

Laser-Cut String Art: Design and Construction in the Mathematics Classroom

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Abstract: *This work presents an innovative approach to integrating mathematics, digital design, and laser fabrication through the creation of laser-cut string art. The project employs mathematics software to generate mathematically inspired designs, which are then converted into vector graphics and fabricated on layers of acrylic sheets using a laser platform. The approach was implemented in an undergraduate abstract algebra course, where students explored group-theoretic structures by designing and constructing their own string art pieces.*

1. Introduction

String art is a creative technique that involves arranging colored threads around fixed points, often pins or nails mounted on a wooden board, to form intricate motifs, designs, or patterns. Originally introduced by mathematician Mary Everest Boole as a method for exploring mathematical patterns [3], string art has since evolved into a powerful tool for visualizing abstract mathematical concepts and enhancing students' appreciation of mathematics in conjunction with the arts.

In mathematics classrooms, string art has been employed to illustrate a wide range of mathematical ideas, including modular arithmetic and cyclic groups in number theory and abstract algebra [3, 9], as well as fundamental ideas such as tangent lines and parametric curves in calculus [1, 8].

However, designing and creating traditional string art can be labor-intensive and time-consuming, especially when working with complex patterns. Recent studies (see, for example, [2, 12]) have explored ways to automate the whole process using machines and digital fabrication technologies. Additionally, geometry software has been utilized to help students explore calculus concepts and translate them into string art designs [8].

This paper presents an approach that integrates mathematics software with laser fabrication technology to create string art that enhances student learning in the mathematics classroom. Departing from the conventional method, this approach introduces an improvised technique that uses stereographic projection to generate string art designs on laser-cut acrylic sheets.

2. Stereographic Projection

The *stereographic projection* from the unit 2-sphere $S^2 = \{(x, y, z) \mid x^2 + y^2 + z^2 = 1\} \subset \mathbb{R}^3$ to the *extended Euclidean plane* $\mathcal{R}^2 = \mathbb{R}^2 \cup \{\infty\}$ is defined as the injective mapping $\rho : S^2 \rightarrow \mathcal{R}^2$ given by the formula:

$$(x, y, z) \mapsto \left(\frac{x}{1-z}, \frac{y}{1-z} \right).$$

This mapping sends the north pole $N = (0, 0, 1)$ to the point at infinity ∞ , and any other point $P \in S^2 \setminus \{N\}$ to the intersection of the line through N and P with the plane \mathbb{R}^2 , identified with the plane $z = 0$ in \mathbb{R}^3 .

