Critical Thinking Skills in Online Mathematics Discussion Forums and Mathematical Achievement

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Abstract: *Critical thinking in mathematics problem solving sessions is the focus of the paper. This preliminary research study set out to examine the problem solving sessions activated through online Discussion Forums in Mathematics classes in a first year university course supported by the Blackboard Learning System. The study involved a group of 46 participants and has the following purposes: (a) to adapt a model to evaluate critical thinking, at individual level in mathematical problem solving sessions of online Discussion Forums; (b) to examine the relationship between mathematical achievement, as measured by the final examination grades and critical thinking in online Discussion Forums incorporated into a university mathematics course; (c) to check whether there has been a progression of critical thinking skills based on the discussion forum postings from forum 1 (Week 3) to forum 2 (Week 11) of the 14 week-long course. The analysis based on the model showed an overall increase in the total number of messages in forum 2 over forum 1. But lower phase of critical thinking was seen dominant and a slight dependence between mathematical achievement and student's communication variables was observed.*

1. Introduction

Mathematics can be described as a combination of calculation skill and competence in mathematical reasoning, but neither of these alone characterizes mathematics (Hannula, Maijala & Pehkonen, 2004). Mathematical knowledge answers the question 'What', and one may remember mathematical facts. Mathematical skill answers the question 'How'; which includes, for example, the traditional calculation skill (procedural knowledge). Only mathematical understanding answers the 'Why' question; it allows one to reason about mathematical statements. The influence of critical thinking skills or metacognition on mathematical problem solving has attracted research from Ennis (2005) and Schoenfeld (2005). In contrast to the traditional text book dominated approach, the mathematics classrooms of the present are encouraged to be a place where discussion and collaboration are valued in building a climate of intellectual challenge. The primary goal of mathematics teaching being the development of the ability to solve complex mathematics problems, mathematical instruction should emphasize the process rather than the product (Kosiak, 2004). According to NCTM 2000 (p. 60), "…Communication is a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. The communication process also helps build meaning and the permanence for

ideas and makes them public." Such reform oriented classrooms are described as communities of mathematical inquiry-where students learn to speak and act mathematically by participation in mathematical discussion and solving new or unfamiliar problems (Schoenfeld, 2007). In short, sociocultural perspectives on learning have caused a reform in mathematics education. This is in line with the Vygotskian school of thought, which claims that human thinking is inherently social in its origins (Vygotsky, 2007). Association of American Colleges and Universities (AACU) (2005) calls for institutions of higher learning to produce graduates who think critically, communicate effectively and who employ lifelong learning skills. Web based technology has empowered mathematics teachers, learning content developers, as well as dynamic mathematics computation andeducation service providers, to deliver an unprecedented mathematics learning environment to students and educators. (Wang et al, 2004). "Technology is essential in teaching and learning mathematics ; it influences the mathematics that is taught and enhances students' learning (NCTM, 2000, p11). The authors hold the belief that technology could be used to develop a richer, interactive environment that encourages higher order thinking and motivation in mathematics, which forms the basis of this paper. This paper has been organized as follows: Section 2 is the literature review, Section 3 is the research study and Section 4 is the conclusion.

2. Literature Review

Literature review was done in the following areas as listed below. They are to do with critical thinking in relation to online discussion forums, along with the measurement of it and student's mathematical achievement.

2.1 Critical Thinking and Online Discussion Forums

There is a general consensus in literature that critical thinking is an expected outcome of all college (university) graduates (Oliver, 2001; Perkins & Murphy, 2006). Social constructivists believe that collaborative work can help problem solving or learning performance (Liu & Tsai, 2008). There are few studies which focus on critical thinking (Perkins & Murphy, 2006) and the aspects of critical thinking in online asynchronous discussions (Angeli, Valanides & Bonk, 2003; Khine, Yeap & Lok, 2003). With appropriate course design and instructional interventions, critical thinking skills can be cultivated and maintained in asynchronous discussion forums (Yang, Newby & Bill, 2005). The asynchronous discussion forum affords students the time for thoughtful analysis, composition, negotiation and reflection as their discussion of an issue evolves and allows instructors to model foster and evaluate the critical thinking skills exhibited during the discussion. Though difficult to foster, teaching and learning critical thinking is worth the effort according to research in the area (Yang, Newby & Bill, 2005; Perkins & Murphy, 2006).

