

Learning mathematics on the Internet

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Abstract: *The Internet offers new opportunities for school students to learn mathematics. This paper analyses these opportunities, based on presently available web sites that do not involve subscriptions by users. A typology of six different sorts of opportunities is described, justified and exemplified: (i) Interactive opportunities allow students to interact directly with mathematical objects; (ii) Reading interesting materials allows supplementation of available textual resources; (iii) Reference materials provide more extensive and accessible sources than usually available to students; (iv) Communication permits students to communicate with other students or with teachers about mathematics across existing barriers; (v) Problem solving opportunities are more extensive than normal classrooms can offer; and (vi) webquests provide structured investigational opportunities that specifically exploit Internet opportunities. The ways in which the Internet is accessed in practice are recognised to be of significance to the prospects for successful educational use of these resources, as are the pedagogies invoked by teachers.*

Introduction

The recent *Position Statement on Technology* [1] of the National Council of Teachers of Mathematics (NCTM) in the USA reaffirmed previous versions in noting that the availability of technology is of key importance to modern mathematics education, affecting both what is taught and learned as well as how it is taught and learned; the Internet is now one important component of available technology for education, offering new opportunities for students to learn mathematics and to learn about mathematics. Furthermore, relatively little Information and Communications Technology (ICT) expertise is now needed by either students or teachers to access relevant websites. Unlike earlier forms of technology, the basic expertise of using a web browser is sufficient to take advantage of the Internet; and is increasingly becoming familiar to most modern students.

The focus of this paper is on the kinds of learning opportunities that are freely available to relatively unsophisticated users, and in particular students and their teachers who are not familiar with developing Internet materials or websites. A major purpose of the paper is to clarify the opportunities now available, as well as to briefly suggest what characteristics of these opportunities appear to offer promise for mathematics education.

(The paper contains some references to websites, to illustrate the points made. Note that websites change addresses and even disappear altogether, so that some links in this paper may not work.)

Access issues

Although Internet access is becoming widely available in more industrialised countries in the Asian region, such as Australia, Japan and Taiwan it is still unevenly spread. For example, according to the most recent Australian census [2], home Internet access increased sharply since the previous census from 2001 to 2006, with home access rising from 35% to 63% over that period. While access is uneven (eg, in Australia two-thirds of homes in major cities have Internet access, compared to only 42% for very remote Australia. Broadband is used by 46% of homes in major cities and 24% in very remote Australia.), it is clear that it is rapidly improving in the region, as a function of growing wealth, declining costs and targeted government policies.

In recognition of the growing significance of technology for learning, and within societies generally, the newly elected Australian Government has begun implementing a policy of providing sufficient laptop computers to senior secondary school students and increasing school access to high-speed broadband from the middle of 2008. While this policy is a general one, not specifically concerned with mathematics, and while it is an Australian matter, such developments are symptomatic of increasing public recognition of the pervasive quality of the Internet in modern life, and the imperatives for it to be taken advantage of for educational purposes.

Access to the Internet in schools continues to be quite varied, with a range of possible mechanisms. Some schools rely on computer laboratories, with individual machines connected to the Internet via a network. In some cases, individual classrooms have access, at least for whole class use via a data projector, possibly also with an Interactive Whiteboard. Some classrooms contain more than one Internet-connected computer, allowing use by small groups of students. In some classrooms (very few in Australia) Internet access is routinely available to all students all the time (through the use of laptops for individuals and wireless broadband networks). These various patterns of use of course have substantial effect on the ways in which the Internet can be used for learning mathematics.

In many cases, access limitations can be reduced by the use of community facilities (such as local libraries, which in Australia often have Internet access for public use) or by offline use (such as when teachers download or purchase materials for class use. Once again, the mechanisms differ considerably both within and between countries in the Asian region.

There is continuing unease in some communities and some countries regarding uncontrolled Internet access by students, because of possible misuses (such as pornography or unwelcome political uses). Responses to such concerns are not the purpose of this paper, but it needs to be noted that schools and school systems have a variety of means of accommodating to such problems in order to increase the educational use of the Internet by students. In some cases, such practices may limit the accessibility of legitimate sites as well as those of concern.