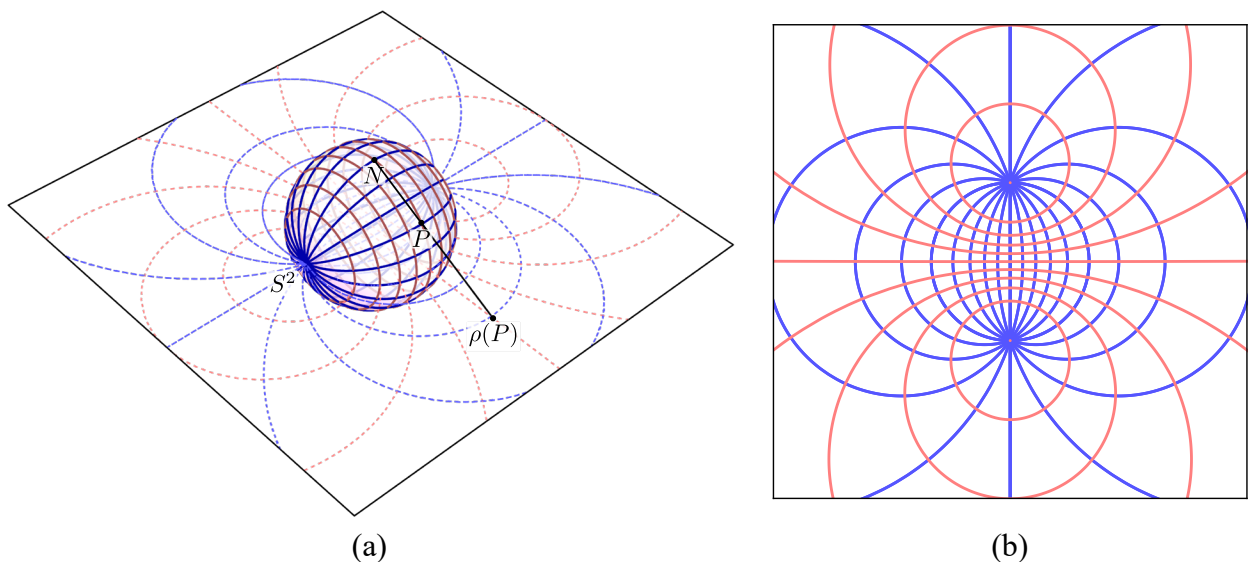


Figure 2.1 The stereographic projection of (a) the point P , and (b) the blue and pink circles on the 2-sphere S^2 , onto the Euclidean plane.

The stereographic projection ρ is *conformal*, meaning it preserves angles between intersecting curves. However, ρ is not area preserving—it distorts the size of regions on the unit sphere, especially near the north pole N . Notably, under ρ , circles on S^2 that do not pass through N are mapped to circles in the plane, while those that do pass through N are mapped to straight lines. Figure 2.1 illustrates the effect of the stereographic projection on a point and certain circles on S^2 .

Because of its distinctive properties, the stereographic projection serves as a powerful tool for visualizing how transformations influence the symmetry of objects within a given coordinate system. In particular, its ability to preserve angles makes it especially valuable in fields such as complex analysis and cartography.

The sequence of programming code in Code 2.1, written using the proprietary computer algebra system *Wolfram Mathematica* [14], was employed to generate the stereographic projection image presented in Figure 2.1(b). Specifically, the code produces a parametric plot of the stereographic projection for each blue and pink circle lying on the unit sphere depicted in Figure 2.1(a).

This figure can be easily reproduced using alternative platforms such as the dynamic mathematics software *GeoGebra* [5], the powerful open-source computer algebra system *SageMath* [10], or *CoCalc* [11], a web-based cloud computing platform that supports the editing and execution of

SageMath worksheets. These mathematics software allow users to directly translate mathematical equations or expressions into editable 2D or 3D graphics files.

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1  (*Stereographic Projection*)
2  stereo[{x_, y_, z_}] := N[ $\frac{1}{1-z}$ {x, y}]

3  (*Blue Circles*)
4  blueCircle[z_, t_] = stereo[{Cos[t]Cos[z], -Sin[t] + 0.001Cos[t]Sin[z], 0.001Sin[t] + Cos[t]Sin[z]}];
5  blueCircles[t_] = Table[blueCircle[z, t], {z, 0, 2π,  $\frac{\pi}{10}$ };

6  (*Pink Circles*)
7  pinkCircle[z_, t_] = stereo[{ $\sqrt{1-z^2}$ Cos[t], -z + 0.001 $\sqrt{1-z^2}$ Sin[t], 0.001z +  $\sqrt{1-z^2}$ Sin[t]}];
8  pinkCircles[t_] = Table[pinkCircle[z, t], {z, -1, 1,  $\frac{z}{10}$ };

9  (*Parametric Plots*)
10 ParametricPlot[{blueCircles[t], pinkCircles[t]}, {t, 0, 2π},
11   PlotRange -> {{-3, 3}, {-3, 3}},
12   PlotStyle -> {{Lighter[Blue], Thickness[0.006]}, {Pink, Thickness[0.006]}},
13   Axes->False, Frame -> True, FrameTicks -> None, FrameStyle -> Directive[Black, Thickness[0.003]]]

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Code 2.1 Mathematica code to generate the stereographic projection image in Figure 2.1(b).

