Mathematics Teachers' Perceptions and Practice of Computational Thinking

Keng Cheng Ang
kengcheng.ang@nie.edu.sg
National Institute of Education, Nanyang Technological University
Singapore

Marc Yi Fei Yeo
marc.yeo@monash.edu
National Institute of Education, Nanyang Technological University
Singapore

Abstract: *Assessing mathematics teachers' readiness for CT instruction requires an understanding of their CT beliefs, which is inadequately addressed in the literature. The current study fills this gap by examining (i) Singapore mathematics teachers' CT beliefs and conception of CT within mathematics pedagogy, (ii) ways which this conception impacts their teaching practices, and (iii) challenges they encounter in incorporating CT in mathematics instruction. The analysis of survey and interview data collected from in-service Singapore mathematics teachers revealed three main findings: firstly, pedagogy-related beliefs that (i) CT is beneficial for pedagogy, (ii) CT constitutes teachers' existing teaching practices, and (iii) CT is necessary/important for pedagogy; secondly, teachers generally have a limited conception of CT practices and their pedagogical implementation in mathematics lessons; lastly, teachers affirm that CT competence, designing CT lessons, and employing CT pedagogies are challenges in CT integration while also identifying other obstacles and possible solutions for these. This study therefore is an attempt to form an assessment of CT instruction among Singapore mathematics teachers, identifying gaps in CT expertise and obstacles that can be addressed as well as existing practices and beliefs that can be harnessed to facilitate CT pedagogy within a mathematics classroom.*

1. Introduction

Computational thinking (CT) has existed since the mid-20th century as algorithmic thinking [30]. However, it did not attract serious attention until Seymour Papert's [21, p. 23] groundbreaking publication *Mindstorms*, which characterized computers and programming as "objects-to-think-with" that develop procedural thinking, and later, Jeannette Wing's [31, p. 33] seminal piece, which promoted CT as a "fundamental skill" that everyone should possess.

Despite the interest in CT among researchers, there is little agreement on how to best define and assess CT [8]. However, amongst the numerous definitions, problem solving appears frequently [31, 32] alongside algorithms; pattern recognition, decomposition, and abstraction are also cited as critical CT aspects [9, 35].

Several CT frameworks have also been proposed. [28, p. 153] for instance, described CT as comprising decomposition (reducing problems into smaller parts), abstraction (teasing out essential elements), algorithms (constructing sequenced instructions for solving problems), debugging (identifying and resolving errors), iteration (performing processes recurrently), and generalization (adapting CT to solve problems across situations or domains). [4] alternatively suggested nine CT capabilities tailored for individual disciplines such as mathematics. These include data practices (data collection, analysis, and representation), decomposition, abstraction, algorithms, automation, parallelization, and simulation.

Although CT is described as a transdisciplinary approach to solving problems, its synergy with mathematics in particular has not gone unnoticed. In 1980, Papert realized the possibilities of using computational tools to concretize "abstract mathematical concepts" and construct "mental models to understand the world" mathematically [16, p. 890]. Since Papert, numerous similarities between CT and mathematics (more specifically, mathematical thinking (MT)) have been proposed in areas like problem solving and logic [3, 14]. CT has also been linked with mathematical topics like probability, algebra, statistics, numbers, and geometry [20, 22] and mathematical reasoning [15].

In Singapore, students are equipped with computational competencies through a computing subject, coding initiatives, and an emphasis on computational tools in the mathematics syllabus [27]. Its commitment to CT in mathematics education is further evidenced by a mathematics and CT specialization for pre-service teachers. Despite these efforts and other initiatives aimed at developing teachers' CT competence, little is known about the readiness of mathematics teachers in Singapore to engage in CT instruction.

To address this gap, the present study examines the views of in-service Singapore mathematics teachers on CT, and is guided by three research questions:

(i) What are Singapore mathematics teachers' perceptions of CT and their conception of how CT is developed in mathematics classrooms?

(ii) How does their conception of CT influence their pedagogical practice

(iii) What challenges have they faced in integrating CT in mathematics lessons?

This paper reports on the findings of this study, which builds on our previous study that examined mathematics teachers' perceptions of a professional development (PD) course aimed at developing computational problem-solving skills through VBA coding in Excel [2]. The preceding study underscored that understanding teachers' perceptions is crucial in creating PD programs [17, 23]. Given that such perceptions and teachers' level of preparedness for CT instruction form fundamental hurdles in CT integration [5, 34] and that the outcome of this research is to develop CT PD resources for mathematics teachers, assessing teacher beliefs about CT through the current study is an essential first step.

2. Mathematics teachers' understanding of CT

While there is significant research on teachers' perceptions of CT, studies that focus on mathematics teachers' views are limited. The bulk of research documents changes in teachers' CT beliefs and knowledge after participating in PD or teacher education programs, with a few that investigate these issues in isolation (i.e., without involving PD or teacher education courses). One such isolated study is [23], which examined elementary in-service teachers' perceptions of six CT aspects (algorithmic thinking, generalization, abstraction, automation, decomposition, and debugging). The results revealed that teachers associated CT more with mathematics instruction than science and cited a lack of instructional time and cultivating students' higher-order thinking as challenges of integrating CT into classrooms. Another one is [14] which found that the views of mathematics and computer science experts (including teachers) towards CT in mathematics education aligns with the literature in identifying CT components like pattern recognition, abstraction, generalization, algorithmic thinking, decomposition, and logical thinking in mathematics teaching, as well as recognizing problem solving and associated thinking processes as shared between CT and MT.

