Desmos Classroom Activities in the Development of a PGD for Graphing Rational Functions

Christian S. Abasta
christian.abasta@student.ateneo.edu

Lester C. Hao
lhao@ateneo.edu

Department of Mathematics
Ateneo de Manila University
Philippines

Abstract: *The study aimed to understand how the use of Desmos Classroom activities could aid in constructing a preliminary genetic decomposition (PGD) for graphing rational functions. The study adopted Action, Process, Object, and Schema (APOS) theory to investigate students' knowledge constructions in learning how to graph rational functions. Multimedia Learning Theory (MMLT) principles were also utilized to select Desmos Classroom activities in teaching graphing rational functions during the Activity-Classroom Discussion-Exercise (ACE) Teaching Cycle. One key finding on the implementation of the ACE Teaching Cycle revealed that the use of Desmos Classroom helped students interiorize certain actions on a rational function to form processes of vertical and horizontal asymptotes.*

1. Introduction

A graph, as a mathematical representation [1], is vital for students to visualize and sketch due to its role in problem-solving, modeling, and summarizing relationships to understand real-world phenomena [2-6]. In the Philippines, graphing rational functions is a key competency in senior high school [7]. However, despite being introduced in high school, students often struggle to grasp certain behaviors of rational functions [8]. Literature also highlights persistent misconceptions, such as the idea that the graph of a function never intersects its asymptote regardless of whether the asymptote is vertical or horizontal or the way how to find the domain and zeroes of a rational function [9-11]. These findings align with the lead author's observations of Filipino students' conceptual and procedural difficulties, prompting a deeper investigation into how they construct knowledge in graphing rational functions.

1.1. Action, Process, Object, Schema (APOS) Theory

APOS theory is a mathematics education theory that focuses on individual understanding of a mathematical concept, which is developed through an individual's construction and reconstruction of the mental structures, namely, the *actions*, *processes*, and *objects* [12], [13]. Afterwards, these elements are organized into a coherent framework as another structure, called the *schema*. These mental structures are formed through mental mechanisms, such as *interiorization*, *encapsulation*, *de-encapsulation*, *coordination*, and *reversal* [12]. Figure 1 illustrates the interrelatedness of the mental structures and mechanisms [14].

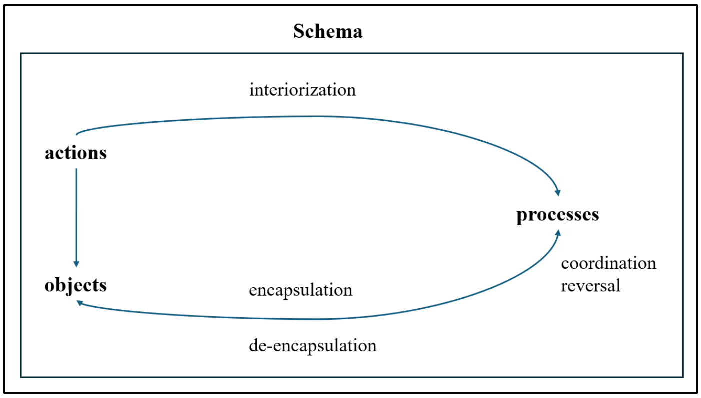


Figure 1. Mental Structures and Mechanisms in the formation of Mathematical Understanding

In APOS Theory, a genetic decomposition (GD) is a theoretical model that explains the construction of mental structures of an individual through mental mechanisms to understand a mathematical concept [14], [15]. Subsequently, the model is referred to as a preliminary GD (PGD) if it has not been experimentally tested or undergone refinement. [15] discussed a research framework that involves the development of a PGD as well as its refinement (see Figure 2) [15].

