Teachers' Narratives on Integrating GeoGebra for Enhancing Conceptual Understanding in Mathematics

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Abstract: *Integrating information and communication technology (ICT) in mathematics education has been increasingly recognized as a powerful tool for increasing students' interest and performance. It is widely discussed that students' understanding of mathematical ideas is not satisfactory, especially in terms of conceptual understanding. Using narrative inquiry as a research method, we examined the experiences of four secondary mathematics teachers—two men and two women—who successfully integrated GeoGebra while teaching geometry and function, an ICT application, into their regular mathematics classes. These classes typically consist of around 20 students aged 13 to 14 years. Through in-depth interviews and reflective narratives, this study explored the teachers' insights on how GeoGebra transforms the learning environment in school mathematics. The participants reported that GeoGebra significantly advances concept formation by allowing students to visualize and manipulate mathematical ideas and concepts such as geometric shapes and algebraic functions. This interactive engagement clarifies abstract ideas and accelerates students' curiosity and motivation, making learning more enjoyable. Further, participants observed that students are more enthusiastic and willing to engage with challenging mathematical problems when using GeoGebra. The study concluded with recommendations for teachers to use GeoGebra effectively in math lessons. It advocates for professional development programs to equip teachers with the necessary skills to integrate ICT tools like GeoGebra effectively and suggests avenues for further research to explore the long-term impact of such technologies on student engagement and learning outcomes.*

1. Introduction

Teaching mathematics at the school level in Nepal is largely conventional and aligned with one-size-fits-all approaches, such as explaining mathematical concepts by teachers and assigning some tricks and techniques to solve routine mathematical problems [2, 20]. This model of teaching and learning mathematics is reflected even in the result of the Secondary Education Examination (SEE). SEE results are not satisfactory in mathematics. More than 50% of students who appeared in the SEE exam failed in mathematics, with less than 35 out of 100 marks, in SEE 2024. Likewise, the results of the National Assessment of Students' Assessment (NASA) conducted by the Educational Review Office (ERO) were below 50% [6]. It shows that the national level of school mathematics is poor in terms of students' academic achievements in mathematics. It is widely discussed that teachers focus more on procedural knowledge of mathematics, in which students are prepared to solve routine mathematical problems following the predetermined steps [12]. The focus on the procedural aspect of mathematics lacks the students' conceptual knowledge [16].

A procedure or process is a methodical approach by which the correct answers must be accomplished. Our everyday routines often follow a predictable pattern, suggesting that human life might be seen as a routine system. Attempting to understand mathematics exclusively through memorization, following a step-by-step approach to find the correct answer, or relying heavily on pre-defined algorithmic problem-solving methods leads to the development of procedural knowledge [15]. Typically, students create a routine and adhere to it from the start to the end of their day, even when they go to bed. We require a systematic arrangement of all elements, which should be carefully organized. Acquiring knowledge that is acquired through a systematic procedure is referred to as "procedural knowledge." This type of knowledge is obtained through a process, as defined by Hiebert and LeFevre [7]. Procedural knowledge refers to the practice of adhering to a tight and predetermined procedure or process in order to obtain a predetermined answer.

On the one hand, conceptual knowledge in mathematics is formed by building upon prerequisite concepts, ideas, and information. We progressively develop mental representations or schemas of something through interactions with the environment and various individuals. These schemas contain significant capacity to generate other notions. On the other hand, gaining a conceptual understanding of mathematics involves acquiring information about the underlying reasons and principles that explain why mathematical concepts and processes work in specific ways [16]. Such pertains not just to existing knowledge but also to how concepts can be understood. It enhances cognitive flexibility by fostering the development of divergent thinking skills, allowing individuals to consider alternative perspectives while approaching a topic.

Above all, it signifies that a student has many approaches to address the topic of mathematics. Conceptual knowledge refers to understanding definitions, formulas, and methods and the ability to justify them. A conceptual understanding of something (mathematics or other ideas) enables one to generate a logical and sound response. In this regard, the School Sector Development Plan 2016-2023 of Nepal prioritized the integration of ICT in teaching and learning methods [11]. This plan has facilitated the advancement of ICT in educational institutions and other governmental entities. The motives behind all of these policies are to incorporate and seamlessly integrate advanced technology into classroom instructions effectively. It will only be possible when teachers retain a high level of proficiency in the technology tools, equipment, and resources used in actual teaching methods. The Government of Nepal, the Ministry of Education, Science, and Technology, has changed the framework and expanded the use of ICT in education. These include using ICT for communication purposes and as a tool for learning. The utilization of ICT in education can be categorized into four major areas: communication in school administration, facilitation of teaching and learning, capacity building of stakeholders, and the inclusion of a separate discipline in the curriculum to enhance students' life skills. Joshi et al. concluded that ICT enhancement programs are needed for mathematics teachers at secondary schools in Nepal [9].

