Using Technology to Develop Inquiry-Based Mathematics Teaching Models

Yuan Yuan
yuanyuan@mail.ntcu.edu.tw
Department of Mathematics Education
National Taichung University of Education
Taiwan

Abstract

This paper explores two models of applying technology to mathematical inquiry. The first model is teacher-guided, aiming to help students discover and understand mathematics through model-based learning. The second model adopts a student-centered perspective, where learners actively construct problem situations to explore mathematical concepts. Both models have been shown to effectively enhance students' mathematical learning. Moreover, using digital tools, students can deepen their understanding of mathematical problems. In traditional mathematics classrooms, instructional approaches often present solutions too quickly, leaving students disengaged from the problem context and missing the joy of mathematical exploration. Integrating technological tools into mathematical inquiry can help students apply mathematical concepts, develop computational thinking, and utilize powerful technological functions to explore and solve complex and unfamiliar mathematical problems.

1. Introduction

The primary goal of school mathematics curricula is to provide students with meaningful mathematical content that fosters problem-solving and critical thinking skills. Inquiry is widely recognized as a central driving force in learning; through inquiry, students are more likely to develop a deeper understanding of mathematics and strengthen their mathematical reasoning (see [4]). However, traditional mathematics instruction often falls short in offering students opportunities for genuine inquiry.

In many mathematics classrooms, teachers assume the role of expert problem-solvers, demonstrating procedures and solutions in a structured and linear fashion. While such instruction may appear efficient, it frequently leaves students disengaged, uncertain about the purpose of mathematical proofs, and struggling with problem-solving—often resulting in a sense of learned helplessness. Gradually, students may become passive participants in the classroom, losing interest in mathematics altogether.

Inquiry-based teaching refers to teachers' intentional design of activities to guide students through knowledge construction and conceptual understanding processes. A comprehensive inquiry process typically involves four key stages: formulating conjectures or hypotheses, devising plans and strategies, collecting and analyzing data, and drawing conclusions. To illustrate the contrast with traditional instruction, consider the following example commonly seen in classrooms: a teacher presents a problem on the board and proceeds directly to its solution.

Given the circle C1: $(x-2)^2+y^2=1$ and the vertical line $L: x-5=0$

Find the equation of the parabola on which the center of a circle lies, if this circle is tangent to both C1 and the line L.

After presenting this problem, the teacher immediately began solving it based on the given conditions and proceeded to elegantly derive the two equations of the parabolas as the final answer. However, in such a teaching process, students are deprived of the opportunity to explore and to formulate their conjectures. The teacher quickly advanced to the stages of planning and executing a solution strategy, bypassing the initial phases of sense-making and inquiry. As a result, even though students may be able to follow the algebraic procedures and arrive at the correct answers based on the geometric relationships in the problem, they may still lack a genuine understanding of the problem context and its purpose.

To address this instructional issue, it is essential to first provide students with opportunities to explore and make sense of the problem themselves. Only then can they begin to develop meaningful engagement with the task. Technology can play a critical role in supporting this kind of mathematical exploration, enabling students to visualize, manipulate, and experiment with the problem in ways that promote more profound understanding. These technologies offer students opportunities to interact with mathematical objects in ways that are impossible with paper and pencil (see [1]).

The purposes of this study are threefold. First, it examines how inquiry-based mathematics teaching can be meaningfully supported through integrating technological tools, enabling students to engage in conjecturing, planning, analyzing, and drawing conclusions. Second, the study aims to illustrate two complementary approaches to technology-enhanced inquiry—learning with models and learning to model—and to provide concrete classroom examples demonstrating how each approach fosters students' exploration and conceptual understanding. Third, the paper intends to contribute to the design of mathematics learning activities by highlighting the teacher's role in orchestrating inquiry tasks that balance guidance with student autonomy, thereby cultivating deeper engagement, reasoning, and problem-solving skills.

2. Learning with Models

In mathematical exploration, observing invariance within dynamic transformations is often essential. For example, consider that the sum of the interior angles of any triangle is always 180 degrees. In elementary school classrooms, teachers often ask students to measure and record the angles of various triangles. While this activity may lead each student to obtain a sum close to 180 degrees, even when performed by the entire class, the result only confirms the property for as many triangles as there are students. Moreover, students must contend with potential measurement errors during the process.