Mathematics teaching has always exalted its goal of aiming to develop the ability to solve a variety of complex mathematics problems (NCTM, 2000 in Kosiak, 2004). Mathematical instruction thus should ideally emphasize the process (how to solve) rather than the product (getting the final answer). Problem solving has been operationally defined as a 'process' by which students apply previously acquired skills and knowledge to new and unfamiliar situations (Branca, 1980; Kosiak, 2004; Krulik & Rudnick,1989; NCTM, 2000). Problem solving engages higher order skills and is believed to be among the most authentic, relevant, and important skills that learners can develop. Thus, critical thinking is often associated with problem solving. Critical thinking is a form

of problem solving, but a major difference between the two is that critical thinking involves reasoning about open ended or "ill structured" problems, while problem solving is usually considered narrow in scope (Kurfiss, 1991; Tapper, 2004). Pellegrino (2003) discussed critical thinking involved in solving complex problems as: representing the problem in context, formulate sub problems, testing, presenting, justifying the solutions chosen. But much more than analyzing arguments, critical thinking is a larger process which includes not only discovery (the intuitive and creative processes), but also justification (the evaluative and logical-reasoning processes).

2.2 The Measurement of Critical Thinking

The past decade has seen popular methodologies being used in computer mediated communication – survey research, case studies, and content analysis of the online transcripts. The content or interaction analyses allow researchers to investigate the nature and quality of the online communication during small group learning. Researchers have tried to answer questions on the quality of the online course, whether knowledge is constructed within the groups by means of online exchanges, whether individual achievement in problem solving is related to the online communication phase level of the individual. The Community of Inquiry model (see Archer, Garrison, Anderson & Rourke, 2001; Garrison, Anderson & Archer, 2001) focus on "critical thinking within a group dynamic as reflected by the perspective of a community of enquiry" (Garrison, Anderson & Archer, 2001, p. 11). This focus on the group dynamic is pertinent when the goal is to examine evidence of critical thinking in the online community as a whole; however, this approach would not be relevant in cases where the focus is on the individual member of the online community.Perkins and Murphy (2006) have attempted to provide a model of critical thinking that could be used efficiently and easily to derive and present individual profiles of engagement in critical thinking on online transcripts. Their preliminary study demonstrated the potential usefulness and importance of identifying critical thinking for individuals in online asynchronous discussion forums.

Table 1. Summary Of Critical Thinking Models

Authors	**Steps Proposed**				
	Step 1	**Step 2**	**Step 3**	**Step 4**	**Step 5**
Norris & Ennis (1989)	Elementary clarification	Basic support	Inference	Advanced clarification	Strategies and tactics
Henri (1992) Clulow & Brace-Govan (2001)	Elementary clarification	In-depth clarification	Inference	Judgment	Strategies
Newman, Webb & Cochrane (1995)	Clarification	In-depth clarification	Inference	Judgment	Strategy formation
Bullen (1997)	Clarification	Assessing evidence	Making, judging inferences	Making appropriate strategies, tactics	-
Garrison, Anderson & Archer (2001)	Triggering events	Exploration	Provisional	Resolution	-
Perkins & Murphy (2006)	Clarification	Assessment	Inference	Strategies	-

Table 1 shows a summary of various models available in the literature, to measure critical thinking in online asynchronous discussion forums. Most of the models include five steps: elementary

clarification, elementary/advanced clarification, inference, judgment and strategies/tactics. The same basic processes have been combined by the different proponents to suit their analysis.

2.3 Mathematical Achievement and Critical Thinking

The relationship between the quality of online communication and mathematical achievement has been the subject of many studies. Beaudrie (2000) has examined the relationship between the amount and level of communication and achievement of students enrolled both on- and off-campus in an upper-level geometry course. Kosiak (2004) examined the relationship between mathematical achievement in a college algebra course, as measured by a procedural final examination, and the quality of online mathematical communication of the students as measured by the Interaction Analysis Model. Kosiak's study has been monumental in designing the methodological frame work for this study.

3. The Research Study

Perkins and Murphy (2006) have attempted to provide a model of critical thinking that could be used efficiently and easily to derive and present individual profiles of engagement in critical thinking. Their preliminary study demonstrated the potential usefulness and importance of identifying critical thinking in online asynchronous discussion forums.

3.1 Methodology

This research study set out to examine the problem solving sessions activated through online Discussion Forums in Mathematics classes in a first year university course supported by the online Learning Management System (LMS) called the Blackboard (http://www.blackboard.com). The study involved a group of 46 participants. Two forums were activated - Week 3 (Forum1) and Week 11 (Forum 2) of a 14 week university course. This study was in many ways inspired by the work of Perkins and Murphy (2006), and has the following purposes: (a) to check on the nature and quality of the critical thinking at individual level , as measured by an adapted model (Jacob & Sam, 2007) to evaluate critical thinking, in mathematical problem solving sessions of online Discussion Forums; (b) to examine the relationship between mathematical achievement, as measured by the final examination grades and critical thinking in online Discussion Forums incorporated into a university mathematics course; (c) to check whether there has been a progression of critical thinking skills based on the discussion forum postings from forum 1 to forum 2.