3. Design and Construction of a String Art Piece

The primary objective of this paper is to present a procedure for creating a string art piece that visually represents a geometric or abstract mathematical concept. In our approach, we depart from traditional string art—typically composed of colored strings woven around pin or nails mounted on a wooden board—and instead use strings threaded through laser-cut holes of an acrylic sheet, which serves as the base. Since a string art piece naturally partitions the planar base into distinct regions, we adopt the multi-layered acrylic technique to construct tiling models, following the approach discussed by the authors in [7].

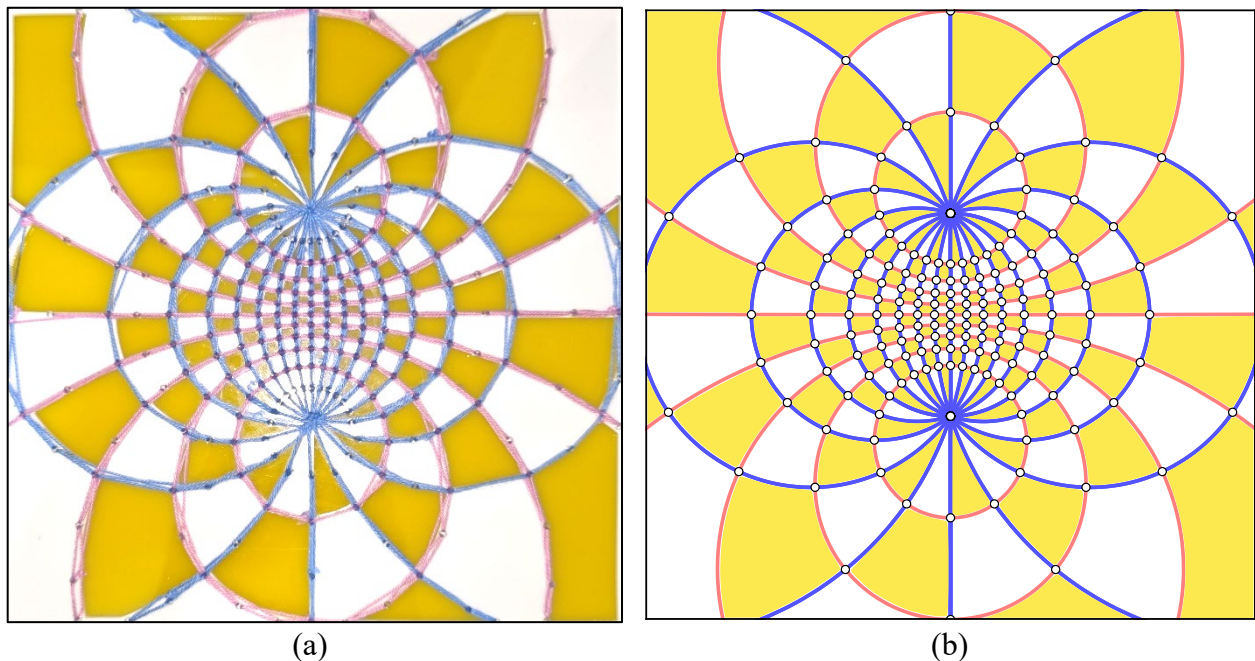


Figure 3.1 (a) String art representation and (b) computer-generated sketch of the stereographic projection image in Figure 2.1(b).

To demonstrate this process, we employ the pattern shown in Figure 2.1(b), resulting in the string art piece in Figure 3.1(a). For this piece, we used three layers of acrylic sheets, each measuring 20 cm x 20 cm x 3mm. The top layer is made of clear acrylic, where holes were laser-cut and colored strings were threaded through. The middle layer is white acrylic, marked with specific regions designated for laser cutting at a later stage. These cut-out areas reveals the bottom yellow acrylic layer, creating a tiling pattern with the colored strings as edges (see Figure 3.2).