By contrast, with dynamic geometry software, a triangle displayed on the screen can be freely manipulated, altering its three angles dynamically—yet the sum of the interior angles remains unchanged (see Figure 1). Through such technology, teachers can design learning tasks that reflect the conditions of the given mathematical situation and guide students to explore specific phenomena through direct manipulation. This approach exemplifies learning mathematics with models—a technology-enhanced mathematics teaching application that enables students to observe and investigate mathematical invariance more effectively.

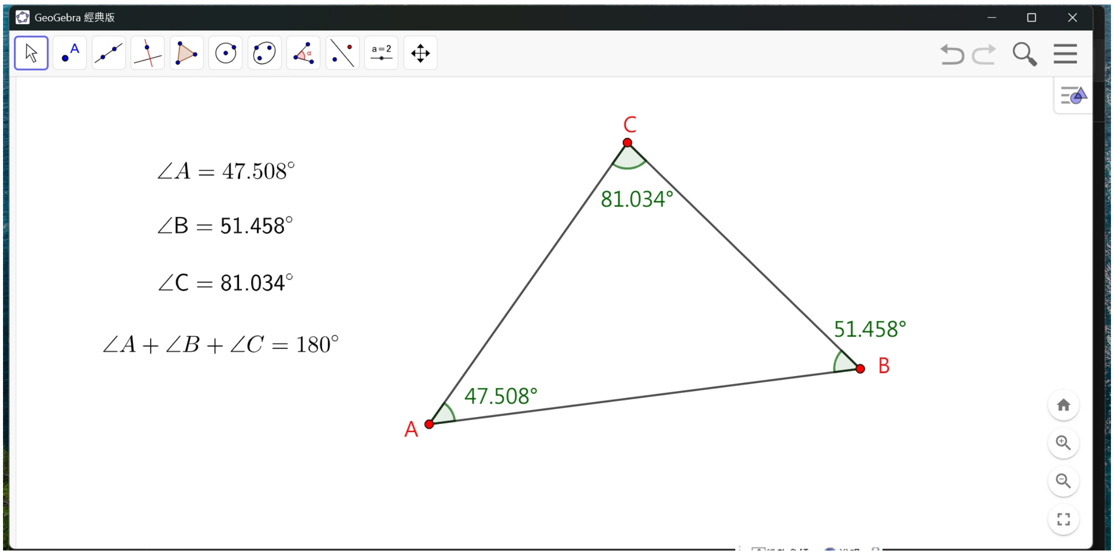

Figure 1: A dynamic triangle showing constant angle sum

To further illustrate the idea of learning mathematics with models, two examples where students explore mathematical concepts through model-based activities will be discussed as follows:

2.1 Area Problem Involving Pathways within a Rectangle

The question discussed below is a commonly explored problem in Taiwanese elementary mathematics textbooks. The scenario involves a rectangular plot of farmland with a given length of 12 units and a width of 10 units. Two straight pathways of equal width are constructed within the rectangle (the width is 1 unit, i.e., $\overline{EF}$ 1). The goal is to determine the available land area for planting crops (see Figure 2).

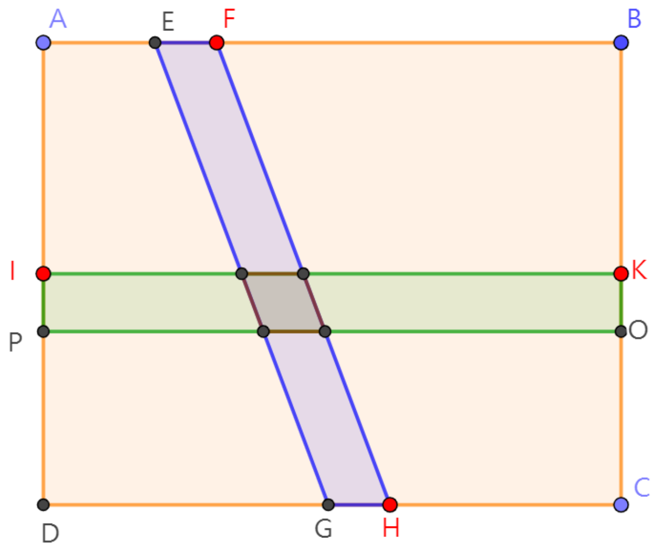

Figure 2. Area Problem of Plantable Land within a Rectangle

When solving this problem, it may seem natural to combine the four small plots of farmland into a single rectangle and calculate the area accordingly (see Figure 3). However, this consolidation method does not always hold in every scenario. For example, different pathway placements can affect the area for planting crops.