The problem solving sessions were activated through the posting of an ill structured problem by the lecturer on the Discussion Forum in Week 3 and Week 11 of the 14-week long course. The course was a compulsory first semester mathematics course for Engineering students in a university in Malaysia. The Discussion Forum sessions formed part of the internal assessment for these students, and worth 10% marks. The problem was taken from a standard textbook of mathematics applications in Engineering (James, 2004). The sessions were designed, adapting from Chang et al (2008), with the following principles in mind: (a) promote active participation designed to support peers by encouraging everyone to achieve a common goal; (b) Assess learners' abilities and give them proper learning opportunities according to their abilities; (c) promote critical thinking in the context of the solving of the problem. The problem was given a deadline of one week to solve. The lecturer minimally moderated the forum sessions through posing questions, commenting on the

direction of the arguments etc. The model for content analysis of the postings in the current study was adapted from two models- the model proposed and tested by Perkins and Murphy (Perkins and Murphy, 2006), and the framework for assessing critical thinking developed by Paul and Elder (2006). The reason for adapting Perkins and Murphy's was because Perkins and Murphy developed their model to measure individual engagement in critical thinking in online discussions, as opposed to Garrison's Community of Inquiry model which was aimed to assess groups (Garrison, Anderson & Archer, 2001). They had drawn on the earlier models of the day to create one that can be easily used to support the coding of transcripts in online discussions (Perkins and Murphy, 2006). Most of the models in literature were to measure critical thinking on a group basis. Paul and Elder (2006) have detailed on the aspects of critical thinking and a critical thinker, which has been taken into consideration in developing the model. The new model is shown in Table 1, with the indicators and description to each phase/category. The phases are arranged in the order of higher critical thinking stages. The phases of Inference and Strategies in the model represent the highest in the hierarchy of the phases of critical thinking.

One message (forum posting) was considered as the unit of analysis. The messages were coded into the four phases/categories, using the indicators in the model as guides. In cases where more than one critical thinking process appeared within a message, only one code was associated, which seemed to be the most important in the context.

Table 2. Model for Identifying Engagement in Critical Thinking during Problem Solving in online discussion forums

Phase 1-Clarification			
Formulates the problem precisely and clearly.			
Analyses, negotiates or discusses the scope of the problem	Identifies one or more underlying assumptions in the parts of the problem	Identifies relationships among the different parts of the problem	Defines or criticizes the definition of relevant terms
Phase 2-Assessment			
Raises vital questions and problems within the problem.			
Gathers and assesses relevant information.	Provides or asks for reasons that proffered evidence is valid or relevant.	Make value judgment on the assessment criteria or argument or situation.	
Phase 3-Inference			
Reasons out based on relevant criteria and standards			
Makes appropriate deductions from discussed results.	Arrives at well thought out conclusions	Makes generalizations from relevant results.	Frames relationships among the different parts of the problem.
Phase 4-Strategies			
Thinks and suggests open mindedly within alternative systems of thought.			
Propose specific steps to lead to the solution.	Discuss possible steps.	Evaluate possible steps.	Predicts outcomes of proposed steps.

The average phase level of each individual is calculated to measure the nature and quality of the critical thinking at individual level, as measured by the adapted model. The phase level of an individual was measured in three ways (Kosiak, 2004): (1) the average communication phase level calculated by dividing the total amount of coded messages of the individual by the total number of messages sent; (2) the number and percentage of high level messages coded in phase 3 and 4 of the adapted model; and (3) the total number of messages sent.

Only quantitative analysis was possible to measure the process of critical thinking and no qualititative analysis attempted, given the short time for the study which finished towards the end of 14 weeks (when the course was over).

Mathematical achievement was measured through the scores received by the students for the final examination of the 14-week long mathematics course. A second focus of this research study was to examine the relationship between the quality of an individual's communication variables (as measured by the Discussion Forums) and mathematical achievement, or the individual's procedural skills in mathematics. To determine if there were significant relationships between the two, a Pearson's Product Moment Correlation Coefficient was used.

3.2 Results of the Research Study

Research Question 1: To check on the nature and quality of the critical thinking at individual level, as measured by an adapted model to evaluate critical thinking, in mathematical problem solving sessions of online Discussion Forums.