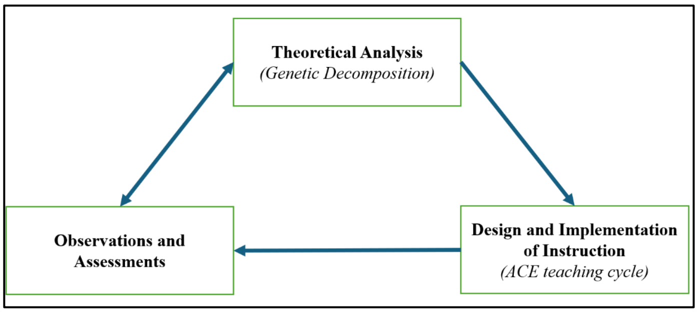


Figure 2. APOS Theory as a Framework for Research and Development

In connection to Figure 2, a PGD is conventionally developed through a researcher's knowledge about the APOS theory, their teaching and learning experiences of and existing academic papers about, a mathematical concept, or the concept's historical development [14]. Afterwards, the model informs the experimental instruction called the *ACE teaching cycle*, which comprises three phases, namely, the activity, classroom discussion, and exercise phases. Lastly, the findings obtained from the instruction through possible observations and assessments are then used to refine the PGD. This cycle can be repeated until the model satisfactorily describes the mental constructions of students in learning the mathematical concept [14].

1.2. Desmos Classroom

One of the latest trends in mathematics teaching and learning at present is the use of dynamic geometry software (DGS), which is a type of software application designed to enable users to construct and explore geometric representations [16]. Some of the widely used graphing software in teaching mathematics are Autograph, GeoGebra, and Desmos [17], [18]. These three DGS enable users to construct, visualize, and manipulate geometric figures (e.g., circles, triangles) and mathematical representations (e.g., graphs of functions), and they also promote interactive learning by allowing users to perform real-time explorations of mathematical concepts, such as graphs of polynomial and rational functions, thereby promoting conceptual understanding of students. Moreover, the three software also offer their respective classrooms, i.e., Autograph Classroom, GeoGebra Classroom, and Desmos Classroom, which are designed to promote interactive education through technology-driven instructions and lessons.

However, Autograph Classroom offers limited access to its features, requiring a paid subscription for full use. In contrast, both GeoGebra and Desmos Classrooms are free and allow teachers and students to create accounts, design interactive activities, and monitor student progress in real time. While the latter two platforms cover a wide range of mathematics topics, the authors found Desmos Classroom more interactive and user-friendly for teaching rational functions and related concepts. Consequently, the researchers chose Desmos Classroom for the present study.

1.3. Multimedia Learning Theory

The multimedia learning theory (MMLT) is a theory of learning that asserts that a combination of words and pictures promotes better learning for students than words alone [19]. Moreover, the theory enumerates several learning principles that help achieve the objective of multimedia learning: (1) *reducing extraneous processing* (i.e., principles minimizing unimportant features of an educational material that do not significantly contribute to attaining learning objectives), (2) *managing essential processing* (i.e., principles suggesting teachers to administer properly the complex characteristics of a presented learning material), and (3) *fostering generative processing* (i.e., principles encouraging teachers to optimize the characteristics of a learning material that help students in information processing).

In connection with the previous discussions, the study utilized the principles of multimedia learning as the basis for the selection of Desmos Classroom activities for the ACE teaching cycle in teaching how to graph rational functions. In particular, the present study seeks to answer the question: *What insights on the use of Desmos Classroom activities were derived from the development of a PGD for Graphing Rational Functions?*

2. Operational Framework

In constructing a PGD, many studies [14], [15], [20-22] tend to solely base their development on the perspective of researchers, i.e., researchers' teaching experiences and literature. While the researcher's teaching experiences and reflections are assumed to include students' input in the development of the PGDs, the cited studies did not consider the actual experiences of students in constructing their respective PGDs. Thus, being more explicit in the development of a theoretical model has been argued by considering the actual learning experiences of students in the research process of developing a PGD. By modifying the research process in Figure 2 and flipping the order of GD and ACE Teaching Cycle, the following operational framework in developing a PGD (in this case, for graphing rational functions) was proposed and is shown in Figure 3 [23].