With the above gap, this article aims to explore the experiences of four mathematics teachers who have incorporated GeoGebra as one of their ICT tools in the classroom. In addition to GeoGebra, numerous ICT tools and applications are available for teaching mathematics. However, GeoGebra is gaining popularity worldwide, including in Nepal, for its ability to facilitate the visualization and manipulation of mathematical concepts. It is a web-based and offline application available in over two hundred languages, allowing users to interact with various mathematical concepts. GeoGebra is also known for its user-friendly interface.

2. Methodology

This study used a narrative inquiry approach to investigate the experiences of four secondary-level mathematics teachers who have used GeoGebra in their mathematical instruction at least five times or more in their mathematics classes. The narrative inquiry is highly appropriate for this research as it enables a thorough investigation of personal and professional experiences, documenting the unique events in which GeoGebra impacts teaching practices and student learning. The participants in this study were purposefully chosen based on their prior use of GeoGebra in their mathematics classrooms. The selection criterion comprised of a) a minimum of three years of teaching experience using GeoGebra at the secondary level, b) proven success in incorporating GeoGebra into their teaching, as indicated by students' verbal comments, c) willingness to engage in comprehensive interviews and share their reflective narratives. Following the three criteria, we carefully chose four participants, specifically two male and two female teachers from two secondary schools in Kathmandu Valley, Nepal. This selection was made to ensure a wide range of opinions

and selection of the research participants. Likewise, narratives were obtained through two methods: comprehensive interviews conducted by the researchers and reflective narratives provided by the participants. During the research process, every participant participated in two consecutive semi-structured interviews, each lasting around 60 to 80 minutes. The interviews were conducted casually, enabling participants to share their experiences and thoughts regarding their encounters with GeoGebra while teaching geometry and functions (to name but a few) based on the themes— (1) students' conceptual knowledge through visualization in GeoGebra, (2) student engagement and ownership and (3) students' learning using GeoGebra. Then, the participants were instructed to compose reflective narratives of their encounters with GeoGebra. These experiences offered further depth and intimate perspectives, enabling participants to express their experiences uniquely. The narratives centered around essential topics such as the acquisition of conceptual knowledge through visualization, student engagement, and the drive to follow further exploration. The narratives produced were subjected to a thematic narrative analysis procedure, which included recognizing, examining, and reporting recurring themes within the data. The scaffolding and More Knowledgeable Others (MKO) ideas of the social constructivist approach [21] have been used for analyzing the narratives and reflective writing of the participants. To ensure the reliability of the results, the study implemented various approaches such as a) Member checking: Participants thoroughly examined the transcripts and produced narratives to confirm the accuracy and authenticity of their reports. b) Triangulation: Data triangulation was done by gathering information from several sources (interviews and reflective narratives) and analyzing these stories to uncover logical patterns. c) Peer debriefing: The study team often held talks to question interpretations and guarantee a thorough analytical process. All participants provided informed permission, indicating their complete understanding of the study's objectives, methods, and entitlements. The participants' names and other identifiable information have been anonymized to ensure confidentiality.

3. Data Analysis and Interpretation

Three key themes emerged after carefully analyzing the participants' narratives and reflective writings. Data presentation and discussion are conducted under each theme. We discovered connections during our review of the theoretical and empirical literature.

3.1 Conceptual knowledge through visualization

A mathematics teacher with more than a decade of experience shared this way;

> *I am always on the lookout for new approaches to help my students understand complex topics such as geometry and function. When I first discovered GeoGebra, I was captivated by its ability to make abstract mathematical concepts more concrete. Over the last three years, I've used GeoGebra in my teachings, particularly when teaching mensuration, focusing on 3D objects and calculating their areas and volumes. In one notable lesson, I used GeoGebra to teach my students about cylinders' volume and surface area. Traditionally, students used to struggle to visualize formulas and grasp why they worked. I used GeoGebra to make a dynamic three-dimensional model of a cylinder. As I adjusted the model in real-time, modifying the height and radius, the students were able to observe how the volume and surface area changed. This visual and participatory approach meaningfully improved their knowledge.*

Next teacher shared about her student;

A student who has previously struggled with geometry showed amazing improvement. Throughout the class, I asked the student to explore GeoGebra on their own devices. He found more engaged as he experimented with the dimensions of various 3-D models. He shouted, "Oh, I get it now!" as he noticed that doubling the radius of a cylinder resulted in a fourfold increase in surface area but an eightfold increase in volume. This moment of exposure had an impact on the entire class, not just on her. I sensed that he developed a deeper level of conceptual understanding.

