The Tangram: A Timeless Puzzle in Modern Education

Vanda Santos
vandasantos@ua.pt
Research Centre Didactics and Technology in Education of Trainers (CIDTFF),
Department of Education and Psychology, University of Aveiro and CISUC
Campus Universitário Santiago, 3810–193 Aveiro
Portugal

Abstract

The Tangram, a traditional seven-piece puzzle originating from China, has long captivated educators with its simplicity and complexity. Beyond its historical roots as a form of entertainment, the Tangram has found a significant place in modern education, leveraging its potential to enhance cognitive and spatial reasoning skills. This paper explores the integration of Tangrams in educational settings, highlighting their application in teaching mathematics, geometry, and fostering creativity. Additionally, it examines innovative approaches such as digital Tangram applications with Scratch. These advancements have redefined how Tangrams are utilized, making them more accessible and versatile for learners. The discussion underscores the Tangram's enduring relevance as a tool that bridges traditional learning methods with contemporary technological innovations, ultimately promoting a more engaging and effective educational experience.

1 Introduction

The Tangram, a seven-piece puzzle originating from China, has fascinated and challenged minds for centuries [23]. Comprised of five triangles, one square, and one parallelogram, these pieces can be rearranged into countless shapes and patterns. The objective is to rearrange these seven pieces using rigid body transformations to fit them into a given pattern. An arrangement is considered a valid solution only if it includes all the pieces without any overlaps [12].

Beyond its initial use as an intriguing puzzle, the Tangram has made a significant impact in various fields. Artists and designers have incorporated the geometric elements of Tangram into their works of art, while mathematicians have explored its properties and connections with various mathematical concepts [25].

In the realm of education, Tangram puzzles have proven to be invaluable as manipulative learning tools and teaching aids. They help young students develop geometric thinking and reasoning skills by providing hands-on experiences that promote active learning. By physically manipulating the pieces, students can better understand geometric shapes, relationships, and transformations [13]. The Tangram's simplicity and versatility make it an effective resource for teaching fundamental mathematical concepts in an engaging and interactive manner.

While traditionally a tool for entertainment, the Tangram has evolved into an educational resource that bridges the gap between traditional learning and modern innovation and technology. In contemporary classrooms, Tangram puzzles are used not only to teach geometry but also to foster problem-solving skills, critical thinking, and creativity [20]. Educators leverage digital platforms to bring Tangram puzzles to life, allowing students to interact with virtual Tangrams through educational software and applications. These digital tools enhance the learning experience by offering instant feedback, diverse challenges, and opportunities for collaborative learning [15].

Furthermore, the Tangram's integration into modern technology extends its relevance and appeal. Digital Tangram applications provide students with a dynamic and interactive environment where they can experiment with shapes and patterns, receive guidance, and track their progress. This integration of technology not only makes learning more engaging, but also equips students with valuable skills in using digital tools for problem-solving and creative expression [15].

2 Tangram in Education

The Tangram is more than a mere puzzle, it's a versatile educational tool that enhances cognitive and spatial reasoning skills [10]. Teachers worldwide have incorporated Tangrams into their curricula to teach various subjects, from mathematics and geometry to art and design (Figure 1). By manipulating the pieces to form different shapes, students learn critical concepts such as symmetry, fractions, and problem-solving strategies [19, 24].

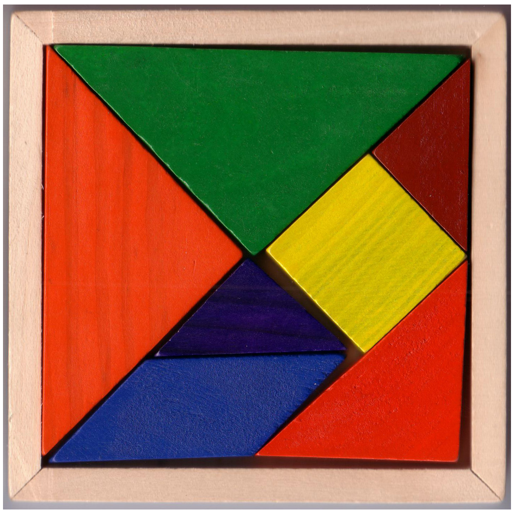

Figure 1: Wooden Tangram (initial square configuration)

According to Miller et al. [16] Tangram stimulates the imagination and creativity, it develops skills and abilities. The seven-pieces can be used in mathematic concepts, in art developing the creativity, such:

Mathematic Tangrams provide a hands-on approach to understanding geometric principles. Students can explore concepts like congruence, area, and perimeter by arranging the pieces into different shapes. This tactile learning experience helps solidify abstract mathematical ideas in a concrete manner.