Typology of potential uses

In this section, six different kinds of Internet uses are identified and exemplified. The emphasis is on ways in which students (as distinct from teachers) might use websites for mathematical learning purposes, although some of these can be used effectively with groups of students by a teacher. (Teacher uses may include other kinds of materials, such as online and electronic journals and resources for lessons.) There is not space in the paper to provide a large number of examples in any of the six categories. Instead, the reader is referred to the website [3] which contains links to many different examples, together with brief commentary on their particular significance. The examples have been carefully chosen to highlight good uses of the Internet (in the author's opinion) and to reduce the need for those interested in such materials (whether students, teachers, parents or others) to rely on browsing with search engines to find valuable resources.

Interactive opportunity

In recent years, the Internet has been used to allow students to interact directly with mathematical objects, in a variety of ways. The experiences provided are very difficult to provide in other ways and engage the students directly in mathematical activity and a corresponding need to think about what they see on the screen. Most of these require that the browser has particular capabilities, such as being Java-enabled or having a plug-in for Flash software.

Virtual manipulatives provide an opportunity for students to interact directly with (virtual) mathematical objects. Manipulations with physical objects have long been considered productive for learning aspects of mathematics, leading to widespread use of materials such as MABs (multibase arithmetic blocks), pattern blocks and Cuisenaire rods. Virtual versions of these share many of the same properties and have the advantage of being in unlimited supply, in sharp contrast to ‘real’ manipulatives. The evidence for the use of these is promising [4], both for young children and for older students [5]. Figure 1 shows an example of a virtual manipulative, allowing students to explore the relationships between Platonic solids and their duals with a Java applet. Students can choose various solids and the ways in which they are represented, and see the relevant duals obtained by joining midpoints of faces. Solids and duals can be rotated. It is usually very difficult to provide activity of this kind in a classroom.

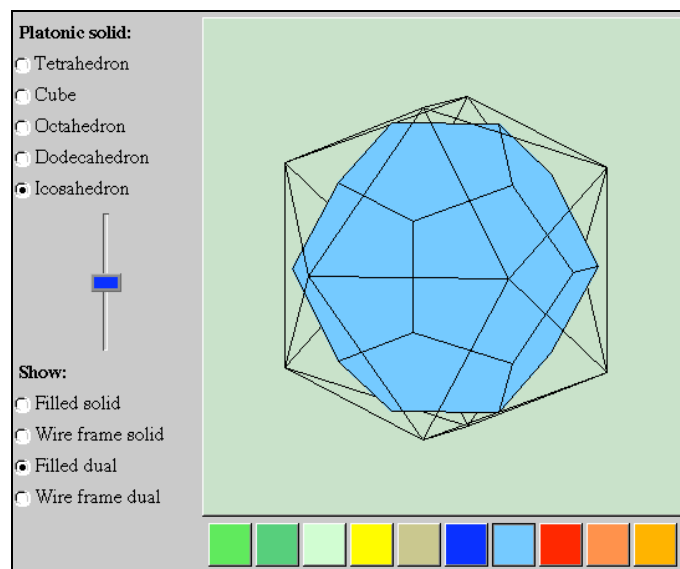


Figure 1: *Platonic Solids – Duals* from the National Library of Virtual Manipulatives [6].

Some websites offer significant advice to teachers as well as students. Advice for students is usually necessary in order to direct student interaction, although advice for teachers is also important, to help teachers see the intentions of the website designers. In some cases, such as the *Illuminations* website [7], individual lesson plans are provided for teachers as well as instructions for students. In other cases, such as the National Library for Virtual Manipulatives, detailed advice is also offered for parents, on the apparent assumption that the Internet is accessed at home as well as at school. Of course, the provision of good advice is no guarantee that it will be read, or followed, as [8] observed in a Taiwanese study into the ways in which some interactive websites were actually used by teachers and their students. Some interactive material provides sophisticated tools for mathematics, rather than individual lessons or activities. A good illustration is [9], which supports an innovative new curriculum exploiting such opportunities.

Interactive objects can be used with a whole class, as well as individually. Indeed, some are designed for this purpose, such as many of the Interactivities on the *Nrich* site in the UK [10]. Some interactive objects take the form of games, for which it is expected that more than one person is involved in the interactions.