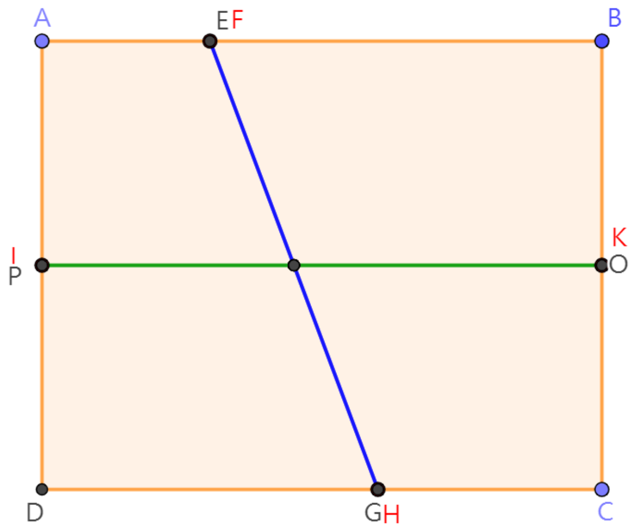

Figure 3. Combining the Four Small Plots into a New Rectangle

To investigate this issue, dynamic geometry software can be used to create a model simulating the problem situation. Through exploration, we find that not all pathway configurations allow the four smaller quadrilaterals to be perfectly rearranged into a rectangle. Sometimes this is possible (see Figure 4), while other times the four quadrilaterals overlap (see Figure 5), and in some cases gaps appear between them (see Figure 6).