Art Beyond mathematics, Tangrams foster creativity and artistic expression. Students can create intricate designs and patterns, encouraging them to think outside the box. This blend of creativity and logical thinking is essential in developing well-rounded problem-solving skills.

Tangram can be used to explore and solve challenging mathematical problems, especially in areas such as geometry, combinatorics and spatial reasoning. For example, given a set of tangram pieces, determine all the possible figures that have the same area but different perimeters. The difficulty is that this problem involves understanding how the different shapes can be combined to create new figures, keeping the same area but changing the perimeter. This requires a good understanding of geometry and the properties of shapes. According, to Yamada et, al. [26] point out that complex tangram puzzles have at least one of the following characteristics "(1) are composed of multiple connected components; (2) contain holes within the puzzle area; (3) require unconstrained rotations for the pieces; (4) require the reflection transformation of the parallelogram" (p.16). Figure 2 reflects what the authors say about the difficulty of constructing the puzzle using the seven pieces without overlapping.

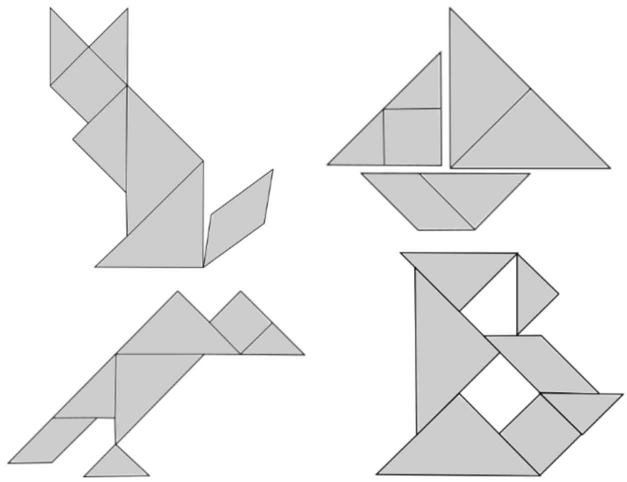

Figure 2: Complex Tangram Puzzle [26]

It is possible to build many shapes with the Tangram, but they all share a common characteristic: the area. Since each figure uses all the non-overlapping Tangram pieces, the areas of these figures are equal.

Decomposing figures is one of the simplest and most intuitive way for comparing the area of polygonal shapes. If two flat figures can be constructed using the same pieces without overlapping, they must have the same area. This principle often leads to intriguing pseudo-paradoxes when working with Tangrams.

The Dudeney's Paradox is a geometric puzzle where, for example, an equilateral triangle is cut into four pieces that can be reassembled to form a square. The paradox arises because the triangle and square have different shapes, yet their areas are identical (Figure 3). The challenge lies in finding the precise way to dissect the triangle for this transformation. It highlights how geometry can create surprising links between different shapes.

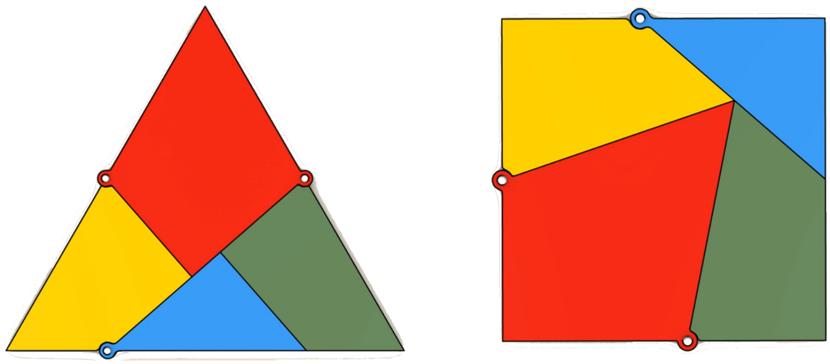

Figure 3: Equilateral triangle and Square (https://en.etudes.ru/models/dudeney-dissection/)

Consider the Figures 4 below:

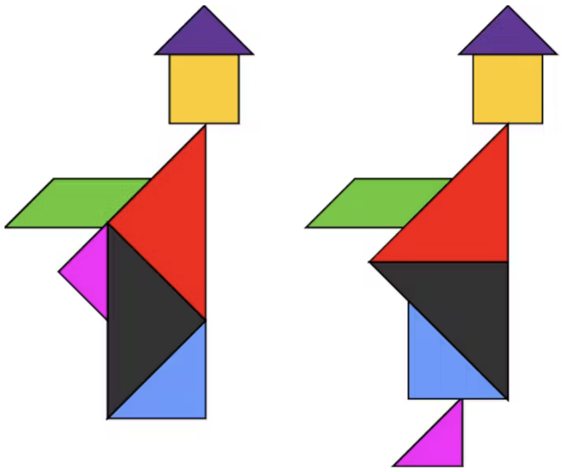

Figure 4: The two monks Pseudo-Paradox Tangram Puzzle (https://tinyurl.com/pparadox)