Some previous studies occurred in a PD or teacher education context. [10] demonstrated an epistemological and attitudinal shift in elementary pre-service teachers who participated in a CT- and mathematics-focused teacher education program with little to no prior CT knowledge – they became aware of the possibilities of incorporating CT into different subjects and how CT tools enhance mathematics learning, while shifting from worrying about CT instruction to embracing its challenges. Similarly, [22] examined how in-service mathematics teachers' perceptions of CT and its incorporation into mathematics education were transformed through PD, from a limited understanding of CT to a more well-formed one aligned with the literature, as well as an awareness of CT integration in mathematics teaching and its obstacles.

There are few studies on Singapore teachers' perceptions of CT, none of which focus solely on mathematics. [17] compared the CT perceptions of STEM and non-STEM pre-service teachers before exposure to CT, observing that the teachers did not possess sufficient knowledge of CT and mainly associated CT with logical thinking/reasoning, programming, technology/computer usage, and problem solving. [11] investigated how in-service teachers perceived CT within and across the contexts of mathematics and computing subjects.

There is, therefore, a gap in understanding mathematics teachers' views on CT and how these reveal their level of readiness for CT instruction. The current study addresses this by assessing the extent to which teachers regard CT as relevant to their pedagogy, whether they possess an adequate understanding of CT and its pedagogical integration, and their perception of barriers which impede CT instruction – areas which are still inadequately addressed.

3. Methodology

3.1 Recruitment of teachers and data collection

This study comprises quantitative and qualitative data from a survey and interviews collected from April to October 2024. The anonymous survey was distributed to mathematics teachers who expressed interest in the study during Professional Learning Community sessions. The survey consists of Likert-scale, multiple-response, multiple-choice, and open-ended questions on teaching experience, prior exposure to CT, beliefs about CT, and practices to integrate CT in teaching, as well as challenges faced in CT integration. Participating teachers were invited to indicate their interest in a follow-up interview, where they could discuss these areas in detail.

3.2 Data analysis

Quantitative findings were tabulated using the percentage of survey participants who: (i) agreed[1] with statements in Likert-scale questions; (ii) selected at least one option in multiple response questions[2]; and (iii) answered multiple-choice questions. The percentages are based on the number of participants (N) for each question, so although N(sample size)=110, N can differ between questions if participants skipped them. SPSS's Crosstabs and Spearman's correlation were employed to explore relationships between options in Likert-scale and multiple response questions. In particular, for the multiple response question that asks participants to indicate whether six given practices are related to CT, their teaching, and/or neither, five are commonly mentioned in CT definitions (algorithmic thinking (AT), pattern recognition (PR), decomposition (DEC), abstraction (ABS), and generalization (GEN)) while the last one, estimation (EST), although not cited as a CT practice, was selected for its links to CT as a crucial problem-solving skill involving a mental approximation of solutions to mathematical problems in the absence of calculations [1].

Where relevant, quantitative findings are corroborated by qualitative findings derived from interview responses and the survey's open-ended questions. 23 participants were interviewed. Through constant comparative analysis [29], codes initially assigned via open coding were refined

[1] Refers to participants who selected Strongly Agree, Agree, or Slightly Agree on a six-point Likert scale.
[2] Percentages were derived from the total respondents for a question vis-à-vis those who selected each question option.

via axial coding by linking them through shared features, after which finalized codes were generated using selective coding. Unless otherwise stated, only codes or themes ranked within the top five (by frequency of appearance) are presented in the findings.

4. Results and discussion

4.1 Participants' profile

Figure 1 presents an overview of the participating teachers' demographic information. 90% have at least 9 years of teaching experience, and the majority have taught Grades 9 to 10 (81%), followed by Grades 7 to 8 (68%), and Grades 11 to 12 (27%). Most have prior exposure to CT (76%), and all without exposure were interested in learning about CT (100%).

