both diagonals constructed. Looking at this figure it is easy to see that the four polygonal areas in this situation (congruent right-angled triangles) have scale ratios of: 1, 1, 1, 1. More interestingly it can be seen that the first figure provides the information necessary to generate the second, namely by dropping a perpendicular from the point of intersection to the appropriate side to locate the midpoint (1/2). This is the point that we used to construct our first figure and its corresponding polygons and areas. In the same 'successor function' fashion we can find the point of trisection of the side of the square (1/3). And so on, *ad infinitum*. This seems a powerful and natural way to introduce mathematical concepts, like the harmonic sequence, that often are not encountered until much more advanced settings.

Interpretation V: Sequences (Ordered Quadruples). Closely related to the sequence described aboved we have a set of ordered quadruples representing the relative areas of the four polygonal areas generated by this set of discrete cases. The first three of these quadruples are:

[1/4, 1/4, 1/4, 1/4], [1/12, 2/12, 4/12, 5/12], and, [1/24, 3/24, 9/24, 11/24] . The 'pattern exploration' and extension possibilities here are significant and non-trivial. This interpretation invites the use of geometry software at this stage to generate an implementation of this situation by having a point capable of moving from one end of the appropriate side to the other (simultaneously exhibiting the relative - in this case represented in decimal form - areas of the four regions in quadruple form) act as the end vertex of the 'non-diagonal' segment. There are some interesting potential 'extremal' and 'continuity' questions here.

Interpretation VI: Application - Thirding. There is a real-world community that has been familiar with some of the properties of our starting point for many years. On occasion they 'apply' its properties to assist them in their constructions. The individuals in question are paper-folders. When doing 'classical' folding they always start with a square piece of paper and then proceed to construct some model based on a sequence of folds. On occasion they need to subdivide their initial square into a set of smaller, congruent squares. It is trivial to generate 4 or 16 smaller sub-squares from a given large square. To generate 9 or 25 is not quite so simple. [In the case of 9, for instance, it is frowned on in purist circles - even if it is mathematically quite pretty - to solve the problem fby tearing off (gnomically) the top and right-hand strips!]. But for 9, our point of intersection, with its strong 'thirding' character, comes to the rescue. Folding top and right-hand edges to it (while keeping the sides parallel) gets one well started. Generalizing is a fascinating challenge.

Interpretation VII: Application - Centre of Mass (Balance). There is no more fundamental 'application' of mathematical and physical theory than that of balance (operating more or less unconsciously, 24/7, for most fortunate humans. Our fundamental point of intersection has an interesting connection to this concept, for in addition to all of its other properties pointed to above, it is also the 'balance point' [centre of gravity, centre of mass …. choose your context for the 'lower half' of the original square [referring to Figure 2.1, that would be triangle ABD]. And, of course, the 'matching' point for the 'upper half' triangle (CBD) is located at the symmetric point along the diagonal. Concatenating/averaging the locations of the two, will, of course, locate the centre of mass for the original square, located, not surprisingly, at the midpoint of the segment between them which is, not coincidentally identical to the point of intersection of the two diagonals.

Interpretation VIII: Big Squares have Little Squares (Fractals). And finally, and perhaps somewhat surprisingly (except, perhaps for those who paid special attention to the enumeration interpretation earlier), there is, buried in the deceptive simplicity of this rich learning task a rather beautiful fractal pattern. To find it, look first of all at the square in the bottom left quadrant of the figure and then at the top-right 'quadrant' of that square (and so on ….) at every stage we have a 'copy' of the original figure at another scale. Looming in the not-too-far distance here is the powerful concept of 'attractor'. Constructively aesthetic indeed.

There may well be more nuggets tucked away in this lode (one thinks immediately of a directly Cartesian approach) but the material outlined above should suffice to illustrate the existence and attraction of rich learning tasks and their potential for bringing some mathematical authenticity, life and depth to classrooms if this approach were imaginatively and energetically implemented.

We end this section with some thoughts on the potential roles of various technologies in a constructive-aesthetic/rich task approach. We have noted above that the general educational environment where this approach might be implemented would be a 'resource rich' one. One major indicator of the extent to which this is a reality would be to itemize the 'materials' students might be able to have access to with minimal effort. Ideally this would run the gamut from classical 'tools' such as paper, pencil and compasses to well-equipped and connected ICT tools. One interesting study would be to look at different teachers and classrooms using the same tasks with different mixtures of technologies. At the extremes for the rich task exemplified above there are two possibilities for the use of digital technologies. At the one end we could have none, or almost no, use of computers and related devices. At the other we could have all of the interpretations outlined above brought to learners in the form of pre-programmed 'microworlds'. There may well be educational strengths associated with both ends of this spectrum. The microworld version, for example, would lend it self much better than the non-ICT version to distance learning.