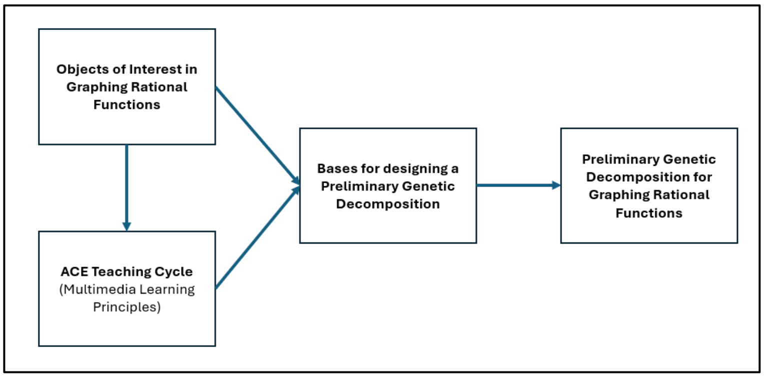


Figure 3. Operational Framework of the Present Study

With this, students' actual experiences are more directly characterized in the research process and eventually in the resulting theoretical model [23]. The present research used past literature to design its ACE Teaching Cycle [24], [25]. Moreover, the *objects of interest* are the fundamental concepts and essential procedures students must grasp to understand and learn graphing rational functions, as well as the flow and development of ideas. These objects inform the design of the ACE Teaching Cycle, featuring the use of Desmos Classroom activities that adhere to principles of multimedia learning [19] and are publicly accessible online. Subsequently, these objects of interest and data from the ACE Teaching Cycle's implementation form the basis for designing the PGD. The gathered information is interpreted alongside APOS Theory to develop the PGD for graphing rational functions.

3. Methods

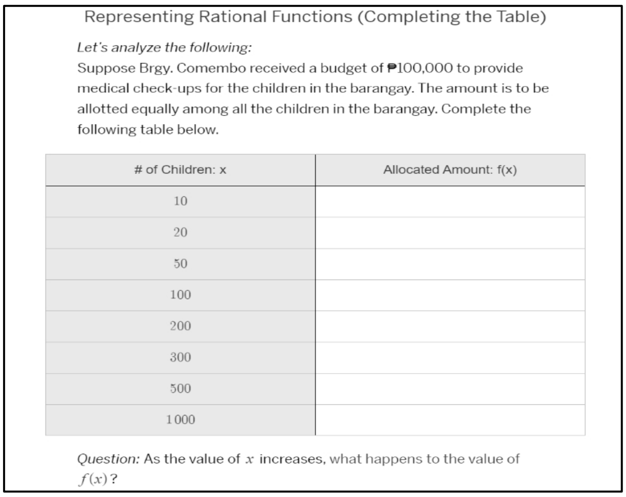
Guided by the framework presented in Figure 3, the study adopted a systematic literature review (SLR) and reflexivity to identify *objects of interest* related to graphing rational functions, including textbooks, learning modules, and the researcher's own teaching and learning experiences. Materials such as textbooks, learning modules, teaching guides, and educational research from reliable sources that have been published in the past 10 years were considered from the SLR, and these resources have content that is focused on teaching or learning how to graph rational functions. The gathered data at this point informed the design of the ACE Teaching Cycle, which was implemented to observe students' classroom participation and analyze their written outputs in learning to graph rational functions. The intervention involved seven Grade 11 students who participated in four ACE Teaching Cycle sessions conducted in a computer laboratory and facilitated by the lead author. The participants were categorized as high-performing (HP), mid-performing (MP), and low-performing (LP) students, and subsequently grouped into three heterogeneous teams based on these classifications. The authors acknowledge that the present research is a small-scale study; the small sample size allowed the researchers to manage and focus on students' responses in depth and greater detail. Although it is sufficient for a qualitative study, it may limit the generalizability of the findings to other groups of students or educational contexts. Moreover, as the facilitator of the teaching cycle, the lead author ensured that it was implemented objectively, without influencing the participants' responses to the potential outcomes of the study. Lastly, data generated from the ACE Teaching Cycle were examined using qualitative content analysis to identify specific *mental structures* and *mechanisms*, which then served as the foundation for developing a PGD for graphing rational functions.