The images are on the same scale, and both can be constructed using the Tangram pieces [4]. However, upon closer inspection, the second figure appears to have a larger area than the first. Where did the extra area, such as the second person's feet, 'come from'?

Dudeney's Paradox is not the most subtle in terms of differences, with careful observation, one can see that one monk is slightly shorter than the other. However, other paradoxes are more difficult to spot.

The surface area of the Tangram pieces always remains constant. The areas have been transferred or shifted. The monks are not truly identical, everything has been moved slightly. This principle applies to all such paradoxes.

Even though all the pieces are used, the paradox appears longer, making it difficult to compare since they are identical in height.

The key to this effect is presenting these puzzles side by side. They appear to be the same height, but their widths differ slightly, which results in a significant part of the puzzle seeming to disappear [22].

3 Innovation in Tangram Usage

The effective use of information and communication technologies (ICT) as a teaching tool aims to encourage students to consolidate and update their technological and scientific knowledge. According to Prensky [17], students process information in a very different way to previous generations. This information-seeking potential associated with new technologies makes it essential to understand the contributions made to enriching learning contexts. The use of ICT by students and teachers contributes to educational development. In this way, appropriate use of ICT makes it possible to enrich, differentiate, individualise and implement all the curricular objectives.

Innovation in education often involves reimagining traditional tools for modern applications. The Tangram is no exception. Educators and developers have integrated Tangrams into digital platforms, creating interactive and engaging learning experiences. For examples:

- Digital Tangrams - With the advent of educational technology, digital Tangram applications have emerged, offering interactive puzzles that can be manipulated on screens. These apps provide instant feedback and step-by-step solutions, making it easier for students to grasp complex concepts [3].
- Gamification - Incorporating Tangrams into educational games has proven to be an effective way to maintain student engagement. Gamified learning experiences motivate students to solve puzzles and progress through levels, reinforcing their learning through repetition and challenge [11].

3.1 Tangram and Technology

Technology has transformed how Tangrams are used in educational settings, making them more accessible and versatile. In Augmented Reality (AR), technology has taken Tangram puzzles to a new level by overlaying digital information onto the real world [5, 21]. Students can use AR apps to see how Tangram pieces fit together in real-time, enhancing their spatial reasoning skills. Also in Virtual Reality (VR), provides an immersive environment where students can interact with Tangram puzzles in a three-dimensional space [2, 5]. This immersive experience helps students understand geometric concepts from multiple perspectives. Or with 3D Printing technology allows for the creation of custom Tangram sets, providing tactile learning tools that can be tailored to individual needs [14]. Teachers can design unique puzzles that align with their lesson plans, offering a personalised learning experience.

3.2 Tangram and Scratch

Scratch is a programming environment developed by a group of researchers from the Kindergarten and Lifelong Learning Group at the MIT Massachusetts Media Lab, under the direction of Resnick. The program allows for simple but efficient work on simple animations in which scenes, characters and sound elements are integrated, whether created by the user in the work environment, developed in other programs or taken from the resource gallery [18].

The integration of Tangram with Scratch represents an innovative approach to education that bridges traditional and modern learning tools. By combining the tactile, hands-on nature of Tangram puzzles with the creative, digital environment of Scratch, educators can provide a rich, engaging learning experience that fosters a wide range of skills.

Studies by Felleisen and Krishnamurthi [9] show that using Scratch in the classroom makes it possible to learn basic mathematical concepts in a fun way and arouses students' interest and motivation in this area of knowledge. The idea is to stimulate students in areas such as maths so that they themselves develop a joy of learning. Felleisen and Krishnamurthi [9] emphasised the concept of 'imaginative programming', considering its importance as an element of programming, due to its close relationship between mathematics and computing.

4 Tangram as Activity

The study presented here takes place in the context of initial teacher training for early years schooling. It involves a class of 53 third-year Basic Education degree students attending the Didactics and Technology of Maths course. The predominant teaching model in this unit is based on carrying out exploratory/investigative tasks, solving problems, and presenting activities, often using manipulatives or technological resources. In this case, the focus is on geometry, and the aim is to introduce the Tangram as a classroom resource. This study examines the use of the Tangram by prospective teachers and its role in deepening their understanding of geometry.