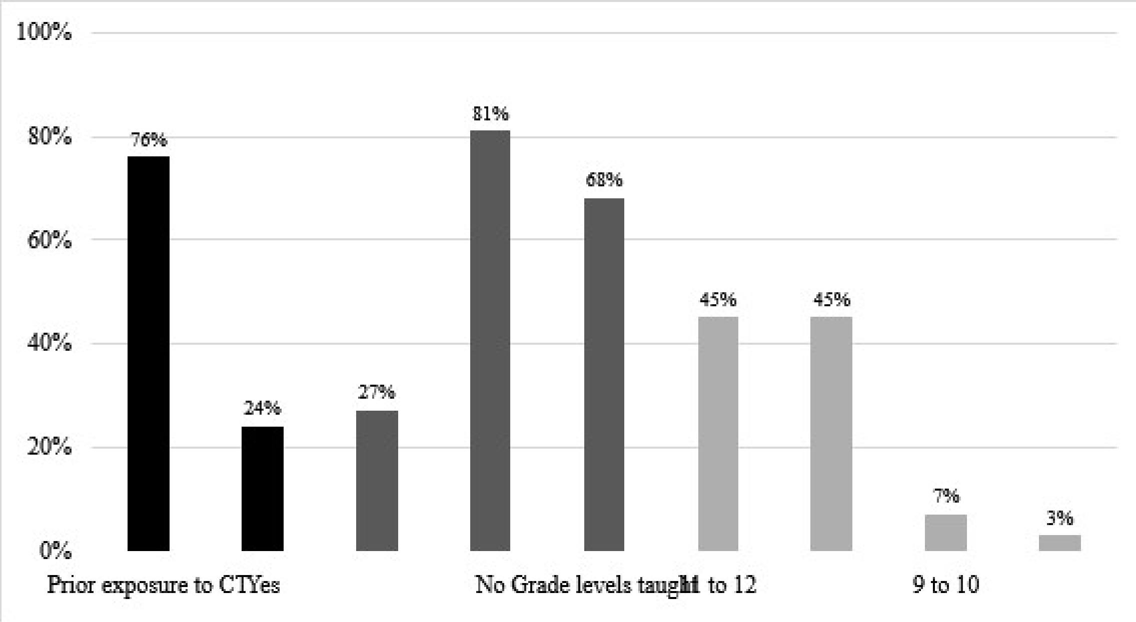

Figure 1. Participants' demographic information

4.2 Mathematics teachers' pedagogy-related CT beliefs

This study reveals some beliefs that teachers held about CT vis-à-vis mathematics teaching, and in this section, we highlight and discuss three such important beliefs: (i) that CT is beneficial for pedagogy; (ii) that CT is part of their existing teaching practices; and (iii) that CT is necessary and/or important for mathematics pedagogy.

4.2.1 CT is beneficial for pedagogy

There is strong agreement with statements regarding exposure to CT (see Figure 2), with a significant proportion of teachers agreeing that it (i) improves (85%) or could improve (96%) their pedagogy; and (ii) motivates them to develop their students' CT in mathematics learning (88%).

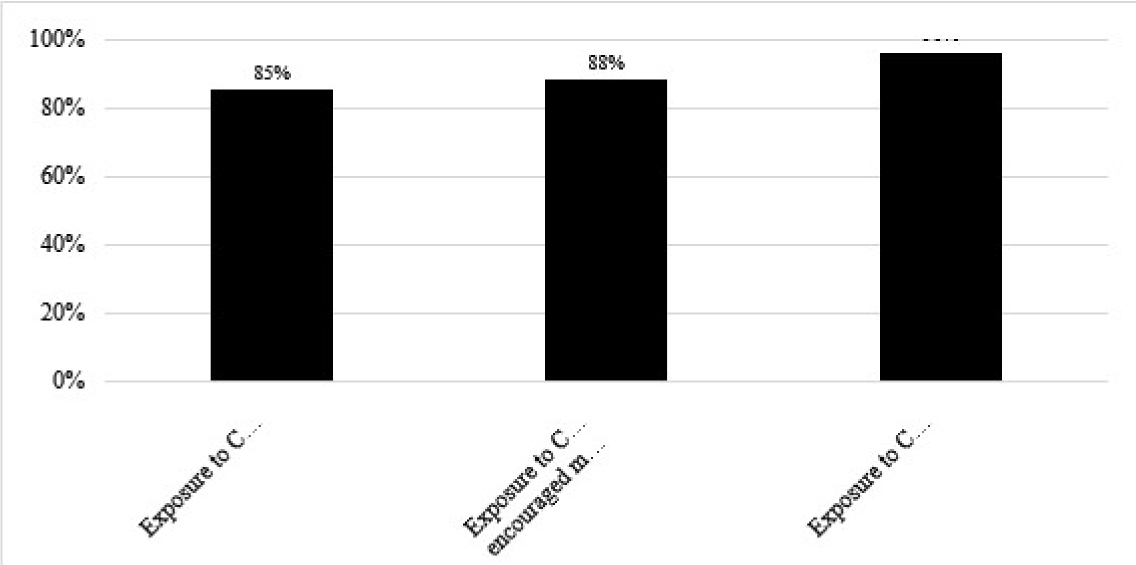

Figure 2. Level of agreement with statements about CT exposure

Such perceptions are supported by a Crosstabs examination of agreement with two statements about the pedagogical advantages of exposure to CT and selection of the six given CT practices as part of participants' teaching. In this Crosstabs computation, we calculate the frequency of responses containing multiple categories, which in Figure 3 refers to responses that agree with the statements and choose CT practices as part of mathematics teaching. A high percentage of those who agreed with the statements also selected the practices as part of their teaching (85-87% and 95-96% respectively for each statement), suggesting that perceptions of CT's positive effects in teaching could influence whether teachers saw CT practices as pedagogically relevant.