3.1. Implementing the ACE Teaching Cycle

The ACE Teaching Cycle commenced with a review phase, which revisited key concepts on functions from the junior high school curriculum. Students were introduced to Desmos Classroom during this session as well. They completed 6 Desmos activities on the following topics: function as a machine, vertical line test, exploring domain, finding the x-intercept(s), finding the y-intercept, and evaluating functions. Meanwhile, the Activity Phase focused on teaching graphing rational functions using computers, in which students engaged in five Desmos Classroom activities facilitated by the lead author: representing rational functions, introducing vertical asymptotes, drawing vertical asymptotes, introducing horizontal asymptotes, drawing horizontal asymptotes. Moreover, although students were not yet grouped according to their performance level, collaboration was encouraged. The lead author also demonstrated manual graphing of rational functions and used Desmos' graphing calculator for comparison at the end of this phase. In the Classroom Discussion Phase, computers were turned off and students completed paper-and-pen tasks based on the Activity Phase. They were grouped heterogeneously to foster collaboration. The lead author facilitated learning and guided group discussions. Later, students used Desmos to graph the same functions and compare them with their manual sketches. Student outputs were subsequently collected. Finally, in the Exercise Phase, each group received some homework on rational functions. Members collaboratively worked on assigned functions and later presented their solutions to the class. The researcher also collected these outputs for analysis.

3.2. Implementing Desmos Classroom Activities

The highlights of using Desmos Classroom activities in the implementation of the ACE Teaching Cycle in learning how to graph rational functions are presented in this section. The participants were engaged in the discussion about a rational function and its definition and domain. Afterwards, students were provided with a real-life situation, posed in a Desmos Classroom activity as shown in Figure 4, where they were tasked to complete a table of values. The said table was designed to enable students to determine the equation of the rational function that models the given real-life situation. Due to time constraints, the researcher allowed the students to use scientific calculators online to complete the table in the Desmos Classroom activity slides.


Representing Rational Functions (Completing the Table)

Let's analyze the following:
Suppose Brgy. Comembo received a budget of ₱100,000 to provide medical check-ups for the children in the barangay. The amount is to be allotted equally among all the children in the barangay. Complete the following table below.

# of Children: x	Allocated Amount: f(x)
10	
20	
50	
100	
200	
300	
500	
1000	

Question: As the value of x increases, what happens to the value of $f(x)$?

Figure 4. A Desmos Classroom Activity on Representing Rational Functions

Moreover, with the intention to allow the students to see the connection between the table of values and the equation of the rational function, they were instructed to type the points and equation onto the Desmos graphing calculator. Using the generated graph of the rational function, students were provided with a visual representation of the table of values for some points of the graph of the given rational function, as shown in Figure 5.

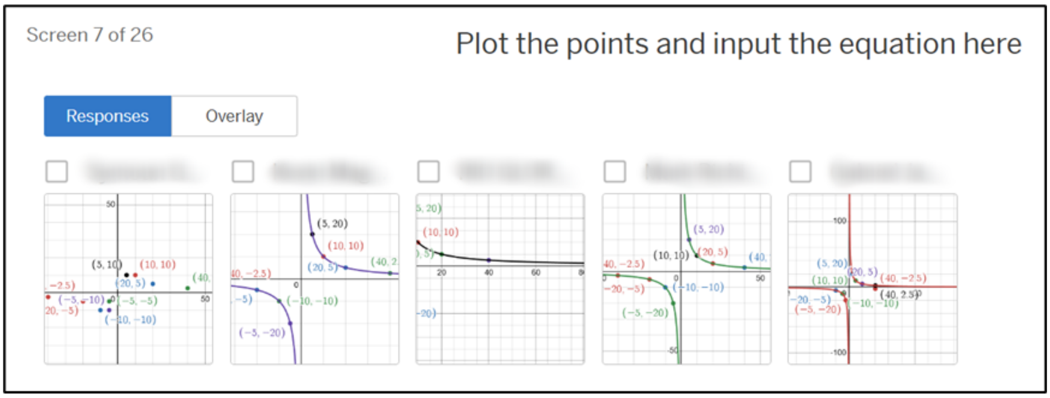
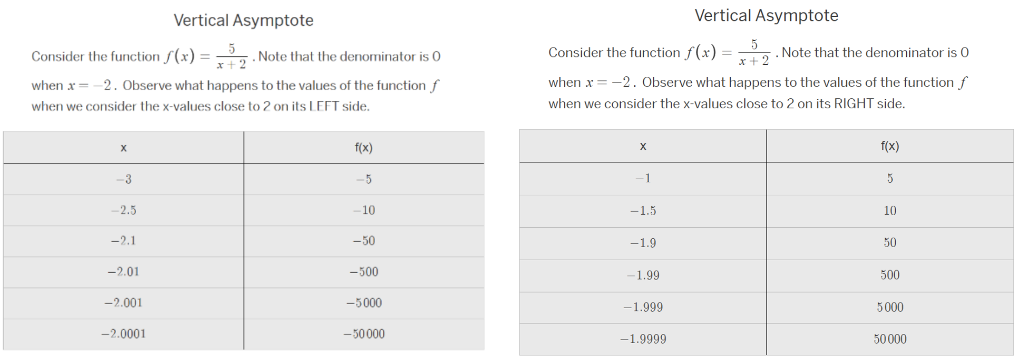