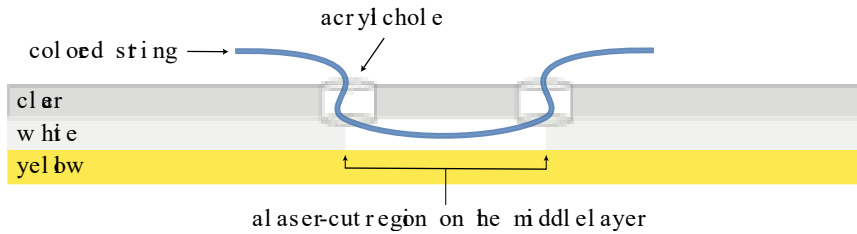


Figure 3.2 Arrangement of the three acrylic layers used in the string art piece in Figure 3.1(a).

For the laser-cut string art pieces featured in this paper, we used the Universal Laser Systems (ULS) VLS 6.75 laser platform [13] (see Figure 3.3), located at the Eugenio Lopez Jr. Makerspace, our university’s dedicated fabrication space [4].

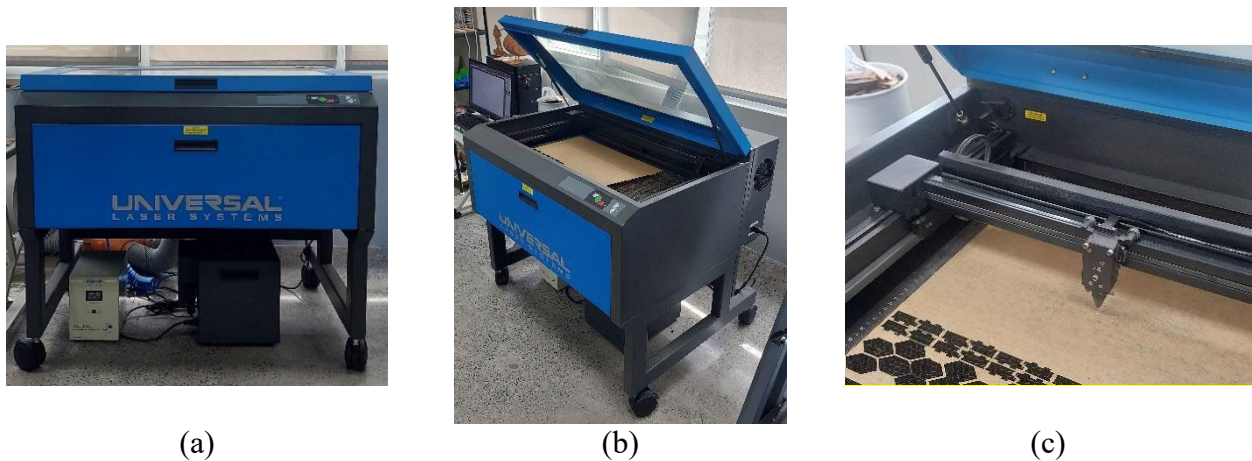


Figure 3.3 The ULS VLS 6.75 laser platform located at the Eugenio Lopez Jr. Makerspace.

3.1. Digital Design Using Math Software

The procedure for creating this string art piece begins with the computer-generated sketch shown in Figure 3.1(b). Producing a sketch prior to the actual construction allows for thoughtful planning and easy revisions, ultimately saving both time and materials.

The sketch must include clearly marked circular holes, centered at the intersection points of each pair of curves. These holes are designated for laser cutting and are generated programmatically by updating lines 9 to 13 of Code 2.1, using the modifications provided below in Code 3.1. The yellow regions in this sketch were manually filled using the Paint Bucket tool in the vector graphics editor *Inkscape* [6]. These regions correspond to the areas in the middle white layer that will be laser-cut to reveal the bottom yellow layer.