Reading interesting materials

There are many interesting materials related to mathematics on the Internet, in sharp contrast to many school libraries. Many libraries restrict themselves to mathematics textbooks, which are often not very interesting to students, especially if they already have a textbook of their own. Of course, the nature of school libraries varies immensely across the Asian region, for a range of practical reasons, not the least of which is the cost of books and the availability of suitable materials for students of different ages and in appropriate languages. Another factor in the availability of reading materials in many countries is the ease with which they can be found: few mathematical magazines are available widely and even many bookshops hold limited reading materials of interest to students, such as works in popular mathematics. (The author's experience is limited to materials in English, which are probably more extensive at present than those in other languages, but hence of most significance to those with English language competence.)

As well as good written material, some Internet readings may have an interactive element, good illustrations, hyperlinks and so on. Some materials intended for the general public are suitable for students, especially older students, and there are also good materials written expressly for students. As well as being of direct relevance as resources for school projects, high quality readings may kindle interests in mathematics that would otherwise not be sparked by more conventional school experiences. In countries in which mathematical expertise is in short supply, materials that generate interest in mathematics amongst school students may be of critical importance to the future.

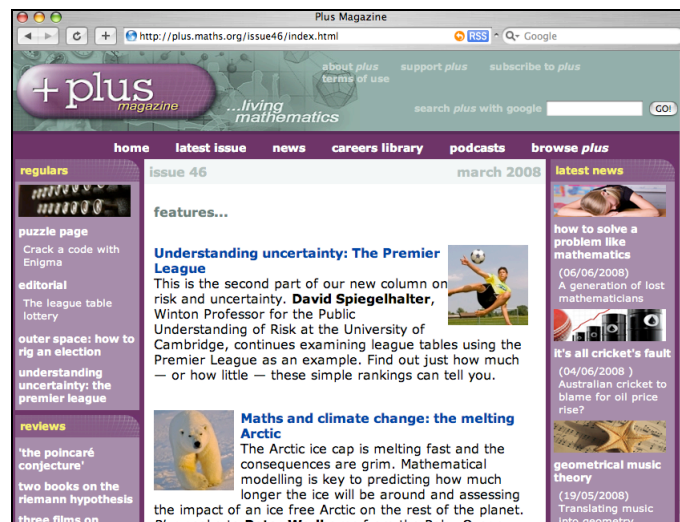


Figure 2: *Plus* magazine, Issue 46 [11].

Better readings on the Internet for school students are likely to be fairly short and liberally sprinkled with images or even interactive elements. Some take the form of regular magazines, such as the excellent companion websites *Plus* and *Nrich*, based at Cambridge University in the UK. Figure 2 shows a recent issue of *Plus* magazine, which provides regular and stimulating mathematical reading material for sophisticated secondary school students (as well as their teachers).

There is a range of materials available in all categories, to suit the needs of a range of students (and others). To illustrate this point, Figure 3 shows some columns from the Mathematical Association of America website, on which there are a variety of readings for an audience that might range from

school students through the general public to teachers and other mathematics professionals. These columns are published regularly and generally include access to previous issues.

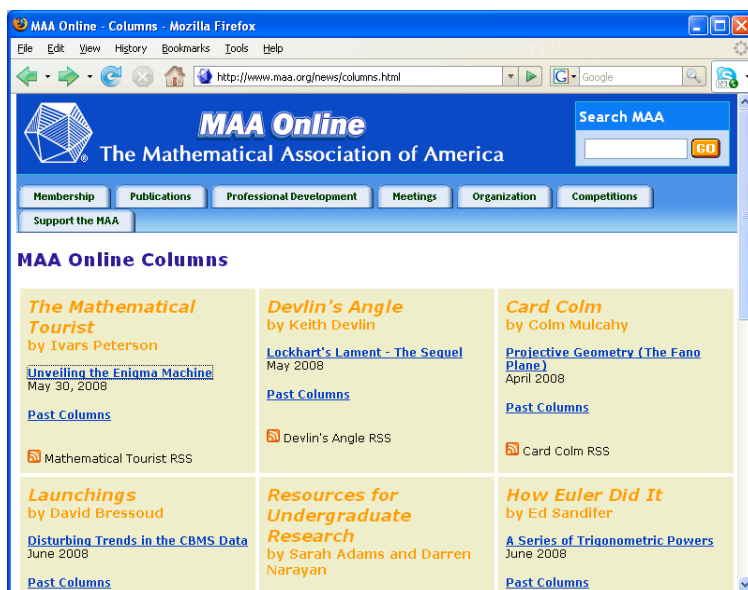


Figure 3: MAA Online columns [12].