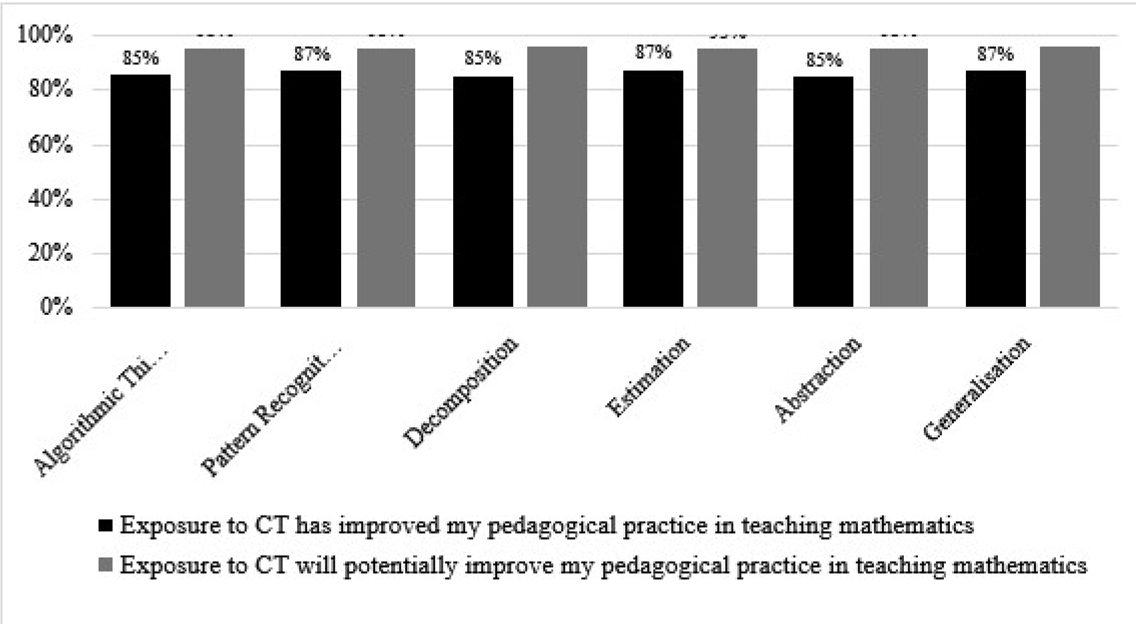

Figure 3. Percentage who agreed with statements about pedagogical benefits of CT and selected CT practices as part of their teaching (Crosstabs)

Spearman's correlation analysis of three statements regarding exposure to and application of CT in mathematics teaching (see Figure 4) implies that perceptions of CT's pedagogical impact could be connected to teachers' self-efficacy in applying CT and motivation/willingness to develop students' CT, as the average coefficient value of 0.7 ($p<0.001$) indicates a strong, positive relationship.

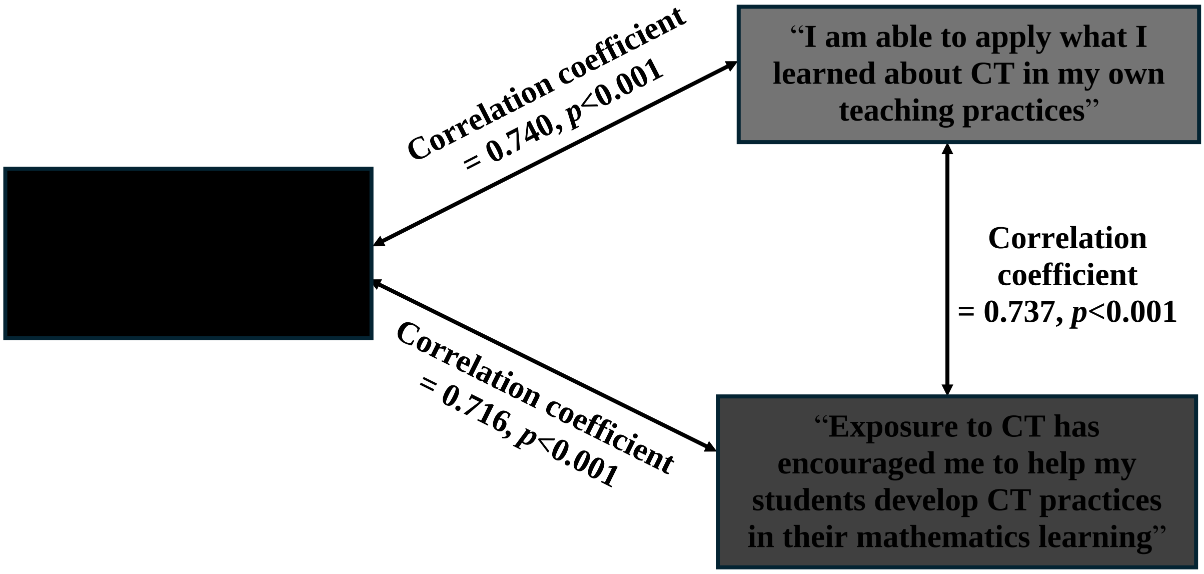

Figure 4. Spearman's correlation analysis of statements about CT exposure and applying CT

It is worth noting that the strong relationship amongst the statements shown above, while not causal, reflects the positive attitude and confidence of the teacher participants in this study. In their view, the exposure they had with CT has made them able to apply CT in their practice and improved their mathematics teaching. They are also confident that such exposure has enabled them to help their students develop CT practices in mathematics learning.

Two main areas in which teachers perceived CT as beneficial for pedagogy are *problem solving* and *algorithm*, which are shared themes cited in interview responses to questions asking about the relevance of CT to participating teachers' teaching and its impact on students' mathematics learning. *Benefits learning* as the second most frequent theme pertaining to CT's impact on students' learning reveals a more detailed picture of ways in which teachers perceive CT favorably in teaching. Two prevalent subthemes and their interview extracts are:

- Better understanding of mathematical concepts/processes
  - "I think [the use of flowcharts] helps them to understand certain concepts better and processes as well, mathematical processes." (Interview 17)
- Correct misunderstandings about mathematics
  - "I find that their understanding of math is a misunderstanding that it's a lot of calculation, and if we teach them CT, maybe it can help to shift them away from this misunderstanding" (Interview 11)

In Interview 17, the teacher participant mentions how flowcharts help his students better understand mathematics concepts and processes, while in Interview 11, the interviewee notes that CT corrects students' misconceptions that mathematics involves only calculations.