The students, who are prospective teachers divided into small groups (4/5 each group), were asked to carry out an activity using the Tangram. The first part of this activity involved using physical materials, while the second part required implementing the proposed activity in Scratch. They were asked to explain the educational level, educational objectives, specific objectives, transversal skills, and provide a description of the activity they were proposing.

The aim of this activity is to mobilize and deepen knowledge and skills related to elementary mathematics, while mastering computer tools suitable for exploring the mathematical topics covered. Additionally, it seeks to develop autonomy in carrying out training activities, promote teamwork in group projects, and encourage continuous self-training in mathematics education. Students are expected to develop mathematical, didactic, and technological skills that will enable them to plan meaningful mathematics learning experiences in an informed manner, taking advantage of computer technologies.

4.1 Proposed Tasks

The task proposals enable the observation of mathematical concepts emerging in contexts that promote understanding and the (re)construction of meanings. Therefore, designing mathematical tasks is a crucial aspect of initial teacher training.

4.1.1 Proposed Tasks - Part I

The 12 groups of students presented their proposals for activities, which are divided into educational levels. The students proposed activities for the 1st, 2nd, 4th, 5th and 6th grades. The groups worked on the theme of geometry and measurement. Only a few of them will be mentioned here.

The group A develop an activity for the 1st grade, worked with the essential learning [6] in the topic plane figures. This topic the student has to:

- Recognise triangles, squares, rectangles, pentagons, hexagons and circles and using appropriate representations;
- Recognise congruent figures, using different strategies and resources to explain their ideas.

This group worked the topic operations with figures. This topic the student has to:

- Constructing, representing and comparing composite plane figures;
- Composing and decomposing a given plane figure, using physical or virtual manipulatives.

This group proposed the objectives of the activity, such as, to identify and recognise basic geometric shapes, like triangles, squares, and parallelograms; to develop skills in classifying and categorising shapes; to explore and understand spatial relationships between geometric shapes; to stimulate logical thinking and problem-solving; to promote creativity and artistic expression through manipulating the pieces. The transversal skills involved are critical thinking, problem-solving, communication and collaboration, autonomy and initiative.

The description of the activity is the following:

- First, distribute a uniformly coloured Tangram to each student (Figure 5). Individually, the student should be able to recognise the congruent pieces and paint them in the same way; the different pieces should be painted differently. The students should also compare the different triangles, checking that although the size is different, the properties remain the same (number of sides and angles). After this, design figures to be built with the Tangram, starting with easier figures and progressively increasing the degree of difficulty (Figure 6). At this stage. the students can work collaboratively. Once they've finished the figures they've asked for, they should create new ones and present them to the rest of the class.

Group B established relationships between the Tangram pieces (Figure 7):

- The two pairs of triangles establish a proportional relationship by the AA criterion (angle-angle);
- The square, the trapezoid and the medium triangle can be obtained by joining the 2 smaller triangles, depending on how they are arranged;