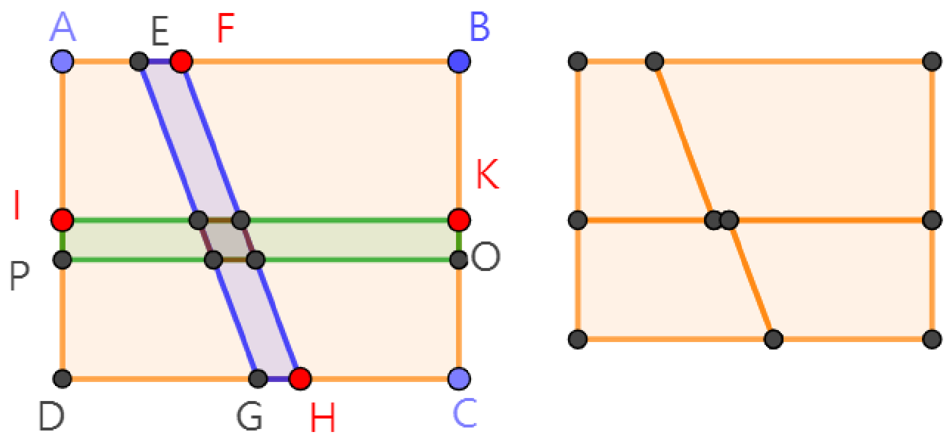

Figure 4. The Four Small Plots Perfectly Combined into a Rectangle

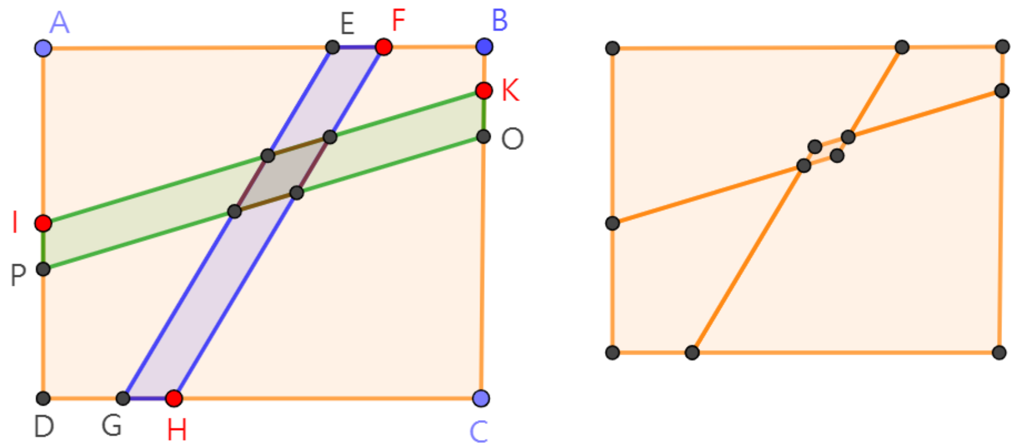

Figure 5. Overlapping Areas After Combining the Four Small Plots

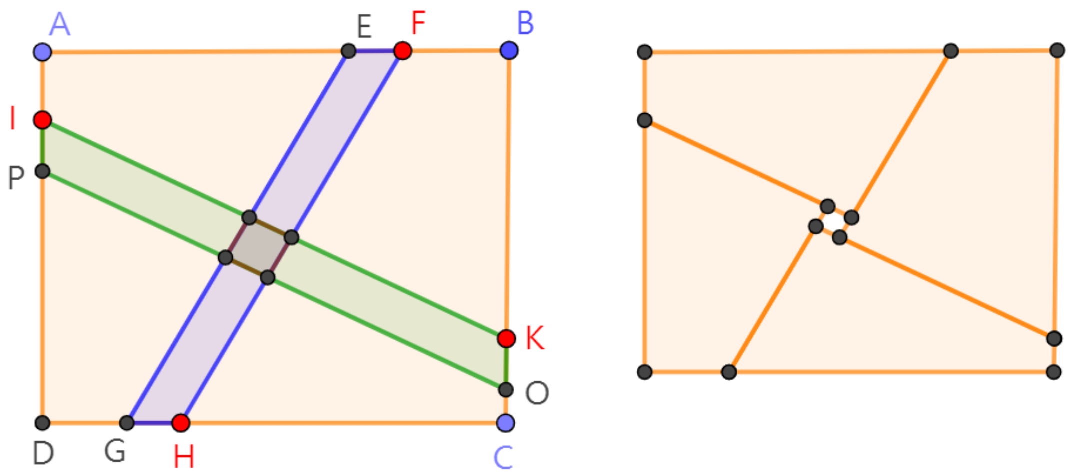

Figure 6. Gaps Appearing After Combining the Four Small Plots

By manipulating the positions of the pathways, we can observe that as long as one of the pathways is perpendicular to the original rectangle's side, the four quadrilaterals can be combined into a complete rectangle. Otherwise, overlapping areas or gaps will occur. This intriguing exploration also serves as a reminder for elementary school teachers to carefully consider the arrangement of pathways when designing assessment problems.

If teachers want students to explore this problem, they can use A4 paper to simulate the scenario. By drawing configurations where the four plots cannot be combined into a rectangle and asking students to cut out the pathways and rearrange the four quadrilaterals, students can better understand that not all cases can be solved by simply combining the plots into a rectangle to find the farmland area.

In this example, students begin by predicting whether the farmland can always be reorganized into a neat rectangle after removing the pathways (Conjectures/Hypotheses). With guidance, they decide to test multiple pathway configurations, first by drawing or cutting shapes, then by manipulating dynamic geometry software (Plans/Strategies). Then, students gather "evidence" by observing overlap, gaps, or perfect alignment in different configurations. They notice a condition: perpendicular pathways yield neat rectangles, while angled ones do not (Data Collection/Analysis). Finally, students conclude that rearrangement into a rectangle is not universally possible; it depends on the orientation of the pathways (Conclusions).

2.2 Finding the Largest Vertex Angle among Triangles with the Same Base and Height

In elementary school mathematics, the concept that shapes with the same base and height have equal areas is fundamental. While exploring this problem, we observed a special phenomenon among these triangles with the same base and height. Teachers can create a model based on this problem scenario to facilitate student exploration (see Figure 7).