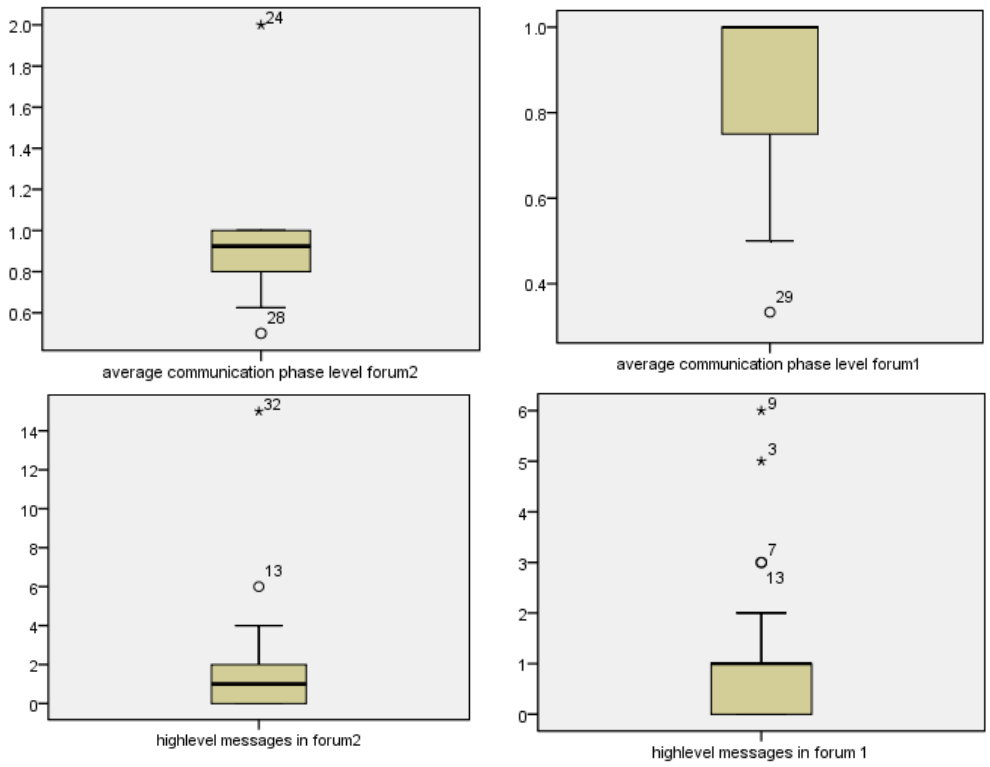

Figure 3.1 Box plots showing the average communication phase level, number of high level messages in forum 1 and forum 2

The three calculations were made for each of the 46 students: (1) average communication phase level calculated by dividing the total amount of coded messages of the individual by the total number of messages sent; (2) the number of high level messages coded in phase 3 and 4 of the adapted model; and (3) the total number of messages sent.

From Figure 3.1, the average communication phase level of forum 2 varies between 0.6 – 2 and with a median of about 0.9. For forum 1, the same varies between 0.4-1 and is skewed to the left. The high level messages in forum 2 varies from 0-17 per student, with a mean of 2; in forum 1, the same varies from 0-6 per student, but very scattered. The average communication phase level has clearly increased from forum 1 to forum 2. The high level messages seem to have increased in forum 2 comparatively, but the variation was contributed to by two students (marked as 13 and 32 on the box plot).

Table 3 shows the statistics of the messages in different phases as measured by the model. Some messages were not classified or ranked into any of the four phases since they did not show any evidence of critical thinking as shown by the indicators in our model. Hence the column titled "ranked1" and "ranked2" represent the ranked messages in forum 1 and forum 2 relevant to our analysis. The columns represent the respective statistics of total messages in forum 1, ranked messages in forum 1, clarification phase in forum 1, assessment phase in forum 1, inference phase in forum 1, strategies phase in forum 1, total messages in forum 2, ranked messages in forum 2, clarification phase in forum 2, assessment phase in forum 2, inference phase in forum 2 and strategies phase in forum 2. The mean number of ranked messages posted by an individual student is 7, but with a standard deviation of 6, of the 143 ranked messages in forum 2. In forum 1, the mean is only 4 and standard deviation of 3. The standard deviation shows the total number of messages is quite varied and not consistently distributed among the 46 students. The dominance in the number of messages is in the 'Clarification' and 'Assessment' categories. High level messages are very few in both forums.