Reference materials

The Internet has been likened to a massive encyclopaedia, and can be used as a means of looking up various kinds of mathematical information for various purposes. A few different examples of this species of web use are shown at [3]. These might be used by students directly from home, especially as few homes will have a mathematical reference source such as a dictionary or encyclopedia. They might also be used in school, by both individual students and teachers or by a whole class, seeking clarification or information of a reference kind.

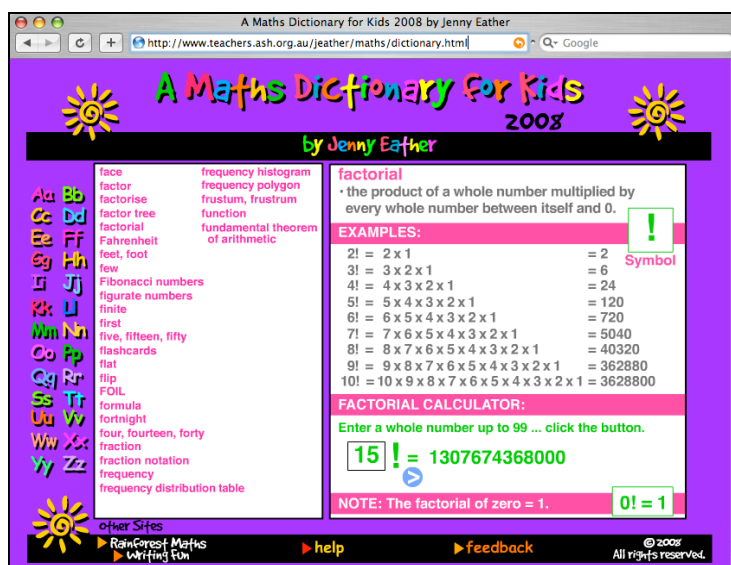


Figure 4: Mathematics dictionary [13].

While printed mathematics dictionaries are available for both young readers and sophisticated professional mathematicians, it is unusual for students to have routine access to these. Indeed, it continues to be unusual for school textbooks to routinely include a glossary or sometimes even an index to mathematical terms and concepts. For this reason, Internet mathematics dictionaries might well be more accessible to students than other kinds of mathematics dictionaries, and may be more helpful than standard dictionaries, because of the possibility of cross-linking of entries and even dynamic interactive definitions. Internet mathematics dictionaries in English are presently available at a range of levels across the school years. The example shown in Figure 4 is intended for young children [13].

Encyclopaedias provide more detailed and extensive information than dictionaries, which (at least in paper forms) focus on the meanings of particular terms. Encyclopaedias provide more than merely meanings, but offer support to readers to locate ideas in contexts, including historical, practical, theoretical and social contexts. On the Internet, there is a gradual blurring of the distinctions between dictionaries and encyclopaedias, with increasingly each of these having some of the characteristics of the other. For this reason, students using quality mathematical reference materials on the Internet will generally be offered more information and support for learning mathematics than they expected, or would be likely to obtain, by consulting paper-based sources.

Two encyclopaedic websites are worthy of special mention. The popular *Wikipedia* website [14] offers a window into mathematics that is helpful at least at the top level, providing a broad background to mathematical ideas and their significance, even though there might be concerns at deeper levels about a website that can be edited by users. The extraordinary *Mathworld* website [15] will satisfy the needs for detailed mathematical information of even very experienced mathematicians, in many cases much too sophisticated for school students. However, such a website locates mathematical work in the public domain in ways which have not previously been imagined, and may serve a wider purpose than those to do with school education.

History of mathematics is frequently neglected in school curricula, yet is relatively accessible to students on the Internet. An outstanding website is [16], which has won a number of awards for excellence. This website provides opportunities for students to trace the development of mathematical ideas into the twenty-first century, and to see the roles played by mathematicians and others over the centuries. Such a perspective is rarely provided by school mathematics curricula, which also suffer from the problem of appearing to be unaware of mathematics of modern times. Engagement by students in websites of this kind will provide a sense of perspective as well as a realisation that mathematics is both an ancient and modern discipline, of widespread significance across all historical epochs and cultures.

Communication

The Internet offers opportunities for students to communicate with other students or teachers, regardless of geographical location (but limited to those speaking the same language, most likely English, regrettably). For students, some opportunities to be part of a wider mathematics community are provided. An example of this aspect of Internet use is the Ask Dr. Math site [17], illustrated in Figure 5. Students can ask mathematical questions of the fictitious Dr. Math, who will provide answers to many of these posted onto the website. Previous questions (as well as their answers, often more than one answer) are archived on the site to avoid repetition of questions and answers.