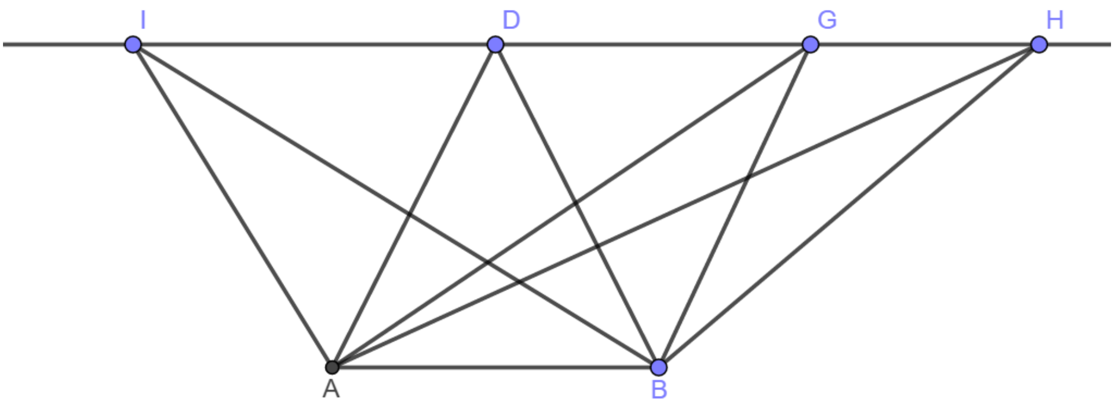

Figure 7. Four Triangles with the Same Base and Height but Different Vertex Angles

In a dynamic geometry learning environment, students can easily observe that although triangles with the same base and height have different shapes, their areas remain equal due to the constant height. During the exploration, we also discovered that while these triangles differ in shape (i.e., they have equal area but are not congruent), the vertex angle tends to decrease as it moves further away from the perpendicular bisector of the base segment AB. For example, in Figure 7, angles H and I are relatively smaller.

This observation raises the question: Is there a largest possible vertex angle among these triangles? Through dynamic manipulation, students can sense that the closer the vertex is to the perpendicular bisector of segment AB, the larger the vertex angle becomes.

Building on this exploratory finding, students can formulate the conjecture that among triangles with base AB and fixed height, the isosceles triangle has the largest vertex angle. Regarding the proof, we can draw a circle passing through points A, B, and D. Once the vertex moves away from point D, one can observe an angle equal in measure to ABD (namely, AHB), which is an exterior angle of triangle GHB. Therefore, AHB is greater than AGB, and consequently, ABD is greater than AGB (see Figure 8).

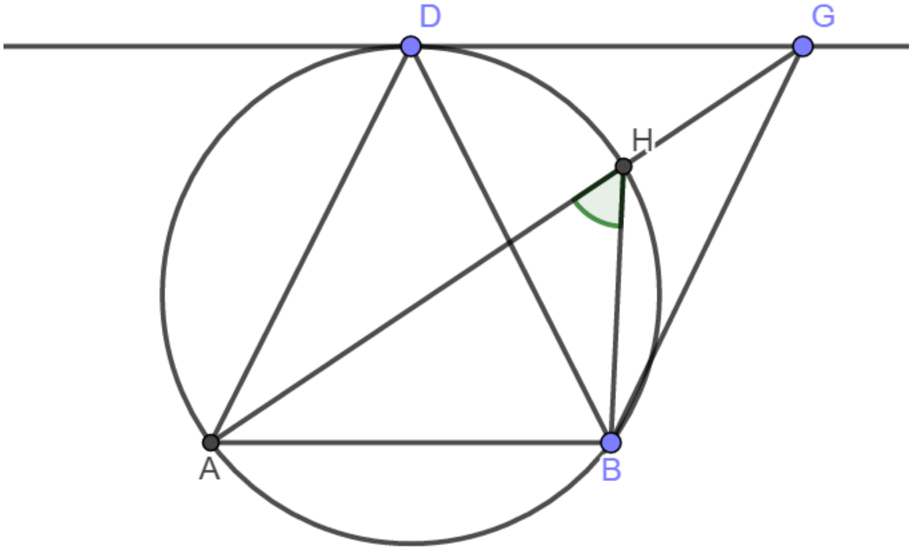

Figure 8. Dynamic Illustration Showing ADB as the Largest Vertex Angle among Triangles with the Same Base and Height

In this example, students first predict which triangle shape yields the largest vertex angle when base and height are fixed (Conjectures/Hypotheses). They manipulate a dynamic geometry model to move the vertex point while maintaining the height, tracking how the angle changes (Plans/Strategies). And repeated observation suggests a pattern: the vertex angle increases when the vertex approaches the perpendicular bisector. Students collect empirical evidence by comparing extreme positions (Data Collection/Analysis). Finally, they confirm (and later prove) that the isosceles triangle maximizes the vertex angle.