Table 3. Table showing the Statistics of the Messages in Forum 1 and Forum 2

Statistics	Total1	Ranked1	C1	A1	I1	S1	Total2	Ranked2	C2	A2	I2	S2
No. of students	46	46	46	46	46	46	46	46	46	46	45	46
No. of messages	231	143	47	59	19	18	340	299	154	85	36	24
Mean	4	3	1	1	0	0	7	7	3	2	1	1
Median	3	2	1	1	0	0	6	5	3	1	0	0
Mode	1	1	1	1	0	0	4	4	3	1	0	0
Std. Deviation	3	3	1	2	1	1	6	6	2	2	2	1
Minimum	0	0	0	0	0	0	2	1	0	0	0	0
Maximum	18	18	8	9	4	2	40	37	11	13	14	3

Research Question 2: To examine the relationship between mathematical achievement, as measured by the final examination grades and critical thinking in online Discussion Forums incorporated into a university mathematics course.

The dependent variable for these hypotheses was an individual achievement variable (the final examination score). The individual achievement variable has a mean of 63.8, SD of 18.3, maximum of 93 and minimum of 11. The respective scores for the 46 students were not normally distributed. The independent variable was the individual's communication scores (total number of messages sent, the average number of high level messages). The ANOVA table reveals significant linear relationship at 10% significance level (p-value = 0.063 < 0.10, r square = 0.174) between an

individual's procedural skills as measured by the final examination and an individual's communication scores using the Pearson's Product Moment Correlation Coefficient. Or, the communication scores of students was a good predictor of their mathematical achievement, at 10 % significance level. The table 4 (SPSS output) of regression coefficients show that the average number of high level messages, and total number of messages in forum 2 individually do not contribute to the linear regression model . But the total number of messages in forum 1 does contribute individually to the model. Thus mathematical achievement, as measured by the final examination grades and critical thinking in online Discussion Forums was seen as linearly related, though not strongly (not significant at 5%).

Table 4. Table of regression coefficients

Model	**Unstandardized Coefficients**		**Standard Coefficients**		
	B	**Std. Error**	**Beta**	**t**	**Sig.**
(Constant)	58.264	4.641	—	12.553	0.000
Total no. of messages_forum1	1.681	0.685	0.383	2.453	0.019
Total no. of messages_forum2	-0.138	0.418	-0.049	-0.330	0.743
Average no. of high level messages	0.077	0.170	0.070	0.452	0.654

Research Question 3: To check whether there has been a progression of critical thinking skills based on the discussion forum postings from forum 1 to forum 2. The table 5 shows the percentage of ranked messages in Forum 1 and Forum 2. There has been an increase from 143 to 299 messages in forum 2 from forum 1. Both forums show dominance in the lower phases of Clarification and Assessment. There was noticed not much progression of critical thinking skills from forum 1 to forum 2, though there has been an increase in the total number of messages. It is important to be noted here that the problem in forum 1 and that in forum 2 were from two different topics of the course, which could be one of the reasons for the varied responses from students.

Table 5. Comparison of Messages in forum 1 and forum2

Phase	**Total Ranked Messages**		**Percentage**	
	Forum1	**Forum2**	**Forum1**	**Forum2**
Clarification	47	154	32%	52%
Assessment	59	85	41%	28%
Inference	19	36	13%	12%
Strategies	18	24	12%	8%
Total	143	299	100%	100%

4. Conclusions

The study was a good experience for the authors to examine the effects of problem solving sessions through online Discussion Forums. The students were first time users of the forums for

such purposes, hence the reason for less number of messages (postings) in forum 1. Forum 2 showed a boost in the total number of messages, compared to forum 1. No inter rater reliability was calculated concerning the classification of the messages into the four phases of the model, since only one instructor was involved in the study. But the majority of the messages were in the lower levels of critical thinking-clarification and assessment. The average communication phase level (calculated by dividing the total amount of coded messages of the individual by the total number of messages sent) was about 0.8-1 for most of the students. There was noticed an increase in the total number of messages , but not a marked improvement in the phases of critical thinking from forum 1, which happened in Week 5 to forum 2 in Week 11. There were some exceptional students who contributed to the high levels of critical thinking, as is seen form the outliers in the box plots in section 3.2. The study reveals similar results as in Kosiak (2004). The research was a pilot study into the possibilities of encouraging critical thinking among first year mathematics students. Further research is intended taking into consideration the majority of low level messages found in the two forums. The moderation strategies would be looked into and the discussion would be planned in heterogeneous groups of high, low and medium ability students (based on an initial assessment) for the future work. The authors are aware of the socio-cultural settings of the study and the particular group of students in the university. More research is needed to check on the validity and reliability of the study with another group in another university or region.

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