A Case Study of Lessons Incorporating Mathematical Activities for Teacher Training Program

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Abstract:

In Japan's primary and secondary education, mathematical activities are utilized in numerous settings and are generally viewed positively. However, most university mathematics classes, especially in teacher training, remain lecture-based. There are several student-led mathematical activities and classroom practices that utilize ICT. This paper examines how combining ICT and mathematical activities in university math classes can enhance future math teachers' mathematical understanding and their future teaching skills. Drawing on examples from analysis classes taught by the author, this proposal highlights the value of incorporating such activities into teacher training and provides specific implementation methods.

1. Introduction

Japan's curriculum guidelines (Ministry of Education, Culture, Sports, Science, and Technology) define mathematical activities as understanding phenomena mathematically, identifying mathematical problems, and solving them both independently and collaboratively. These activities are widely used in primary and secondary education and serve as the foundation for inquiry-based learning.

However, interviews with students revealed that in university education, especially in mathematics teacher training programs, opportunities for students to actively participate are limited, and lesson design that incorporates Information and Communication Technology (ICT) is insufficient. Many students first learn how to use ICT after entering the teaching profession, and their busy schedules often restrict learning opportunities. Given this situation, experiencing mathematical activities that utilize ICT at the university level not only deepens students' understanding of mathematics but also helps develop their teaching skills for future practice. This paper introduces classroom experiences and assesses the importance of ICT-based activities in teacher training courses.

2. Methods

The study was conducted in mathematics teacher training courses for third- and fourth-year undergraduates. Students engaged in problem-solving tasks that combined mathematical reasoning with ICT-based visualization using GeoGebra. Activities were designed to solve differential equations, explore the properties of complex functions, and analyze conformal mappings. In this paper, each activity includes both analytical

approaches and ICT-supported explorations. We provide interactive activities using the software [4] in the Supplementary Electronic Material sections.

3. Specific classroom practices

3.1 Analysis III (Differential Equations) - Third-Year Mathematics Majors

In this class, students not only learned the process of solving differential equations but also used GeoGebra (see [4]) to graph and verify the solutions.

Problem 1

Find the solution to the following first-order differential equation.

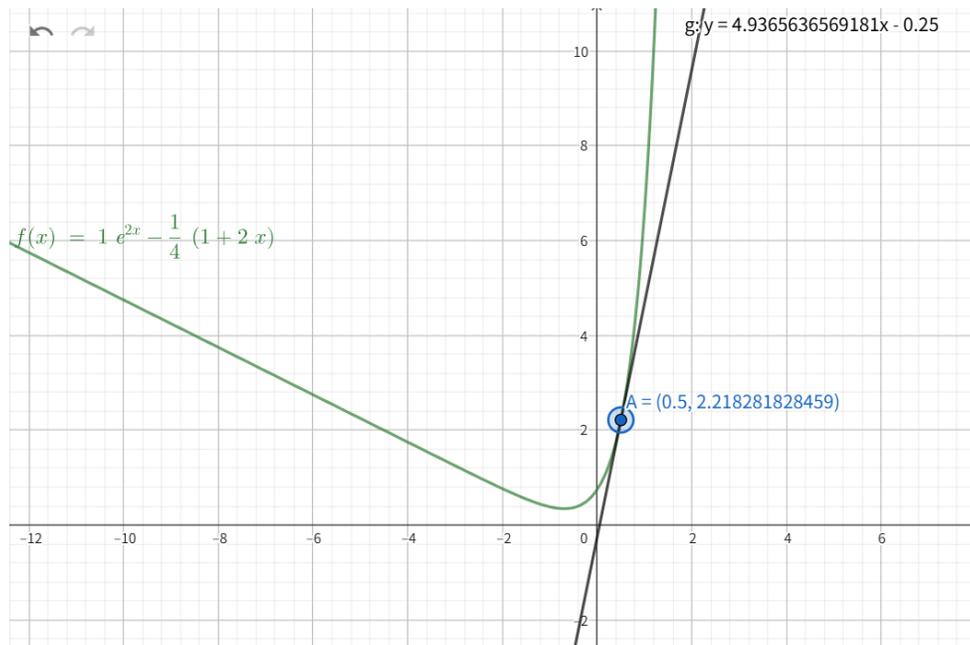$$y' - 2y = x \qquad (2.1)$$

Solution:

Activity 1:

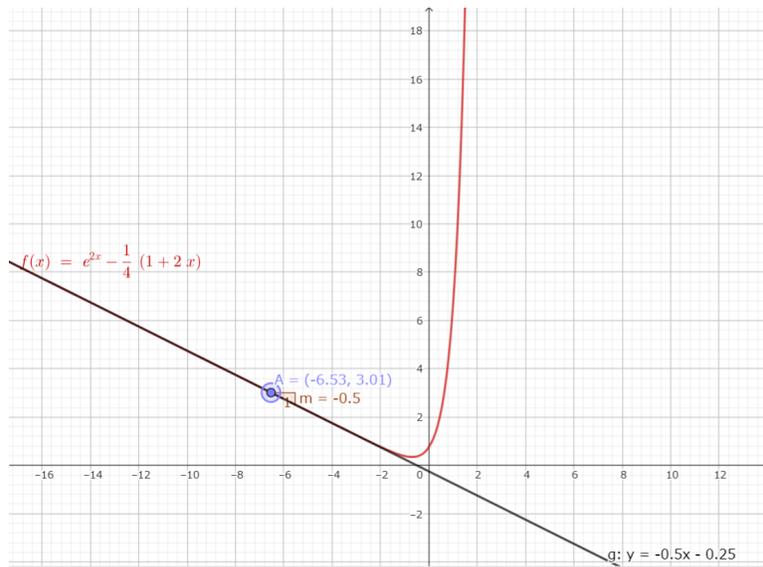Find an analytical solution using the integrating factor method.
Obtained solution:

$$f(x) = e^{2x} - \frac{1}{4}(1 + 2x) \qquad (2.2)$$

Activity 2:

Use GeoGebra to draw a graph of equation (2.2), and substitute multiple points (x, y) on the graph into equation (2.1) to confirm that the equation always holds.

Figure 2.1 Graph of the solution of the differential equation (2.1)

$$f(x) = e^{2x} - \frac{1}{4}(1 + 2x)$$

A = (-6.53, 3.01)
m = -0.5

g: y = -0.5x - 0.25

Figure 2.2 Graph of the solution of the differential equation (2.1)

Interactive activity files for Figures 2.1, 2.2, and 2.3 can be found at [S1], [S2], and [S3], respectively. Through this work, students were able to visually and logically understand the concept of differential equations.

3.2 Analysis IV (Complex Functions) - Fourth-Year Mathematics Majors

This class conducted an activity to visually understand the properties of complex functions through comparison with real functions.

Problem 2

The real function $f(x) = 1/(x^2 - 1)$ has a maximum value of $f(0) = 1$ at $x = 0$ inside the closed region $-1/2 \le x \le 1/2$.

Also, if we consider the complex function $f(z) = 2/(z^2 - 1)$ in the closed region $|z| \le 1/2$, at what point does the maximum value of $|f(z)|$ occur?

Solution:

Step 1: Mathematical Considerations

(1) For Real Functions

By differentiating $f(x) = 1/(x^2 - 1)$,

it has a maximum value at x = 0 within the range -1/2 ≤ x ≤ 1/2,

and it can be easily shown that f(0) = 1 is also a maximum value.

(2) For Complex Functions

This function $f(z) = 2/(z^2 \quad 1)$ is differentiable within the closed domain $|z| \le \frac{1}{2}$. Therefore, by the maximum principle, holomorphic functions have a maximum value on the boundary.

In other words, a point on the boundary can be expressed as $z = \frac{1}{2}e^{i\vartheta}$
In this case,

$$f(z) = 2/(z^2 \quad 1) = 2/(\frac{1}{4}e^{2i\vartheta} \quad 1)$$

Here,

$$\left|\frac{1}{4}e^{2i\vartheta} + 1\right| = \left|\left(\frac{1}{4}\cos(2\theta) + 1\right)^2 + \left(\frac{1}{4}\sin(2\theta)\right)^2\right|$$

This gives us

$$|f(z)| = \frac{2}{\sqrt{\frac{17}{16} + \frac{1}{2}\cos(2\theta)}} = \frac{8}{\sqrt{17 + 8\cos(2\theta)}}$$

Consequently, the maximum value occurs when cos(2θ) is minimum. Therefore, when cos(2θ) = -1, and we have

$$|f(z)| = \frac{2}{\sqrt{\frac{17}{16} + \frac{1}{2}\cos(2\theta)}} = \frac{8}{\sqrt{17 + 8\cos(2\theta)}}$$

$$|f(z)| = \frac{8}{\sqrt{17 - 8}} = \frac{8}{3}$$

Step 2: Considerations of Using GeoGebra on Complex Functions
We will use the three-variable case in GeoGebra. This illustrates the significance of observing the situation where the value approaches a maximum on the real axis as it approaches the boundary. It also helps us understand that not all values reach their maximum value on the boundary.