In the two examples above, the instructor first constructs the problem scenarios and guides students to perform targeted operations during instruction (such as specifying the movement of certain dynamic points for observation). Alternatively, the teacher may manipulate the model themselves to lead students in exploring possible outcomes of the mathematical problems. Based on these observations, students are encouraged to formulate and verify conjectures. This approach makes the subsequent verification process more meaningful and purposeful.

3. Learning to Model

In the process of mathematical problem solving, it is often necessary to draw diagrams in order to understand the problem context. Technological tools play a significant role in assisting students to represent information visually and to explore the relationships among different elements. Research has shown that many low-achieving mathematics students struggle with diagrammatic representation of problem information or make errors in this process, which hinders their ability to solve problems effectively (see [2]).

Suppose students can master the functions and features of technological tools and construct problem environments based on the problem context. In that case, they can better observe patterns and conduct exploratory problem-solving. Furthermore, during model construction, students may apply mathematical concepts and engage in mathematical thinking. The following two examples illustrate this approach.

3.1 Exploring the Quadrilateral Formed by Connecting the Midpoints of Any Quadrilateral's Sides

Traditional mathematics instruction has limitations when addressing statements involving "any" or "arbitrary" cases, since the term "any" refers to all instances satisfying given conditions. Teachers often present only a single example or a finite number of examples when representing this concept, making it challenging to capture the variability within the problem context fully.

In a dynamic geometry environment, students can display an arbitrary quadrilateral that can be continuously manipulated within the interface. The midpoints of each side can be immediately identified using built-in software functions (such as locating segment midpoints), and their positions adjust dynamically as the quadrilateral's side lengths change. Notably, the properties of segment midpoints remain invariant despite changes in the shape.

When the four midpoints are connected to form a new quadrilateral, students can manipulate the original quadrilateral and observe the characteristics of the newly formed figure (see Figure 9).

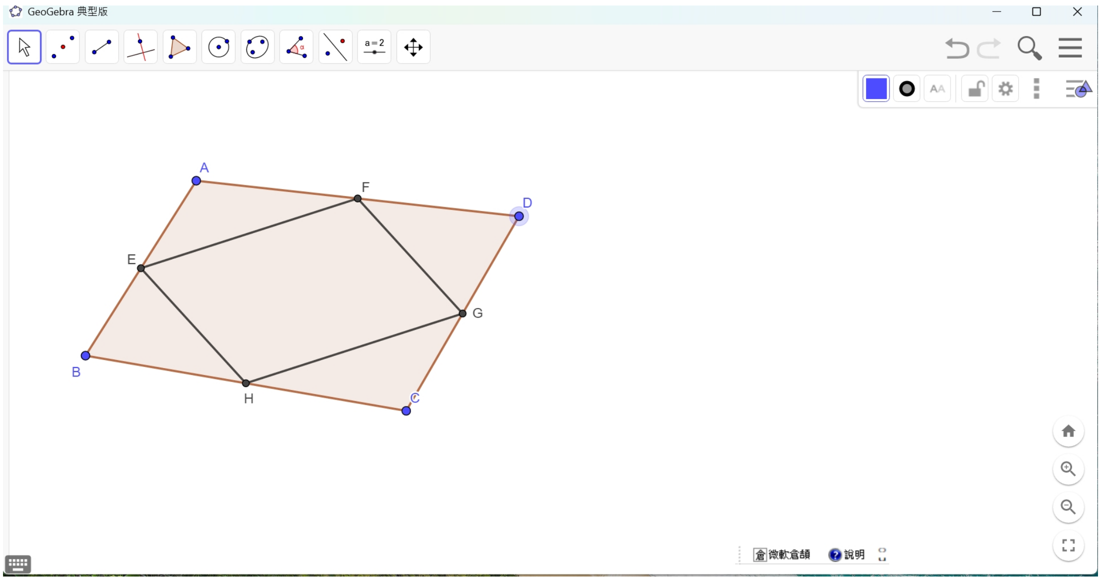

Figure 9. Exploration of the Quadrilateral Formed by Connecting the Midpoints of Any Quadrilateral