ICT integration in mathematics pedagogy improves conceptual understanding of mathematics and achieves a higher ability for Bloom's taxonomy, such as creating. However, including ICT in Bloom's taxonomy of learning stages in mathematics education is challenging. For example, suppose a teacher uses GeoGebra to teach mathematical concepts like parabola. In that case, the teacher should know what stages of learning may be completed and what the following stages of learning require. However, visualization techniques help teachers and students understand mathematical concepts and solve problems. There are different ways to develop visualization techniques. One of them is a concept map [24]. While using GeoGebra in teaching factorization, using algebraic tiles in GeoGebra helps students visualize the ideas of factors of algebraic expressions.

Atnafu and Zergaw mentioned that the process of developing students' mathematics visualization involves four steps: (i) creating lessons that incorporate visualization techniques; (ii) instructing students in mathematical concepts using visualization techniques; (iii) providing opportunities for students to practice visualizing mathematical concepts; (iv) assessing students' progress in visualizing mathematical concepts [1]. Next, Lim argued that visualizations are crucial ideas to maximize learning [19]. In one of the reflective notes of teachers, it was found that the students visualized the ideas of vertical height and slant height of a pyramid while showing them through GeoGebra. Furthermore, it was also widely discussed that technology can be used to scaffold mathematical problem-solving [25]. The ideas of Vygotsky, in which scaffolding is one of the effective ways of learning, are well connected to the use of GeoGebra in mathematics learning. Studies such as [4, 10, 18] further elaborated on the use of GeoGebra for visualizing mathematical concepts. For our study, visualization involves the interpretation of meaning from visualization objects or introspective visualizations in relation to "the person's existing network of beliefs, experiences, and understandings." [13, p. 26]. While using GeoGebra, secondary teachers shared that both interpretive and introspective visualizations happen for students.

3.2 Student Engagement and Ownership

The student's engagement and ownership were other themes generated based on the teachers' narratives and reflective notes. One female teacher wrote a reflection this way:

I have found GeoGebra to be a useful tool for promoting student engagement in my geometry course. One day, I assigned my students the responsibility of investigating the surface area and volume of a variety of three-dimensional shapes using GeoGebra. The class was an explosion of activity as students independently found relationships and manipulated shapes. Hem, a student who is known for his reserved behaviors, took the leadership role in his group and proudly shared his findings. In addition to enhancing the mathematical concepts, this hands-on approach additionally created a collaborative learning environment in which students felt empowered and took responsibility for their learning.

While conducting interviews with one male teacher, he responded.

I observed a major shift in my students' learning attitude when I introduced GeoGebra to calculate irregular objects' volume. Students, for example, Bibisha, who typically expected a series of instructions, began to experiment with the GeoGebra independently. Bibisha developed her 3-D models, changing the dimensions to observe their effect on volume. She proudly shared her discoveries with the class, showcasing a sense of ownership and confidence in her learning. The interactive nature of GeoGebra facilitated the development of a more personal and deep understanding of mathematics by allowing students to take charge and explore concepts at their own pace.

Engagement in mathematics is supposed to be an indicator of meaningful mathematics learning. Several researchers in Nepal [22, 23] argued that the lack of engagement in mathematics classrooms is one of the major issues in mathematics education in Nepal. We, as educators, have observed mathematics students for several years, and we have seen that students' engagement in mathematics learning is notably below average. There are several ways to make learning engaging. Ishaq et al. argued that ICT-integrated pedagogy provides an extended collaboration between students and creates ownership in learning [8]. Moreover, learners get several opportunities to explore problems engagingly [3]. Pynos concluded a study and concluded that the use of personal devices contributes to student engagement and that the use of devices by students to help with their learning is largely the same regardless of whether the item in question a personal device or a device is that the school provides [14]. In our study, teachers also mentioned that few students practiced GeoGebra on their personal devices parents at home as the use of GeoGebra grabbed students' attention in the school.

According to Vygotsky, the zone of proximal development (ZPD) is the distance between the actual developmental level of the learner of solving the (mathematical) problem independently and the level of their potential development as determined through problem-solving in mathematics under some guidance or collaboration with more capable peers and teachers [21]. In this study, the interactive exercise supported students' ZPD achievement by making them engaged in learning. In the reflection of a participant teacher, it was written;

Making geometric proofs through logical arguments was difficult for students. When I used GeoGebra and demonstrated the experiment, students started to argue from different perspectives. I felt that it helped to create ownership in learning as they started to search by themselves.

The teachers' narratives focused on how ICT-integrated pedagogy helps create ownership in learning. The use of ICT in teaching mathematics can make the teaching process more effective and improve the students' capabilities in understanding mathematics concepts.

3.3 GeoGebra Motives Students for Learning

This is another theme we developed from participating teachers' narratives and reflective notes. One of the major issues of school education in Nepal is the lack of students' motivation in general and in mathematics learning in specific.