To illustrate what the teacher participant in Interview 17 meant by "better understanding of mathematical processes", consider the following example.

Example

Part 1: Design a flow chart to compute the sum,

$$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}$$

for some value of n which is provided by the user as input.
Implement on an Excel worksheet.

Part 2: Modify the flow chart in Part 1 above so that now, the value of n is not provided by the user. Instead, the user provides a small number ε, and the series is summed up to the point where the condition, $\frac{1}{n!} < \varepsilon$ is reached. Implement on an Excel worksheet.

The following are the possible flowcharts for the above problems.

In this example, the task is simply to compute the partial sum of a series. However, if we wish to use a machine or a computing tool to assist with the task, then we need to give instructions to the machine. These instructions are listed as an algorithm, and the flowcharts help to describe the process clearly, step by step. In some sense, flowcharts illustrate and clarify the algorithm used to solve a problem, or carry out a process, and they are often used before we write code on a computer to execute the process.

4.2.2 CT is part of existing pedagogical practices

A high percentage (71-93%) indicated that the six given CT practices are related to their teaching (see Figure 5), suggesting that many perceive these as part of their existing pedagogy.

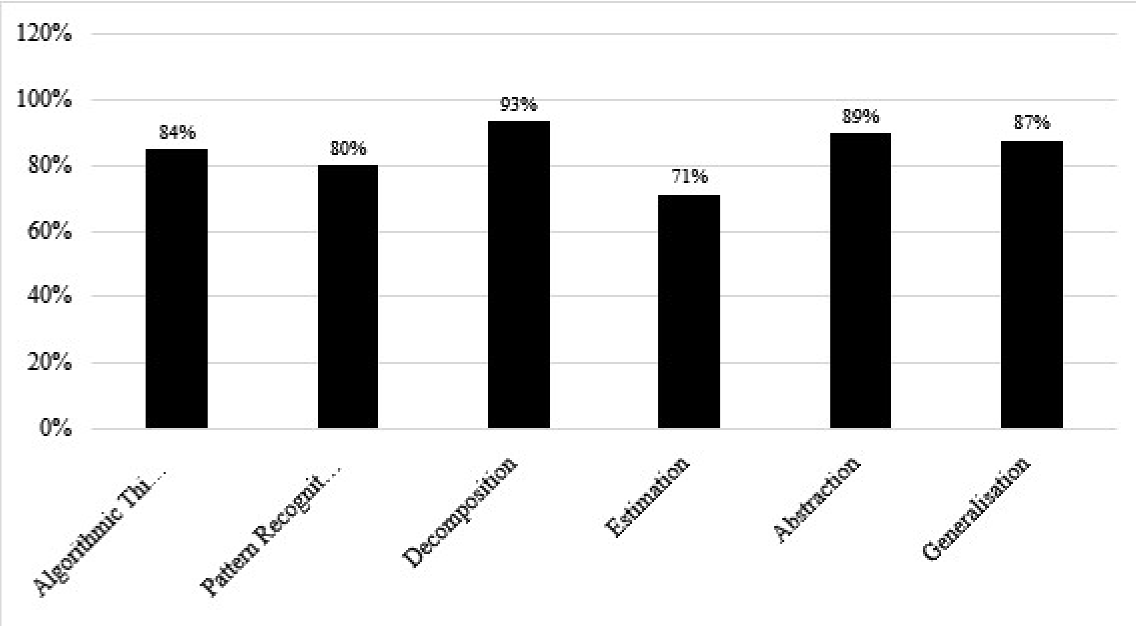

Figure 5. The percentage who indicated that CT practices are related to their teaching

This aligns with the "*Already part of pedagogy*" as the second most mentioned theme relating to CT's relevance in teaching. It has two prominent subthemes:

- Already part of teaching materials
    - "When I flip through the old textbook, they already got flowchart... So all the resources, even before CT, they are already in the books" (Interview 8)
- Already part of teaching practices
    - "These four categories actually lie very well in the subject of mathematics, because we have been doing all this." (Interview 23)

The teacher in Interview 8 mentioned that even before CT was popularized, CT resources like flowcharts were already in mathematics textbooks, while the teacher in Interview 23 noted that four aspects of CT (algorithmic thinking, pattern recognition, decomposition, generalization) tie in with mathematics because they have always been practiced in the subject.

4.2.3 CT is necessary/important for pedagogy

There is a high level of agreement (93-97%) with statements relating to the importance and necessity of CT in mathematics teaching for both teachers and students (see Figure 6).

This perception is supported by a Crosstabs examination in Figure 7 which shows that a high percentage of those who agreed with two statements about CT's importance also indicated that the given CT practices are connected to their teaching (94-97% and 96-98% respectively for each statement), suggesting that perceptions of CT's importance/necessity in teaching could influence whether teachers perceive CT practices as relevant to their pedagogy.