At the beginning, students wonder: What shape is formed by connecting the midpoints of any quadrilateral (Conjectures/Hypotheses)? Instead of checking just one or two quadrilaterals, they use dynamic geometry software to generate many arbitrary cases (Plans/Strategies). By dragging vertices, they observe that the new quadrilateral always "looks like" a parallelogram, no matter the parent quadrilateral **(**Data Collection/Analysis). Finally**,** they conjecture and justify that the midpoint quadrilateral is always a parallelogram (Conclusions).

3.2 Locus exploration

Why does the locus of points satisfying the given conditions form a parabola in the initial mathematical problem? Without initial exploration, students fail to develop an intuitive sense of the problem and struggle to understand its deeper meaning. With the aid of technology, students can construct the problem environment based on the problem statement. The construction process requires first considering how to find the circles that satisfy the given conditions (see Figure 10).

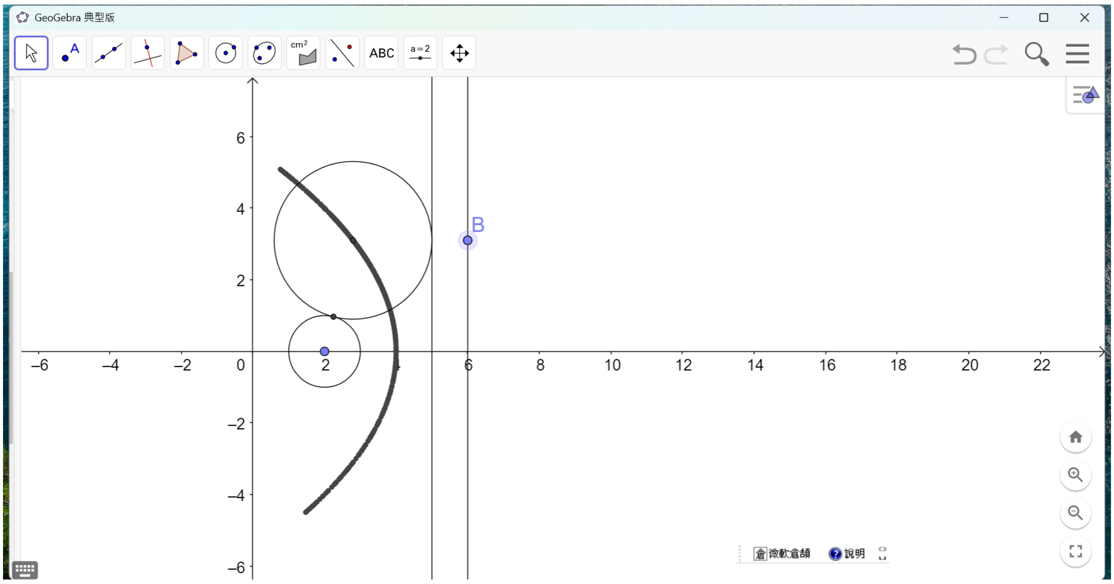

Figure 10. Constructing Circles That Meet the Problem Conditions and Displaying the Locus of Their Centers

To construct this model, students first must use the software to draw the unit circle with center at (2,0) and the vertical line x=5. Constructing a circle tangent to the given circle and the line presents a key challenge. Where would the center of such a circle be located? This becomes a focal point of the inquiry (requiring the identification of the line x=6 and the placement of a dynamic point on it), which parallels the considerations involved in solving the problem algebraically.

By constructing the model, students can understand the relationships among the geometric figures. This experience also facilitates subsequent algebraic problem solving. Furthermore, the model constructed with the support of digital tools helps students visually confirm that the locus of the circle centers is indeed a parabola.