Problem 3

The real function $f(x) = 2/(e^x \quad e^{-x})$ has a maximum value of f(0) = 1 at x = 0 inside the closed region -1 ≤ x ≤ 1. Now, consider the complex-valued version of this function. If $f(z) = 2/(e^z \quad e^{-z})$ is considered in the closed region |z| ≤ 1, at what point does the maximum value of |f(z)| occur?

Solution:
Step 1: Mathematical Consideration: We leave the exploration of real functions to the readers.
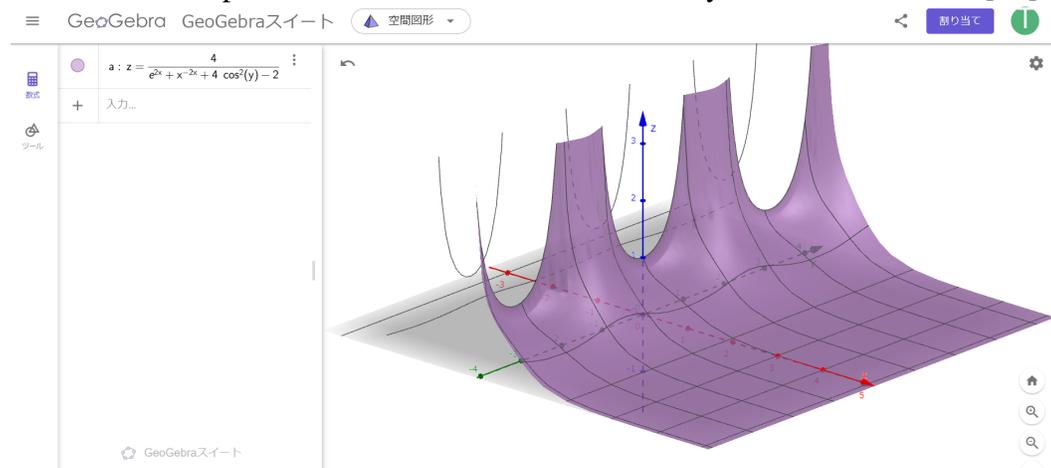
Step 2: Consideration Using GeoGebra
(1) The Case of Real Functions: Considerations Using GeoGebra, interactive activity file can be found at [S3].
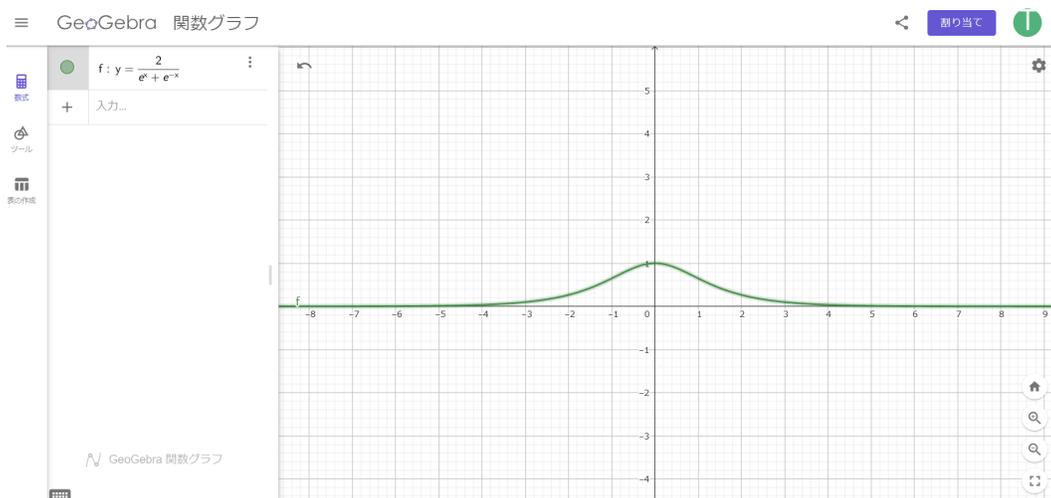
Figure 2.3 Graph of the Real Function $f(x) = 2/(e^x \quad e^{-x})$

(2) The Case of Complex Functions. The interactive activity file can be found at [S4].

Figure 2.4 Graph of the Complex Function $f(z) = 2/(e^z \quad e^{-z})$

3.3 Understanding Conformal Mappings

This activity aims to deepen understanding by visually grasping differential equations and comparing them with real functions.