Figure 5. Sample works of students in Desmos Classroom

In introducing a vertical asymptote, the participants were presented with a rational function and its table of values, and provided with several guide questions to arrive at the correct interpretation of the table as shown in Figure 6.



Vertical Asymptote

Consider the function $f(x) = \frac{5}{x+2}$. Note that the denominator is 0 when $x = -2$. Observe what happens to the values of the function f when we consider the x-values close to 2 on its LEFT side.

x	f(x)
-3	-5
-2.5	-10
-2.1	-50
-2.01	-500
-2.001	-5000
-2.0001	-50000

Vertical Asymptote

Consider the function $f(x) = \frac{5}{x+2}$. Note that the denominator is 0 when $x = -2$. Observe what happens to the values of the function f when we consider the x-values close to 2 on its RIGHT side.

x	f(x)
-1	5
-1.5	10
-1.9	50
-1.99	500
-1.999	5000
-1.9999	50000

Figure 6. A Desmos Classroom Activity on Introducing Vertical Asymptote(s)

As the lead author asked probing questions, some students in chorus were able to guess the correct behavior of the function as the variable x approaches a number from the left or from the right, although not precisely. Afterwards, the definition of a vertical asymptote was presented. Moreover, students were asked to determine the vertical asymptote of the given function and use the line tool in the Desmos Classroom activity to sketch the asymptote as shown in Figure 7.

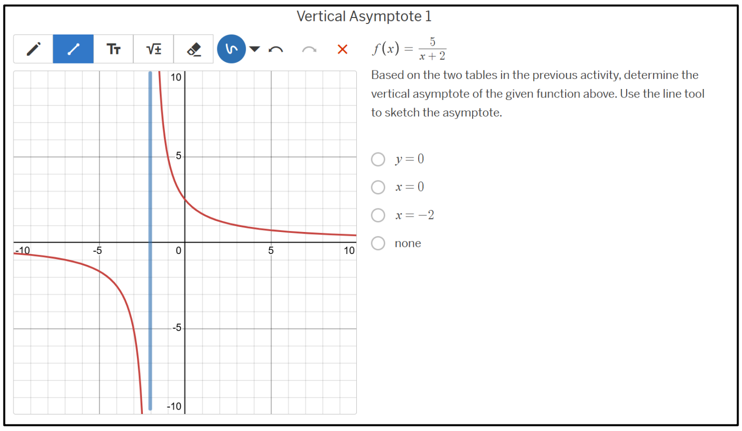


Figure 7. A Desmos Classroom Activity on Drawing a Vertical Asymptote

In the succeeding activities, students were instructed to use the line tool to sketch the graph of each linear equation found in the choices of the activity. From this, students were asked to identify the equation of the vertical asymptote of the given graph of a rational function. The activities also showed that a rational function may have more than one vertical asymptote or no vertical asymptote at all.

On the other hand, the same flow and similar activities were presented when it came to introducing a horizontal asymptote of a rational function. In the context of basic rational functions for Grade 11 students, the activities also showed that a rational function may only have one horizontal asymptote or none at all. Furthermore, the activities on asymptotes allowed the students to encounter the case where the graph of a rational function intersects its horizontal asymptote, as shown in Figure 8. It was observed that students were able to see the visual connection between the behavior of the function and its corresponding asymptotes using graphs.