Based on the informal testing of this approach in a few settings (see Upitis et al, 1997, for example) it would seem that what is wanted is a balanced approach with extensive but appropriate (noting that that may well not be a simple criterion to define) use of both classical/concrete and digital technologies. Both 'sides' of the term 'constructive aesthetic' have strong connections to the physical or concrete. The etymological roots of the term aesthetic are, for example, close to the idea of 'perception by the senses' [as in the medical use of 'anaesthetic']. It would seem necessary, therefore, for any proponent of this approach to have a strong physical/manipulative component (as, for instance, with the folding and balancing interpretations above) in their planned activities/resouces. This is not to say that there is any bias against digital technologies in a constructive-aesthetic approach but only that their use may well emerge later in the overall evolution of curriculum unit than they would in approaches based on differenct sets of conceptual frameworks. It is perhaps worth noting that this view is, perhaps contrary to popular conceptions of his ideas, consistent with the formative version of Logo and related Papertian ideas. Recall the early work (captured under the term 'syntonic' learning) with young children imagining themselves to be 'turtles' and physically tracing out paths related to certain instructions.

Two other important roles for digital technologies in this approach are as highly effective tools for two central educational activities. The first of these is as a tool for exploration. Contemporary software packages and materials for geometry are unprecedented and unequalled as tools for bringing, particularly young, learners to an appreciation for the exploratory dimension of mathematics. The second essential activity is communication and again digital tools offer exceptionally fluent and powerful avenues for development in this area.

5. Conclusions/Summary

This paper has been created as a preliminary attempt to respond to Semyour Papert's challenge to participants at the 17th ICMI Study Conference in 2006 to generate alternatives to existing ways of bringing mathematics to learners. Starting from a position of agreement with the position that this needs to be done, it has argued that any such reconceptualization needs to be explicit about the conceptual framework on which it is built. It then outlined one such alternative, a constructive aesthetic approach with a brief description of its intellectual underpinnings. This approach gives primacy of place to an analysis of basic human actions in natural settings and to a set of educational experiences based on a set of carefully chosen rich learning tasks. An example of such a task was given together with a set of proposed responses that pointed in the direction of a number of central and powerful mathematical ideas. It was suggested that digital technologies could play an important role in this 'reconceptualized' approach but that they might be somewhat more 'delayed' in their implementation than in other approaches. In concluding this contribution two points need to be made. The first is, as noted earlier, that this response is quite preliminary. The second is that Papert's plea is probably much more important than it might appear at first glance. The multiple challenges of the contemporary 'global problematique' are connected to an extent not frequently acknowledged in our field to the limited conception of our discipline that has been promulgated from the earliest days of public education. Consider, for example, the devastating effects of the exclusive emphasis on the (ultimately arithmetic) concept of "the bottom line" in the dominant corporate view of values. Professor Papert may have been uncharacteristically conservative in suggesting that we need to spend something of the order of 10% of our time in the search for alternatives.

Endnotes

(1) The proceedings of the Hanoi meeting have (as of the mid-2008 drafting of this paper) yet to appear. It is planned that they will constitute Number 13 in the New ICMI Study Series to be published by Springer in 2009. This quotation comes from the notes of a participant at the Hanoi meeting. It appeared in the preliminary announcement for Working Group 'D' ("Communication and Mathematical Technology Use throughout the Postsecondary Curriculum") at the 2008 Meeting of the Canadian Mathematics Education Study Group [see http://publish.edu.uwo.ca/cmesg/pdf/Program2008_english.pdf].

(2) The keynote speaker in question was Seymour Papert of MIT, undoubtedly the best known and influential (1980) of 'progressive' computer educators. His involvement in a very serious traffic accident in Hanoi shortly after delivering this address has saddened his many friends (see http://www.boston.com/lifestyle/articles/2008/07/12/in_search_of_a_beautiful_mind/?page=full) from around the world.

(3) These examples are, in fact, variations of a problem the author first encountered in Barbeau's (1995) fine recreational mathematics collection. As becomes obvious later in the paper, the selection if these 'two' questions is far from arbitrary, and attempts to underline the importance of teacher attitudes and conceptual frameworks. I would contend, however, that questions almost identical to these could easily be found in mathematics texts.

Dedication This paper is dedicated with respect and admiration to three scholars, whose knowledge, compassion, commitment, imagination and wisdom have indelibly shaped, for the better, the worlds of mathematics, science and technology education: David Hawkins (1913 - 2002), University of Colorado; Seymour Papert (1928 -), Massachusetts Institute of Technology; and David Wheeler (1925 - 2000), Concordia University.

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