Problem 4

Confirm that conformality is maintained when

$$x = 1, y = i$$

is mapped by $f(z) = z^2$. Analyze to show that conformality (conservation of angles) is maintained.

Solution:

In Problem 1, to confirm conformality, we need to analytically prove that the mapping of the complex function $f'(z) = 2z$ preserves angles at

$$x = 1, y = i.$$

It is clear that the function $f(z) = z^2$ is regular for any complex number, $z \in \mathbb{C}$.

Also, since $f'(z) = 2z$,

$$f'(1) = 2 \neq 0, f'(i) = 2i \neq 0$$

This ensures conformality at these two points.

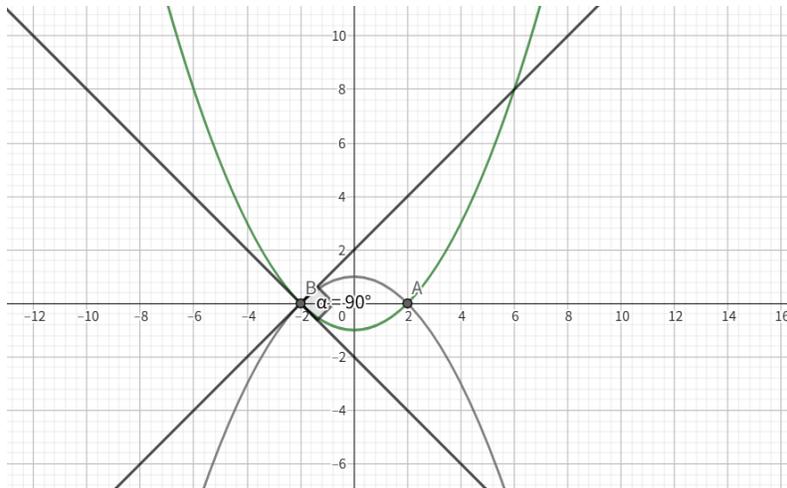**Problem 5**

Generalize Problem 1 to the case where

$$x = a, y = bi.$$

Solution:

As we have seen in Problem 1,

$$f'(a) = 2a \neq 0, f'(bi) = 2bi \neq 0,$$

We also allow students to explore using the interactive file, see [S5].

Figure 2.5 Confirming conformality using $f(z) = z^2$

Problem 6

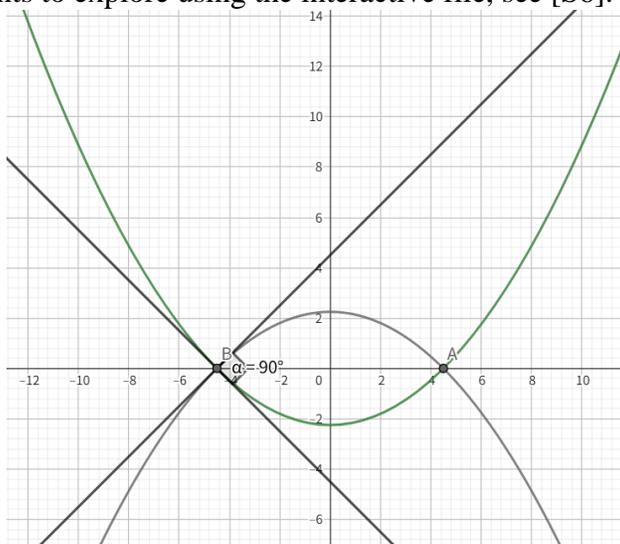Generalize Problem 1 to the case where

$$x = a, y = bi$$

Solution:
As seen in Problem 1,

$$f'(a) = 2a \neq 0, f'(bi) = 2bi \neq 0$$

We also allow students to explore using the interactive file, see [S6].