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1 (*Holes*)
2 intPt[k_, z_] := stereo[{{sqrt[1-z^2]Cos[kpi/10], -z + 0.001sqrt[1-z^2]Sin[kpi/10], 0.001 z + 1.001sqrt[1-z^2]Sin[kpi/10]}]}
3 intPts = N[Flatten[Table[intPt[k, z], {k, 0, 19}, {z, -1, 1, 1/5}], 1]];
4 holes = Graphics[{{EdgeForm[Black], FaceForm[White], Table[Disk[pt, 0.04], {pt, intPts}]}];

5 (*Parametric Plots*)
6 Show[ParametricPlot[{blueCircles[t], pinkCircles[t]}, {t, 0, 2pi},
7 PlotRange -> {{-3, 3}, {-3, 3}},
8 PlotStyle -> {{Lighter[Blue], Thickness[0.006]}, {Pink, Thickness[0.006]}},
9 Axes->False, Frame -> True, FrameTicks -> None, FrameStyle -> Directive[Black, Thickness[0.003]]],
10 holes]

```

Code 3.1 Mathematica code used to replace lines 9 to 13 of Code 2.1 in order to generate hole positions designated for laser cutting.

3.2. Prototyping and Production

Based on the computer-generated sketch in Figure 3.1(b), we prepared two laser-ready vector graphics files—one for the top layer (see Figure 3.4(a)) and one for the middle layer (see Figure 3.4(b)). These files were designed in accordance with the platform’s specific guidelines for marking areas to be cut or engraved. In our configuration of the ULS VLS 6.75 laser platform, regions designated for engraving are marked black, while areas to be cut are marked red.

For the top layer, we created 2 mm red disks centered at each hole indicated in the sketch and converted the colored curves into thin, dashed black curves to serve as thread guides. Additional red disks were strategically placed to help guide the strings in forming smooth curves rather than straight segments.

For the middle layer file, we selected all yellow regions from the sketch, recolored them red, and resized them to approximately 97% of their original dimensions. This slight reduction ensures that the acrylic sheet remains a single connected piece after laser cutting. Without this adjustment, the process could result in completely detached *acrylic islands*.

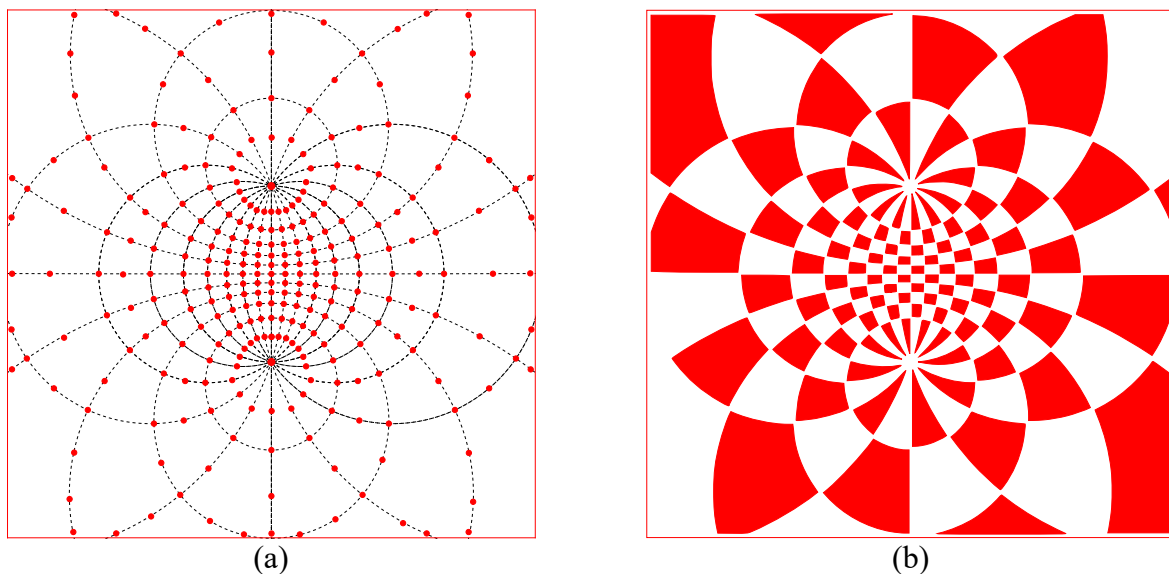


Figure 3.4 Laser-ready vector files derived from Figure 3.1(b) for (a) the clear top acrylic layer and (b) the middle white acrylic layer.

The next step is to import each vector image shown in Figure 3.4 into the laser platform’s user interface to begin the engraving and cutting process. To allow potential revisions during prototyping, these laser-ready files must remain easily editable. For instance, adjustments may be needed based

on the thickness of the thread, which can affect the appropriate hole size. Similarly, the number and placement of the holes may also require modification.