In this context, a participating teacher shared;

The use of GeoGebra in my classroom has immensely affected my students' motivation. Teaching the properties of quadratic functions was one remarkable experience. Students were

enthusiastic about the instant visual representation of the equations they provided. For example, Anil, who usually had difficulties with algebra, was taken in by the capacity to manipulate the coefficients and observe the parabola's transformation in real-time. This interactive experience sparked his curiosity and transformed abstract concepts into tangible ones. He started his own experiments with various equations, driven by a desire to comprehend the fundamental principles. GeoGebra changed a previously considered difficult subject into an engaging and motivating learning experience.

In a similar way, another teacher wrote a short reflective note.

GeoGebra provides a source of inspiration for students and promotes individual inquiry. Following a session on the measurement of the capacity of prisms and cylinders, I observed that several students, including Maya, chose to remain after class to engage in additional experimentation with the program. Maya, who had previously expressed difficulties with mathematics, was now engaged in the creation of her own three-dimensional objects and following the calculation of their volumes. She shared her discoveries with her classmates, inquiring and establishing relations that expanded beyond the confines of the formal curriculum. Her independent investigation demonstrated a notable change in her attitude to learning. GeoGebra inspired her curiosity, motivating her to explore mathematical ideas more deeply.

From the above narratives and reflective notes, it was observed that students feel motivated when they get opportunities to interact with mathematical concepts through GeoGebra software. Rahman et al. conducted a study and found that students were observed to be drawn to the concept of self-access learning and the slider function during the learning process [15]. Moreover, mathematics teachers are encouraged to enhance their teaching and learning methods by utilizing GeoGebra, a dynamic mathematics program. They should build a learning module that integrates this software with written materials. This approach will be particularly beneficial for underachieving students, as it will help to raise their motivation in the classroom.

Mathematics is considered a difficult discipline, and most people do not have positive perceptions of mathematics [20]. One participating teacher wrote:

Students in my schools think that mathematics is made for smart people and can be taught and learned only by a group of smart people. It is too abstract for many people.

As per our experiences, society manifests mathematics as difficult and more abstract. This has demotivated students to learn mathematics. When students don't understand mathematical concepts, it leads to demotivation, and students consider mathematics difficult. On the other side, when teachers use different ICT tools such as GeoGebra, students make sense of mathematical ideas and gradually develop a conception that mathematics is easy and can be learned in a fun way. Thus, ICT-integrated pedagogy helped students to develop positive perceptions of mathematics. Dhakal and Sharma conclude that ICT is helpful in enhancing engagement and interaction [5]. ICT helps students to change their perception of it.

4. Final Remarks

First, the teachers' narratives demonstrated that GeoGebra helps students understand concepts by allowing them to observe and play, in some cases animating with geometric contents and algebraic functions (to name but a few). It helps to bridge the gap between abstract ideas and meaningful understanding. Second, the study found that using GeoGebra makes students interested and responsible for their meaningful learning of mathematical ideas and concepts. The interactive nature of the application motivates secondary-level students to take an active role, not just passively receiving information from the mathematics teachers but also actively constructing their knowledge in their context, pace, and speed. This kind of participation is very important for making the classroom more student-centered. It gets students more involved in the process of learning and makes them more responsible for their learning, as shared by the four secondary-level mathematics teachers. Lastly, one important theme that came up through the teachers' stories was how GeoGebra motivated students. Students experienced fun-filled learning while learning using software because of its dynamic and interactive features. In conclusion, using GeoGebra in secondary math classes in general and other mathematics classes is a comprehensive way to improve students' conceptual understanding, motivation, and involvement for quality learning and engagement. In the context of a highly technology-friendly society, it's becoming more and more important to use technology such as GeoGebra to support students and improve their teaching and learning process. The teachers' positive experiences in this study demonstrate that GeoGebra is useful for reaching these learning objectives by helping students do and perform better in math.

5. Limitations and Future Study

This study has several limitations. Firstly, the small sample size limits the generalizability of the findings. With only four secondary-level teachers participating, the findings may not represent the broader educator population. Additionally, the focus of the study on a specific group of teachers may introduce selection bias, as these individuals might have unique characteristics that do not reflect the diversity of teaching practices and experiences on integrating GeoGebra while teaching conceptual concepts in mathematics. To address these limitations, future research could expand the sample size to include a more diverse group of teachers from various educational settings. This would enhance the generalizability of the findings and provide a more comprehensive understanding of the phenomena under investigation. Moreover, incorporating a longitudinal design could offer insights into how the observed effects evolve over time, providing a deeper understanding of the long-term impacts. Furthermore, future studies could explore additional variables that may influence the outcomes, such as different teaching methods, student demographics, and school environments.

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