Figure 2.6 Confirming Conformality with $f(z) = z^2$

Problem 7

Generalize Problem 5 to the case where

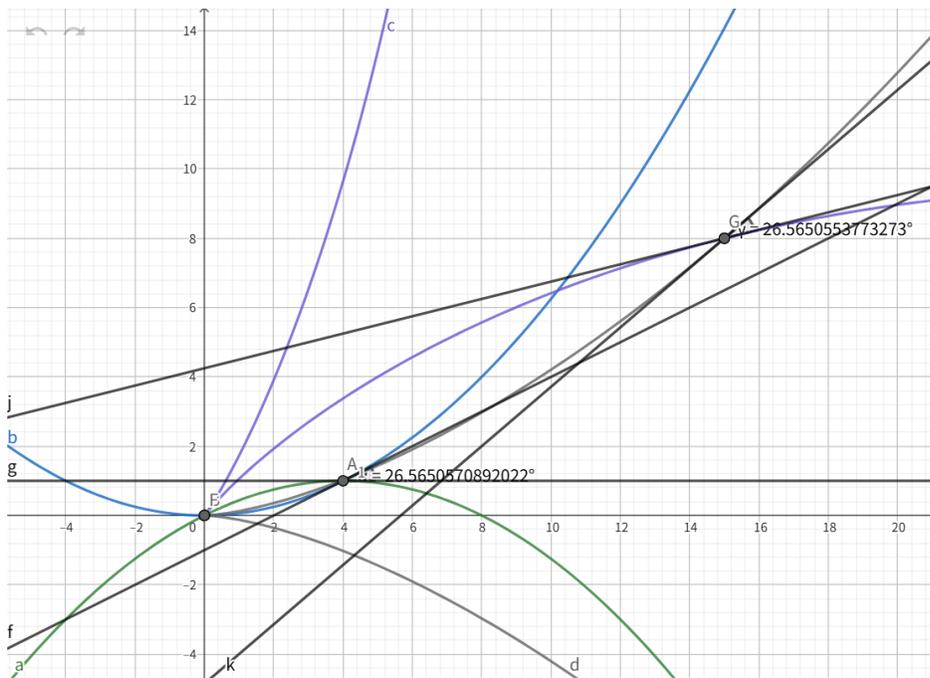$C1; 4t + (2t - t^2)i, (t > 0)$

$C2; 4s + s^2i, (s > 0)$

Solution:
Use GeoGebra to show that conformality is maintained when

$C1; 4t + (2t - t^2)i, (t > 0)$

$C2; 4s + s^2i, (s > 0)$

is mapped with

$f(z) = z^2$.

Figure 2.7 Graph of Problem 7

Visually confirming the angular relationships after mapping enhanced students' understanding of conformality. The interactive activity file can be found at [S7].

4. Results and Discussion

These case studies yielded the following educational implications:

1. ICT is still underused in mathematics teacher training, making systematic university-level ICT development vital for students to apply these skills in classrooms.
2. Visual and interactive tasks with tools like GeoGebra helped undergraduates grasp abstract math concepts and boosted their motivation. In *Analysis III*, students used GeoGebra to check analytical solutions to differential equations. In *Analysis IV*, comparing complex and real functions clarified maximum principles and boundary behavior.
3. ICT-based activities, particularly in analysis and statistics, enhanced students' conceptual understanding instead of focusing solely on procedural skills. Student feedback revealed that integrating analysis with ICT enabled them to comprehend complex topics that they had previously regarded as overly abstract.
4. Informal surveys indicated greater confidence in using differential equations and complex analysis. While formal test scores were not tracked, early comparisons showed slight gains in problem-solving accuracy after ICT-based activities. Future studies should use quantitative methods to verify these findings.
5. Some test items were revised after initial implementation to focus on conceptual understanding rather than rote procedures, emphasizing the iterative process of instructional design.

Remarks: This study does not include comprehensive quantitative outcome measures. Future research could address this by conducting controlled studies that compare ICT-supported and traditional approaches.

5. Conclusion

This paper examined how mathematical activities and ICT enhance university mathematics education, especially in teacher training. Systematizing these practices and applying them to broader areas will help improve teacher preparation quality.

6. Supplementary Electronic Materials

[S1] Figure 2.1: https://www.geogebra.org/calculator/mt5v2yxr
[S2] Figure 2.2: https://www.geogebra.org/calculator/sfut5brj
[S3] Figure 2.3: https://www.geogebra.org/calculator/rqhdtqtw
[S4] Figure 2.4: https://www.geogebra.org/calculator/xahkuyrn
[S5] Figure 2.5: https://www.geogebra.org/calculator/m3dwdtpf
[S6] Figure 2.6: https://www.geogebra.org/calculator/dbc8dyte
[S7] Figure 2.7: https://www.geogebra.org/calculator/s2mzs7wb

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[2] Boyce, W. E., & DiPrima, R. C. (2017). *Elementary Differential Equations and Boundary Value Problems* (11th ed.). Wiley.

[3] Conway, J. B. (1978). *Functions of One Complex Variable I.* Springer.

[4] GeoGebra Team. GeoGebra Software. Retrieved from https://www.geogebra.org/

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