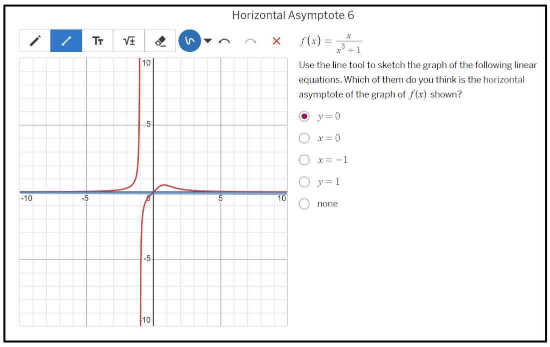


Figure 8. A Desmos Classroom Activity on Drawing Horizontal Asymptote

4. Findings

In the classroom discussion phase, one meaningful observation was recorded when a group of three members was able to exhibit an understanding of the number of vertical asymptotes concerning the function $h(x)=(x-4)/(x^2+x-6)$. The students noticed that the domain has two restricted values of x (see Figure 9). Therefore, the vertical asymptotes must be two (since the given function is irreducible).

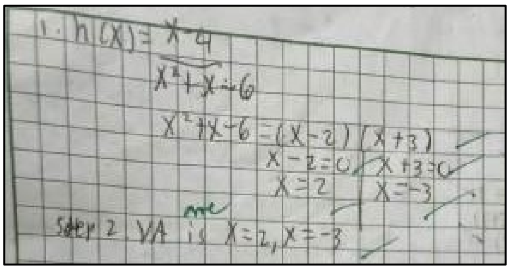


Figure 9. Sample work of a student in finding the vertical asymptotes of the function *h*

Also, they were able to find the horizontal asymptote of the function analytically (see Figure 10), and realized that the graph of the function crosses the horizontal asymptote at its x-intercept (see Figure 11). When the participants of the group were informally asked about it, they were first hesitant to draw the graph of the function that crosses its horizontal asymptote, i.e., $y=0$.

Figure 10. Sample work of a student in finding the horizontal asymptote of the function *h*

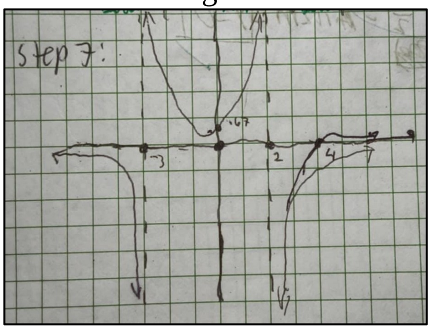


Figure 11. Sample work of a student in sketching the graph of the function *h*

However, they eventually realized that the line *y=0* may cross the graph of *h.* When the participants of the group were informally asked about it, the students mentioned that these realizations (i.e., determining that there could be more than one vertical asymptote and horizontal asymptote may intersect the graph of a rational function) were attributed to the activities they had in the computer laboratory (see Figures 7 and 8), in which they were given scenarios of a function having more than one vertical asymptote and a function with a horizontal asymptote being intersected by its graph.

In light of the aforementioned, using APOS theory, it was found that the use of Desmos Classroom activities complemented the construction of several *actions* of the students, to wit, (1) finding the zeroes of the denominator of a rational function, and (2) comparing the degrees of the numerator and denominator of a rational function. A student constructs an *action* when he works on a physical or mental object (e.g., a rational function) through a series of instructions possibly from a teacher or peer [12-15]. Moreover, at this stage, the student can perform tasks on the *object* with external guidance. In sketching the graph of a rational function, finding the zeroes of the denominator of a rational function and comparing the degrees of the numerator and denominator of a rational function are two of the fundamental steps that students need to perform, which usually require guidance from the teacher (or their peers). Hence, these are considered *actions.*

When a student repeats and reflects on an *action,* it turns into a *process* [12-15]. At this stage, the student can perform the same task without the need for an outside influence. In relation to the study, the participants were given a number of functions to work on, i.e., *f, g,* and *h*, and they performed the aforementioned *actions* repeatedly and reflected on them. This paved the way for *interiorizing* these *actions* into their respective *processes* of vertical and horizontal asymptotes. In particular, the solutions of the student shown in Figures 9 and 10 illustrate the *actions,* which were eventually *interiorized* as *processes* of vertical and horizontal asymptotes. Meanwhile, the graph of the function *h* in Figure 11 embeds these two *processes.*

These findings contribute to the construction of the PGD by connecting *actions* (i.e., finding the zeroes of the denominator of a rational function and comparing the degrees of the numerator and denominator a rational function) to being *interiorized* into *processes* (i.e., vertical and horizontal asymptotes). It should be noted that in the context of this study, these findings are the only cognitive pathways in graphing rational functions that relate to the use of technology. This is seen as a vital step in the construction of other mental structures, such as the *process* of the characteristics of a rational function and the graph of a rational function as an *object*.