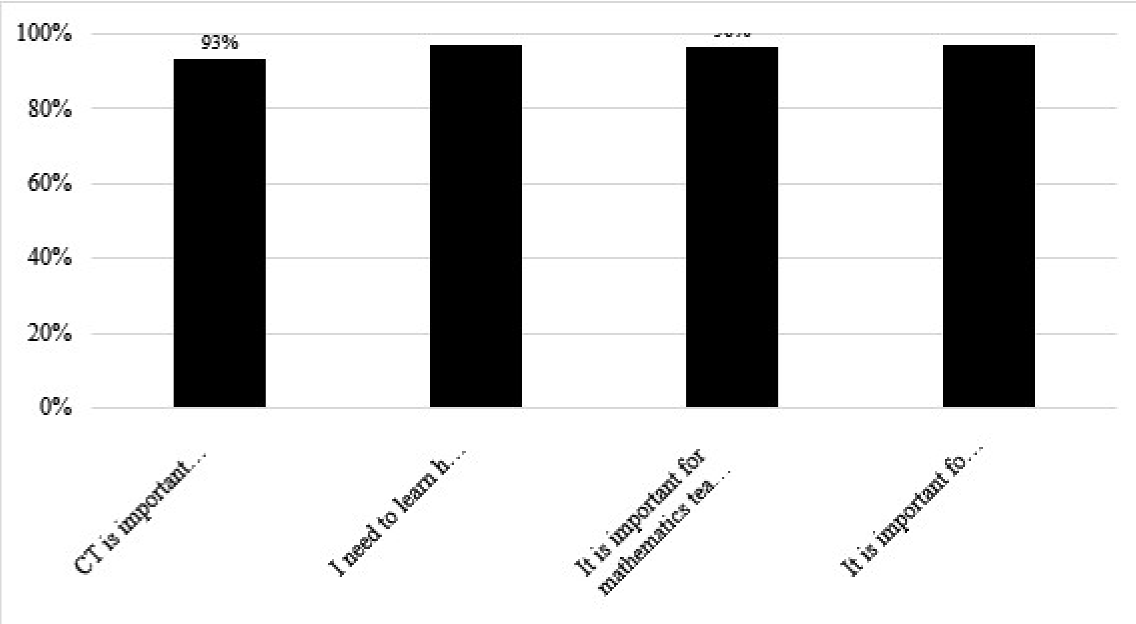

Figure 6. Level of agreement with statements about the importance/necessity of CT

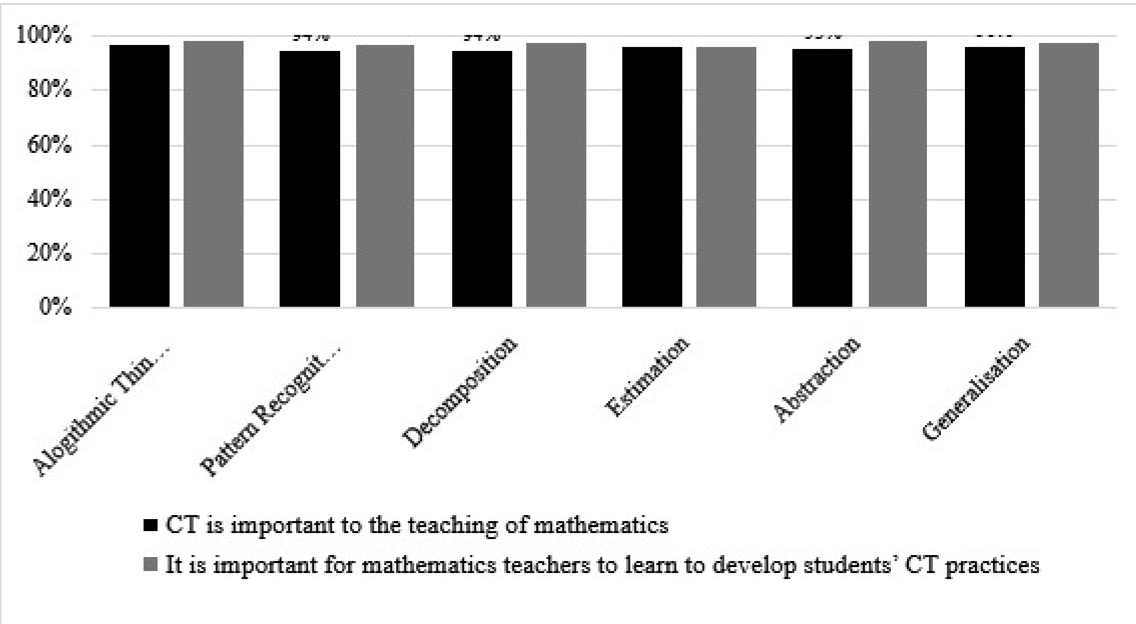

Figure 7. Percentage who agreed with statements about the importance of CT and selected CT practices as part of their teaching (Crosstabs)

4.3 Mathematics teachers' CT practices in the classroom

This section discusses how mathematics teachers in the present study handle or view the practice of CT in the classroom, and details how these conceptions of CT practice are influenced by their beliefs and perceptions of CT. Most of these insights are gathered from the interviews and open-ended responses in the survey, and are given within the context of the participating teachers' experience in and exposure to CT.

Problem-solving seems to form a very important part of the ways in which teachers described the pedagogical implementation of their practice in all the different aspects of CT. Two other areas where teachers felt they could implement CT are "*written words*" (as in written information in mathematics) and "*real-world context*" (mathematical problems in real-life situations). The latter two could be because of the teachers' understanding of "abstraction" as one aspect of CT and in interpreting "word problems" or real-life problems into mathematical problems, there is a certain level of abstraction involved. How this eventually pans out as CT was not well explained or elaborated in the interviews.