In this example, instead of jumping into algebra, students hypothesize where the centers of tangent circles might lie (Conjectures/Hypotheses). They plan to construct circles using dynamic software and track the locus of their centers (Plans/Strategies). Students notice the emerging parabola by iteratively drawing tangent circles and observing the moving center. This experimental process provides a visual anchor and a bridge to symbolic reasoning (Data Collection/Analysis). Finally, students conclude that the locus of centers forms a parabola, which connects to the formal algebraic solution (Conclusions).

In the two examples presented above, students apply their existing mathematical knowledge and utilize the information in the problem to construct a model of the problem environment. By engaging with this model, they deepen their understanding of the problem and discover potential patterns or clues through observation and exploration. This approach lends greater exploratory significance and value to mathematical problem solving.

4. Discussion and Conclusion

In a Taiwanese high school mathematics textbook for the second year, there is a problem stated as follows:
"Do the graphs of $y = a^x$ and $y = \log_a x$ *intersect?"*

The textbook illustrates shows that when $0<a<10 < a < 1$, the graphs of $y=a^x$ and $y = \log_a x$ intersect at exactly one point. Therefore, answering this question is straightforward—the two graphs have an intersection point. However, if the problem is reframed as: For which values of the base a do the exponential and logarithmic functions intersect, and how many intersection points are there? The question becomes more complex and challenging.

Without technological assistance, exploration of this mathematical problem is limited. Students who become familiar with dynamic software tools can independently construct the problem scenario to form a learning model. By varying the value of a, they can investigate the intersection points of the two functions. This exemplifies the application of learning to model in mathematical inquiry. Alternatively, when the teacher constructs the problem environment and designs exploratory tasks—such as asking students to change the value of aa and observe how the intersection points vary—students engage in a step-by-step inquiry process to reach a solution, illustrating the learning with models approach.

While using technological tools to solve this problem, some students discovered the existence of three solutions in cases that are difficult to observe intuitively. This finding prompted further investigation by several mathematics educators. Readers interested in the detailed results of this study are referred to the article by Lee et al. (see [3]).

The four examples illustrate how inquiry-based teaching, supported by technology, can guide students through the essential stages of conjecturing, planning, data collection and analysis, and drawing conclusions. In the pathway problem, students' initial conjecture—that farmland plots can always be recombined into a rectangle—was challenged by systematic exploration with dynamic geometry software, leading them to refine their assumptions. In the vertex angle task, technology-enabled real-time manipulation revealed a generalizable pattern, prompting students to test and prove their conjecture about isosceles triangles. Similarly, in the midpoint quadrilateral exploration, the software allowed students to experience the meaning of "any case," observing an invariant property that naturally encouraged formal proof. Finally, in the locus problem, technology transformed a purely algebraic challenge into a meaningful visual inquiry, enabling students to hypothesize and confirm the parabolic locus before engaging in symbolic derivation.

The notion of instrumental orchestration [5][6] provides a valuable lens for interpreting the role of teachers in technology-enhanced inquiry. Instrumental orchestration emphasizes that the effectiveness of digital tools lies not in the tools themselves but in how teachers deliberately configure and manage their use during classroom activity. In the examples presented, such orchestration is evident: in the farmland pathway problem, teachers can direct students' attention to configurations that challenge initial assumptions; in the vertex angle exploration, they can guide systematic manipulation of the vertex and scaffold the transition from conjecture to proof; in the midpoint quadrilateral task, they configure dynamic environments to highlight invariance across cases; and in the locus exploration, they structure model construction to focus students on the emergence of the parabolic locus. Across these cases, instrumental orchestration illustrates how teachers' intentional guidance channels technological affordances toward meaningful mathematical inquiry, ensuring that digital tools serve as genuine instruments for reasoning rather than passive visual aids.

Drijvers [7] emphasized three crucial factors for the effective use of digital technology in mathematics education: the design of the digital tool and its pedagogical potential, the design of corresponding tasks, and the role of the teacher and the educational context. The examples discussed in this paper satisfy these criteria and illustrate how the interplay among tool design, task design, and teacher guidance can create powerful opportunities for mathematical inquiry. Importantly, technology does not replace the teacher but amplifies the teacher's ability to foster deeper reasoning within an inquiry-oriented classroom culture. In this way, the study underscores the potential of technology-enhanced inquiry models to enrich mathematics teaching and learning.

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