5. Conclusion

This study primarily used APOS theory as a theoretical lens to investigate how students construct their understanding of graphing rational functions. Guided by the multimedia learning principles, the study selected Desmos Classroom activities appropriate in teaching how to graph rational functions via ACE Teaching Cycle. By presenting interactive tasks through Desmos Classroom, students engaged in learning how to graph rational functions. One key highlight is the notable contribution of the use of technology to the students' understanding of vertical and horizontal asymptotes of a rational function. In relation to APOS theory, it revealed that Desmos Classroom activities aided students in constructing *processes* of vertical and horizontal asymptotes based on their previous learning experiences in exploring the different situations of rational functions and their asymptotes. This shows how the use of educational technology, such as Desmos Classroom, can not only contribute to a better understanding of a concept but also complement the development of a PGD.

Thus, future research could investigate further how educational technology may shape the learning pathways of students through the lens of APOS theory.

References

[1] G. A. Goldin, "Mathematical Representations," in *Encyclopedia of Mathematics Education*, S. Lerman, Ed., Dordrecht: Springer Netherlands, 2014, pp. 409–413. doi: 10.1007/978-94-007-4978-8_103.

[2] G. Leinhardt, O. Zaslavsky, and M. K. Stein, "Functions, Graphs, and Graphing: Tasks, Learning, and Teaching," *Rev. Educ. Res.*, vol. 60, no. 1, pp. 1–64, Mar. 1990, doi: 10.3102/00346543060001001.

[3] H. Lohi, Mardiyana, and I. Pramudya, "How Students' Difficulty in Implementing Mathematical Representations in Solving Problem of Statistical Content is?," presented at the International Conference of Mathematics and Mathematics Education (I-CMME 2021), Atlantis Press, Nov. 2021, pp. 118–122. doi: 10.2991/assehr.k.211122.016.

[4] S. A. Ozgun-Koca, "The Graphing Skills of Students in Mathematics and Science Education. ERIC Digest," ERIC Clearinghouse for Science, Mathematics, and Environmental Education, 1929 Kenny Road, Columbus, OH 43210-1080, Aug. 2001. Accessed: Apr. 09, 2024. [Online]. Available: https://eric.ed.gov/?id=ED464804

[5] D. Hasanah, I. Hidayah, T. Lestari, L. Oktoviana, and M. Agung, *Investigating Students' Errors in Graphing Polynomial Functions*. 2021. doi: 10.2991/assehr.k.210508.049.

[6] M. Bourne, "Functions and Graphs," Interactive Mathematics. Accessed: June 01, 2024. [Online]. Available: https://www.intmath.com/functions-and-graphs/functions-graphs-intro.php

[7] DepEd, "K to 12 Senior High School Core Curriculum – General Mathematics." Department of Education, May 2016.

[8] W. J. Cook and M. J. Bosse, "Crossing through and bouncing off∞: Graphing rational functions," *MathAMATYC Educ.*, 2019.

[9] G. Dotson, "Collegiate Mathematics Students' Misconceptions of Domain and Zeros of Rational Functions," Dissertation, Universtity of Kansas, Kansas, USA, 2009. [Online]. Available: https://eric.ed.gov/?id=ED532878

[10] G. Nair, "College Students' Concept Images of Asymptotes, Limits, and Continuity of Rational Functions," Dissertation, Ohio State University, Ohio, USA, 2010. [Online]. Available: https://etd.ohiolink.edu/acprod/odb_etd/ws/send_file/send?accession=osu1282259818&disposition=inline

[11] L. C. Schnepper and L. P. McCoy, "Analysis of misconceptions in high school mathematics," *Netw. Online J. Teach. Res.*, vol. 15, no. 1, pp. 625–625, 2013.

[12] E. Dubinsky, "Actions, Processes, Objects, Schemas (APOS) in Mathematics Education," in *Encyclopedia of Mathematics Education*, S. Lerman, Ed., Cham: Springer International Publishing, 2020, pp. 16–19. doi: 10.1007/978-3-030-15789-0_3.