As for problem-solving, it seems to permeate different levels of CT integration, as far as these teachers are concerned. From the teachers' conceptualization of CT to their actual or envisioned implementation of CT in a lesson, this idea of problem-solving prevails. In other words, it appears the purpose and value of CT lies in problem-solving. While this is highly understandable, given that the Singapore mathematics curriculum has mathematical problem solving as a central focus, it is more important for teachers to be able to construct mathematical problems in which a computational approach is a natural way of solution.

Nevertheless, it was found that the participating teachers' knowledge and perception of CT are based on what they had been exposed to through self-study (e.g., via YouTube videos) and workshops or sharing sessions conducted as professional development for them. Mathematics teachers in Singapore do have access to "official" resources provided for them by the local authorities. In this case, the CT resources appear to focus on algorithm design, defining CT as comprising thinking skills for problem solving (which may explain the interviewees' responses as described earlier) as well as foregrounding flowcharts as a main way to engage in designing algorithms for mathematical learning.

The impact of these resources in shaping mathematics teachers' CT conception and pedagogical practices is reflected in the aforementioned features, which teachers characterized as CT and how the majority (18 out of 23) of teachers interviewed talked about flowcharts vis-à-vis mathematics pedagogy (including Flowgorithm, an application software for testing flowcharts) and reported employing flowcharts in class. An example of teachers' focus on flowcharts is the extracts from Interviews 8 and 17 discussed earlier.

Amongst the interviewees, 8 of them connected the use of flowcharts primarily to algorithms, 11 described flowcharts as facilitating a systematic/logical way of thinking that guides decision making in problem solving, and 4 noted that flowcharts make this thought process more "visible" to students. The teacher in Interview 12 went a step further to showcase actual flowcharts used in class for the topics of estimation (i.e., Heron's method as a form of algorithmic thinking in estimating square roots) and trigonometry (i.e., applying the sine or cosine rule given the lengths and angles in a triangle).

In terms of CT implementation, teachers do face some obstacles and challenges. Our findings (see Figure 8) affirm that participants agree to a high level (97%) with statements about CT competence, designing CT lessons, and adopting CT pedagogies as obstacles in CT instruction. These were included in the survey as they are frequently mentioned in the literature [13, 19]. Spearman's correlation analysis of these statements reveals coefficient values of 0.6 and 0.7 (p<0.001), implying a strong, positive relationship between the challenges.

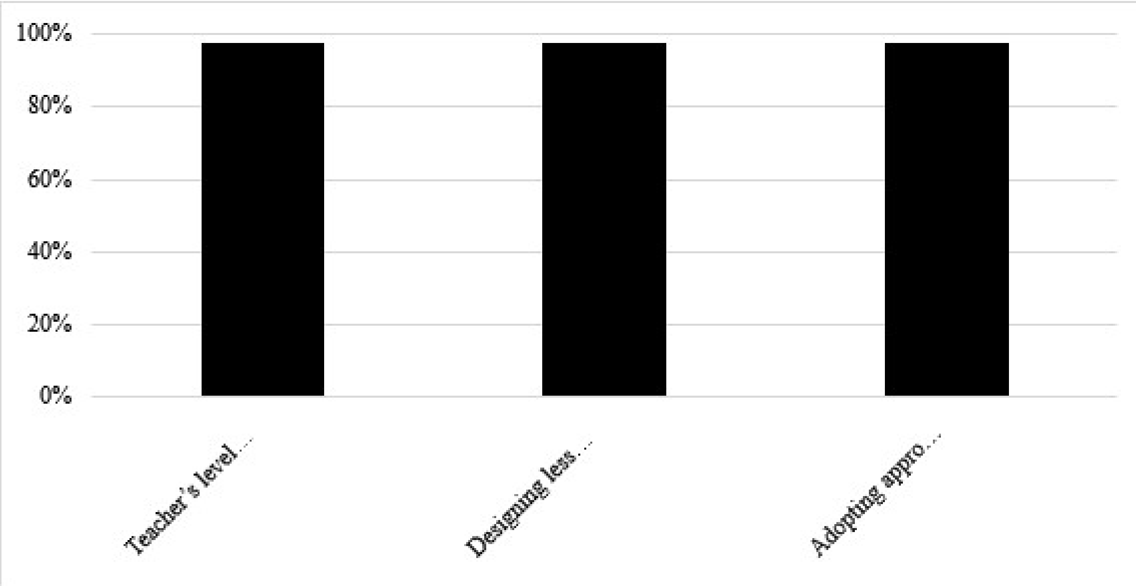

Figure 8. Level of agreement with statements about challenges of CT integration

The results shown in Figure 9 highlight another challenge: a shortage of resources for CT instruction. While an overwhelming majority (87%) agree that the availability of resources is crucial in developing students’ CT, only slightly over half (54%) agree that they can access such resources.