Since this string art technique does not use nails or pins to anchor the thread, it is essential to place holes at the ends of each dashed curve near the edge of the top acrylic sheet to mark the thread's starting points. A starting thread can be secured at one of these points using a temporary knot larger than the hole diameter. The thread is then passed through the adjacent designated holes in such a way that it returns to the starting point, where it can be tied off securely. Additional string art pieces created by the authors' research group using this technique are shown in Figures 4.1(a – b).

4. Sample Student Activity

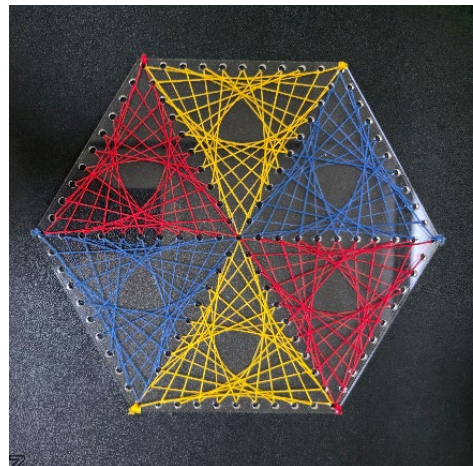
The following is a sample student group activity designed to implement the process outlined in the previous section for creating a laser-cut string art piece. Prior to this activity, students had already been introduced to foundational concepts in group theory, including group operations, generators and relations, and group actions. This theoretical background equips them to translate abstract algebraic ideas into visual representations.

Student Group Activity. This assessment is worth **60 points**:

- Creativity – 20 points
- Quality of Output – 20 points
- Explanation of Group Theory Features – 20 points

Your pair's main task is to create a laser-cut string art that best illustrates a chosen algebraic group G and at least one property of G that you want to feature.

For example, the string art on the right depicts a figure whose symmetry is governed by the dihedral group D_6 , of order 12, along with its subgroups.



Explanation of Group Theory Features

1. Your selected group G may be: finite or infinite, geometric or non-geometric in nature, previously introduced in class or entirely new.
2. Prepare a one-page write-up in pdf format detailing the group-theoretic features represented in your string art.
3. In your write-up include a digital sketch of your string art using graphics software capable of creating or editing SVG vector files, such as [GeoGebra](#) or [Inkscape](#).
4. Use labels sparingly. The string art should primarily convey its mathematical meaning visually.

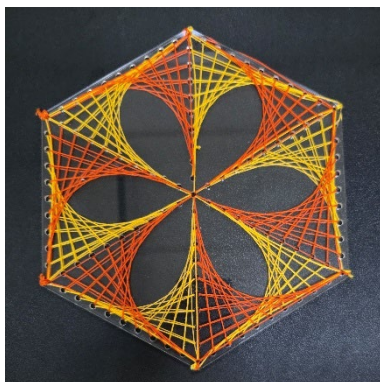
5. Prior to finalizing your write-up consult with your instructor.

Physical Output Requirements

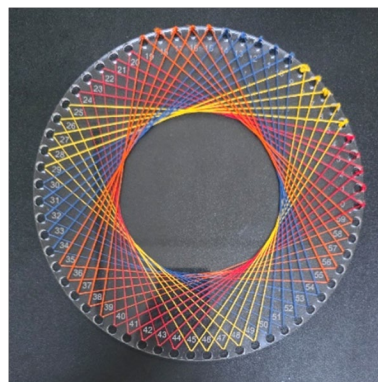
1. The back of the acrylic sheet must be etched with the names of your group members (any font style, size 12).
2. The final output must be a square with dimensions 20 cm x 20 cm.
3. You are not required to submit the digital files used for fabrication; only the physical output and the written report are necessary.
4. Before visiting the Makerspace Lab, consult your instructor for feedback and suggestions.
5. Each group member may spend only a maximum of 3.50 USD for this project.

From a technical standpoint, students should have prior experience with a computer algebra system or graphics software. Additionally, the instructor should conduct a demonstration of the complete construction process, covering the laser-cutting or engraving of acrylic sheets and the threading of strings to form the final artwork. This ensures that students are comfortable operating the laser platform and understand the practical steps involved.