[13] E. Dubinsky and M. A. Mcdonald, "APOS: A Constructivist Theory of Learning in Undergraduate Mathematics Education Research," in *The Teaching and Learning of Mathematics at University Level*, vol. 7, D. Holton, M. Artigue, U. Kirchgräber, J. Hillel, M. Niss, and A. Schoenfeld, Eds., in New ICMI Study Series, vol. 7. , Dordrecht: Kluwer Academic Publishers, 2002, pp. 275–282. doi: 10.1007/0-306-47231-7_25.

[14] I. Arnon *et al.*, *APOS Theory: A Framework for Research and Curriculum Development in Mathematics Education*. New York, NY: Springer New York, 2014. doi: 10.1007/978-1-4614-7966-6.
[15] M. Asiala, A. Brown, D. DeVries, E. Dubinsky, D. Mathews, and K. Thomas, "A framework for research and curriculum development in undergraduate mathematics education," in *CBMS Issues in Mathematics Education*, vol. 6, J. Kaput, A. Schoenfeld, E. Dubinsky, and T. Dick, Eds., Providence, Rhode Island: American Mathematical Society, 1996, pp. 1–32. doi: 10.1090/cbmath/006/01.
[16] M. F. Vazzana, "The Implementation of Technology Into the Mathematics Classroom: A Student-Centered Approach," in *Advances in Educational Marketing, Administration, and Leadership*, T. Mulvaney, W. O. George, J. Fitzgerald, and W. Morales, Eds., IGI Global, 2024, pp. 300–309. doi: 10.4018/978-1-6684-9904-7.ch020.
[17] L. D. Madrilejos, "The Perspectives of Using Desmos for Students' Conceptual Understanding and Procedural Fluency to Solve Linear Equations," Dissertation, The University of Texas Rio Grande Valley, 2024.
[18] I. Karnasih and M. Sinaga, "Enhancing Mathematical Problem Solving and Mathematical Connection Through the Use of Dynamic Software Autograph in Cooperative Learning Think-Pair-Share," *J. Pendidik. Mat.*, vol. 17, no. 1, pp. 51–71, 2014.
[19] R. E. Mayer, *Multimedia learning*, 2nd ed. New York: Cambridge University Press, 2009.
[20] D. Brijlall and S. Bansilal, "A genetic decomposition of the Riemann Sum by student teachers," in *Proceedings of the eighteenth annual meeting of the South African Association for research in mathematics, science and technology education*, Citeseer, 2010, pp. 131–138.
[21] E. Duque-Marín, C. Ramirez-Carrasco, and M. Altamirano-Espinoza, "Construction of a genetic decomposition: theoretical analysis of the concept of the value theorem medium," *J. Phys. Conf. Ser.*, vol. 1702, no. 1, p. 012025, Nov. 2020, doi: 10.1088/1742-6596/1702/1/012025.
[22] H. L. Tarr and A. Maharaj, "A preliminary genetic decomposition for conceptual understanding of the indefinite integral," *J. Math. Behav.*, vol. 63, p. 100891, Sept. 2021, doi: 10.1016/j.jmathb.2021.100891.
[23] C. Abasta and L. Hao, "Using ACE Teaching Cycle in Developing a Preliminary Genetic Decomposition: A Proposed Framework," in *Proceedings of the 9th ICMI-East Asia Regional Conference on Mathematics Education*, Seoul National University, Siheung Campus, 2025, pp. 713–717. [Online]. Available: https://www.earcome9.org/earcome9/05_view.html?sMenu=05&s=l&bIdx=MzM=
[24] I. M. Arnawa, Yerizon, and S. Nita, "Improvement students' achievement in elementary linear algebra through APOS theory approach," *J. Phys. Conf. Ser.*, vol. 1567, no. 2, p. 022080, June 2020, doi: 10.1088/1742-6596/1567/2/022080.
[25] H. Syarifuddin and B. Atweh, "The Use of Activity, Classroom Discussion, and Exercise (ACE) Teaching Cycle for Improving Students' Engagement in Learning Elementary Linear Algebra," *Eur. J. Sci. Math. Educ.*, vol. 10, no. 1, pp. 104–138, Dec. 2021, doi: 10.30935/scimath/11405.