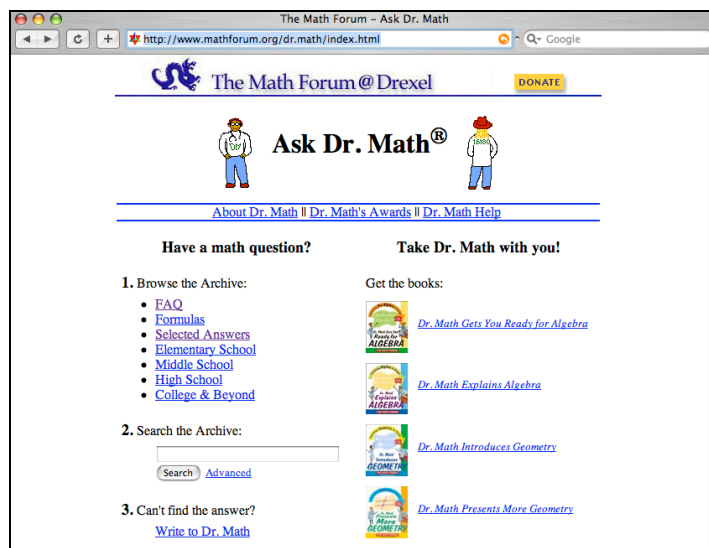


Figure 5: Ask Dr. Math home page [17].

The *Ask Nrich* site (<http://nrich.maths.org/cgi-bin/discus/discus.cgi>). provides opportunities for more direct communication between students and others, as part of the extensive *Nrich* site [10]. In addition to the communication opportunities explicitly provided on the site, there are regular responses from students all over the world to mathematical questions and problems, helping to reinforce the fact that mathematics is of international interest and is universally engaging.

Problem solving

While a great deal of ‘problem solving’ ought to happen in regular classrooms, the Internet can theoretically offer some extra benefits to students, including a regular supply of suitably targeted problems, opportunities to share solutions and even opportunities to get professional feedback on their work.

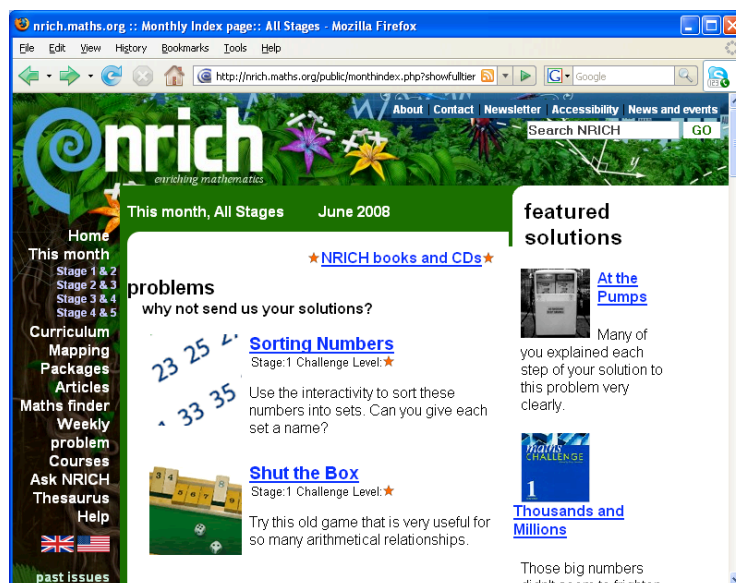


Figure 6: NRich problems for June 2008 [10].

There seems little argument for a set of problems on the Internet that could just as easily be written in a textbook, with nothing extra added. Figure 6 shows an example of how the Internet might offer increased opportunities for problem solving, with the very large *Nrich* site. The typical month shown includes a range of mathematical problems for school children at a range of levels of sophistication. Some of these problems include interactive elements, so they comprise more than merely descriptions of problems (as might be found in textual resources). As well as problems, the site provides hints, a printable version, some notes for students or teachers and opportunities to contribute solutions online. Importantly, the site also publishes solutions from students themselves to previous problems, highlighting the range of ways in which problems can be solved as well as the range of students around the world interested in such activity.