The activity is designed to promote collaborative learning, with students working in pairs and receiving minimal guidance from the instructor. This approach fosters independent exploration and encourages creative engagement with mathematical concepts.



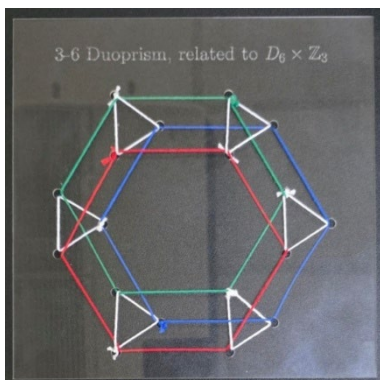
(a)



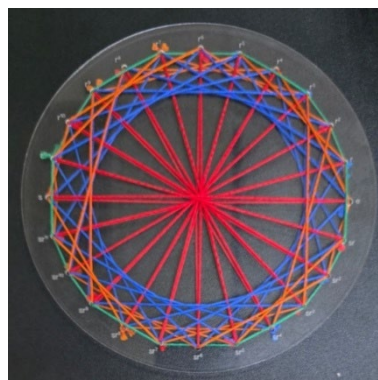
(b)



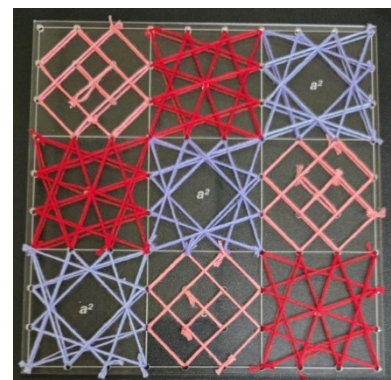
(c)



(d)



(e)



(f)

Figure 4.1 Laser-cut string art pieces: (a – b) created by the authors' research group, and (c – f) submitted by students from an undergraduate abstract algebra course.

A slightly modified version of the student activity described above was implemented by the first author in his *Fundamental Concepts of Algebra* class during the second semester of SY 2024 – 2025. This undergraduate course in abstract algebra, taken by mathematics majors, focuses on group theory. Photos of the laser-cut acrylic pieces submitted by the students in this class are shown in Figures 4.1(c–f).

Each string art piece is accompanied by a brief written explanation detailing the selected abstract group and the mathematical properties represented. This written component ensures that the mathematical meaning behind the visual design is preserved, while also providing students with an opportunity to reflect on how the creative process deepened their understanding of group theory.

Overall, the outcomes of the activity highlight the students' ability to synthesize theoretical knowledge with artistic expression and technological practice. Through the creation of these string art pieces, students were challenged to engage with the algebraic structures discussed in class and translate these abstract ideas into visual representations. Their work demonstrated innovative applications of string art to express a variety of mathematical ideas, extending beyond abstract algebra and connecting to other areas of mathematics. For example, in Figure 4.1(c), one student pair represented the Cayley graph of the group $C_2^2 \times C_3^2$ through string art. In Figure 4.1(d), another pair illustrated a projection of the 3-6 duoprism, a 4-polytope, using string arrangements to highlight its symmetry group.

5. Conclusion

This paper has presented a novel approach to integrating mathematics, digital design, and laser fabrication through the creation of laser-cut string art. Implemented in an undergraduate abstract algebra course, the activity engaged students in exploring algebraic structures by designing and producing their own string art pieces. This hands-on experience aimed not only to deepen their conceptual understanding but also to foster creativity, highlighting the educational value of blending mathematics with technology and art.

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