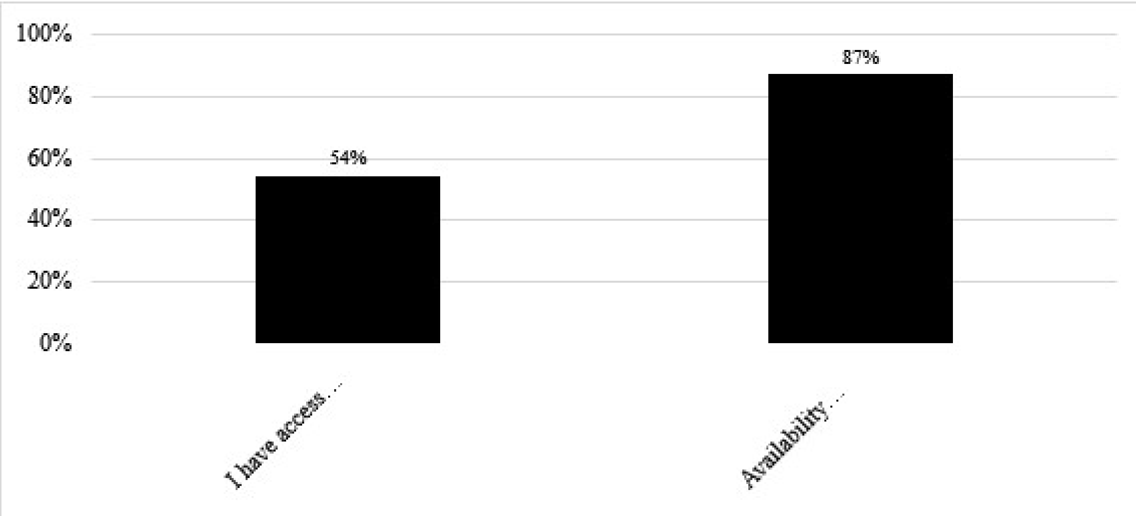

Figure 9. Level of agreement with statements about CT pedagogical resources

Interviewees also proposed other barriers (see Table 1), showcasing diverse curriculum-related (assessing CT and time needed to integrate CT), pedagogy-related (uncertainty/difficulties about and lack of CT integration), proficiency-related (high skill floor of CT resources), and attitude-related (limited perception of CT’s importance/benefits and convincing teachers/students to adopt CT) obstacles. Proposed solutions included developing CT competence, providing customized resources, inducing attitudinal changes, and pedagogical interventions.

Table 1. Suggested challenges of CT integration and possible solutions

Challenges (other than in the survey)	**Solutions**
CT competence	-Cultivate teachers' CT competence -Availability of CT resources
Lack of CT integration efforts	-Targeted integration of CT -Explicit integration of CT
Teachers' uncertainty/difficulties about CT instruction/integration	-Support and reassure teachers about CT integration
Time needed to acquire/incorporate CT	
Assessing students' CT learning	
High skill floor to learn to operate CT resources	-Creating tailored CT resources (different levels, abilities, topics etc.)
Convincing teachers to adopt CT	-Increase teachers' buy-in regarding CT adoption -Develop CT initiatives within schools
Teachers may not perceive the importance/benefits of CT	-Make teachers see the benefits/value of CT

5. Conclusion

This study, which involved Singapore participants, unveiled aspects of mathematics teachers' CT beliefs in relation to mathematics pedagogy: (i) Views on the benefits of CT in pedagogy, existence of CT in current pedagogical practices, as well as the significance of and need for CT in teaching; (ii) An understanding of CT and how it is operationalized in classrooms; and (iii) Perceptions of diverse challenges in CT integration and their solutions.

These beliefs crucially underscore the importance of enabling teachers to see the relevance of CT to their teaching [18, 32] in terms of its advantages, importance, necessity, and synergies with their current teaching practices. Furthermore, the finding about teachers' conception of CT and its integration (for instance, characterizing problem solving as the core feature of CT and flowcharts as the dominant form of CT instruction despite existing diverse ways of CT integration in the literature – a decision likely influenced by resources teachers are exposed to) reflects a common phenomenon of an inadequate understanding of CT amongst teachers [6, 7] and suggests that there remains "much work to be done" in preparing teachers for CT instruction [26, p. 161]. Although this and the numerous challenges of CT integration identified by participating teachers imply that Singapore mathematics teachers do not possess a high level of preparedness for CT instruction, understandings of misconceptions, gaps in practice, challenges etc., gained through analyzing their CT beliefs could be useful in developing PD that addresses these issues [12].

Despite this limited conceptualization of CT, most teachers identified problem solving, algorithmic thinking, pattern recognition, decomposition, abstraction, and generalization as being part of CT and their lessons, which aligns with the most cited CT practices reported. This suggests that although teachers are aware of these practices, they do not appear to have a deep understanding of why these practices are considered CT and how to implement them in the classroom. Apart from these, aspects concerning the implementation of CT, such as "written words" and "real-world context" have been noted in other studies [24, 25].

This study does have a few limitations. Firstly, the findings were based on a relatively small sample and entirely voluntary participation. This means that the results might not be generalizable to the mathematics teacher community in Singapore. In addition, the self-selection process could mean that teachers might have participated based on their own perception that they know enough about

CT, while those who perceive themselves as lacking CT knowledge might have refrained from participating. This could result in some form of bias, although it may not be a significant factor.

Furthermore, there were some incomplete responses to the survey, and the reliance on self-reporting leaves the question of whether and how teachers' CT beliefs translate into classroom practices. For instance, many teachers reported that the given CT practices are part of their teaching, but it is not known whether and how they actualize these practices in the classroom.

Nevertheless, the findings have provided a snapshot of Singapore mathematics teachers' understanding and practice of CT, as well as identified gaps to be filled for more effective CT instruction. Further studies that involve, for instance, teacher observations of CT-based mathematics lessons, will be needed to confirm these findings and shed more light on how teachers' beliefs about CT translate into pedagogical settings [33].

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