Webquests

Webquests are often defined as inquiry-oriented activity in which some or all of the information that learners interact with comes from resources on the Internet. In the case of mathematics, these usually take the form of a guided exploration of a topic, requiring students (or usually groups of students) to undertake some web-based research and to generate a product of some kind (such as a report).

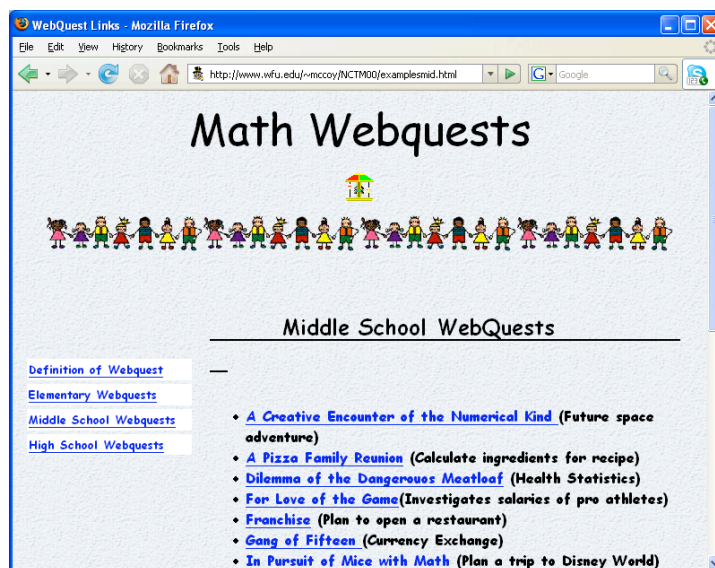


Figure 7: Examples of Webquests [18].

Webquests often have a particular structure, comprising an introduction, the task to be undertaken, appropriate student roles, suggested processes to follow, a formal evaluation rubric and a summarising conclusion. This structure provides systematic and structured advice for a group of students to undertake a webquest by themselves and present the results of their exploratory work to others. A substantial collection of webquests for students of various ages is provided by [19].

By their nature, webquests often have a local flavour, drawing on local resources, and hence are more problematic internationally than other kinds of Internet resources. For this reason, for example, webquests written in the USA for US students may sometimes be of diminished use for students in another country, such as Thailand, even when English language is not a problem, since they may draw on local prices, cultural conventions, social practices, and so on. However, teachers

may derive good ideas for their own community by looking at webquests designed by fellow teachers for other communities and using them as a basis for designing their own webquests for their students.

While some webquests focus on mathematics itself, the major contributions of webquests to student learning concerns the contexts in which mathematics is significant in the world and the opportunities for collaboration between students. Such contexts are necessarily local in flavour and it is also the case that curricula across different countries in the Asian region have different emphases regarding the place of the wider context and everyday applications generally in the mathematics classroom. Similarly, the extent to which collaborative activity among students is explicitly encouraged and valued in school mathematics also varies across the region.

Internet pedagogy

It is one thing to have access to good resources for learning, but another for these to be used effectively. While this paper has already exhausted the available space, a brief indication of some important issues is still needed. It seems likely that the Internet resources will be most effective if they are integrated into the classroom and the curriculum in some systematic way; this will require teachers to be aware of the possibilities and to refer their students to them appropriately. Some resources are designed to be used in particular ways, and advice for teachers (or parents) is offered about this on the website itself. While some resources can be well used as a whole class activity, others are better used with groups of students, while still others are likely to be effective with students working individually, either at school or outside school. Effective use of the Internet requires careful educational planning by teachers to ensure that students obtain the benefits expected [4]. The uses available to teachers are of course constrained by the available facilities as noted earlier; an extensive discussion of the ways in which teachers can use their available classroom resources effectively is offered by [20].

Conclusion

This paper has offered a typology of ways in which the Internet might be used by relatively unsophisticated users (from an ICT viewpoint) to support and augment the teaching and learning of mathematics. The six categories of uses are distinctly different and offer different potentials, which might be constrained to some extent by the different circumstances through which Internet access is available. The typology is exemplified by [3], which contains links to many examples, together with brief supporting advice. There is much to offer both students and their teachers already on the Internet, without subscription charges, and every prospect that the available offerings will be increased in the next few years, as developers make their work available to a global audience. The paper has clarified the range of ways in which the Internet might be used for learning mathematics and learning about mathematics.

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