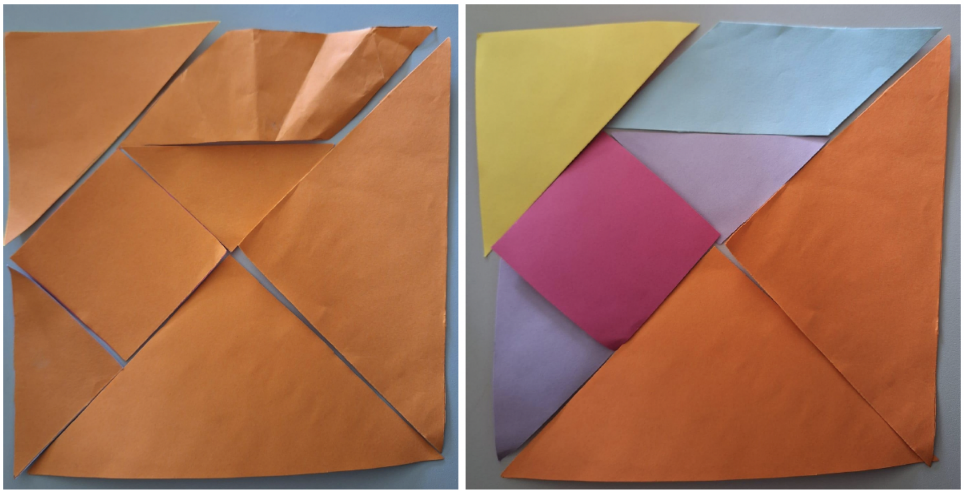

Figure 5: Activity Proposed - Part I

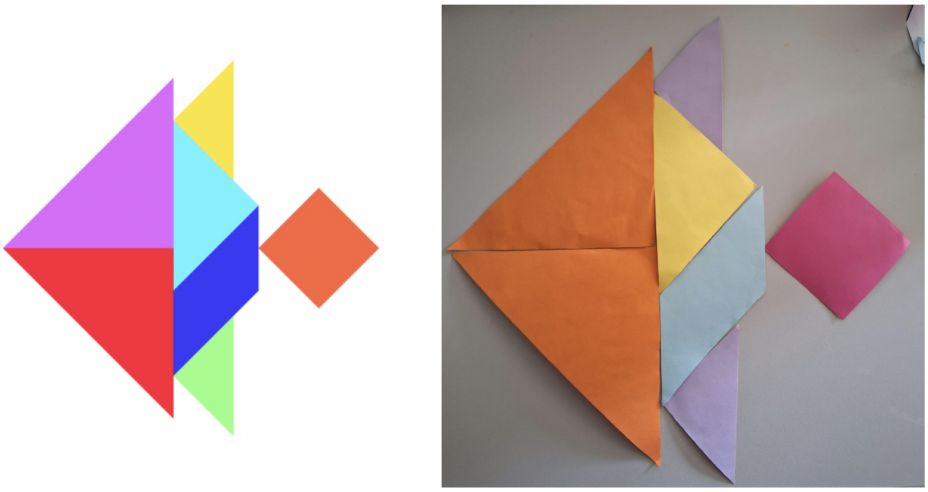

Figure 6: Activity Proposed - Part II (students image)

- The large triangle is made up of the square and the 2 small triangles;
- The sum of the hypotenuse of the two smaller triangles is equal to the hypotenuse of the larger triangle;
- The large triangle makes up 1/4 of the area of the Tangram.

The group B develop an activity for the 6th grade, worked with the essential learning [8] in the topic regular and irregular polygons. This topic the student has to:

- Distinguish between regular and irregular polygons (in the case of this activity, identify regular polygons);
- Solve problems involving regular and irregular polygons.

This group worked the topic sum of the measures of the internal and external angles of a triangle. This topic the student has to:

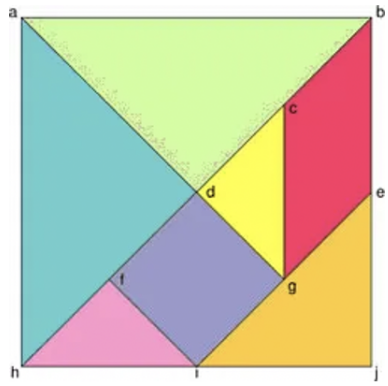

Figure 7: Activity Proposed - Tangram (students image)

- Conjecture about the sum of the internal and external angles of a triangle and explain the relationship found;
- Solve problems involving the properties of triangles.

This group proposed the objectives of the activity, such as, to develop problem-solving skills and logical reasoning; develop basic concepts of geometry; develop spatial perception; realising relationships between geometric figures. The transversal skills involved are patience and perseverance; creativity.

The description of the activity is the following:

- Divide the students into groups and give each group a Tangram. Ask them to find out the area of each Tangram figure, knowing that the measure of the long side is 6 cm. Ask them to justify how they arrived at each result.

4.1.2 Proposed Tasks - Part II

In this second part, the students were asked to implement an activity in Scratch. Two groups developed the activity proposed earlier, on the Tangram, which is presented below.

The group C prepare the activity with a duration of 45 minutes, with the objectives, to develop problem-solving skills and logical reasoning; to develop basic concepts of geometry; to develop spatial perception. The transversal skill to develop are patience and perseverance; creativity and to promote contact with ICT. The topics are analysing and working with geometric shapes and constructing patterns.

The description of the activity is the following:
After reading the book Tangram Cat (Figure 8) by Maranke Rinck and Martin van der Linden, divide the students into groups of 3 and let them explore the game 'Tangram remix copy' (Figure 9).

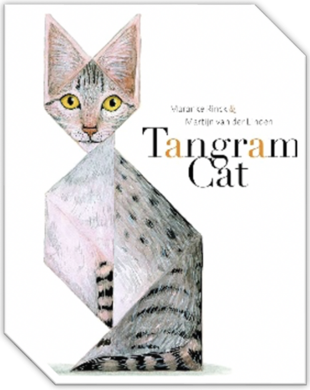

Figure 8: Tangram Cat (students image)

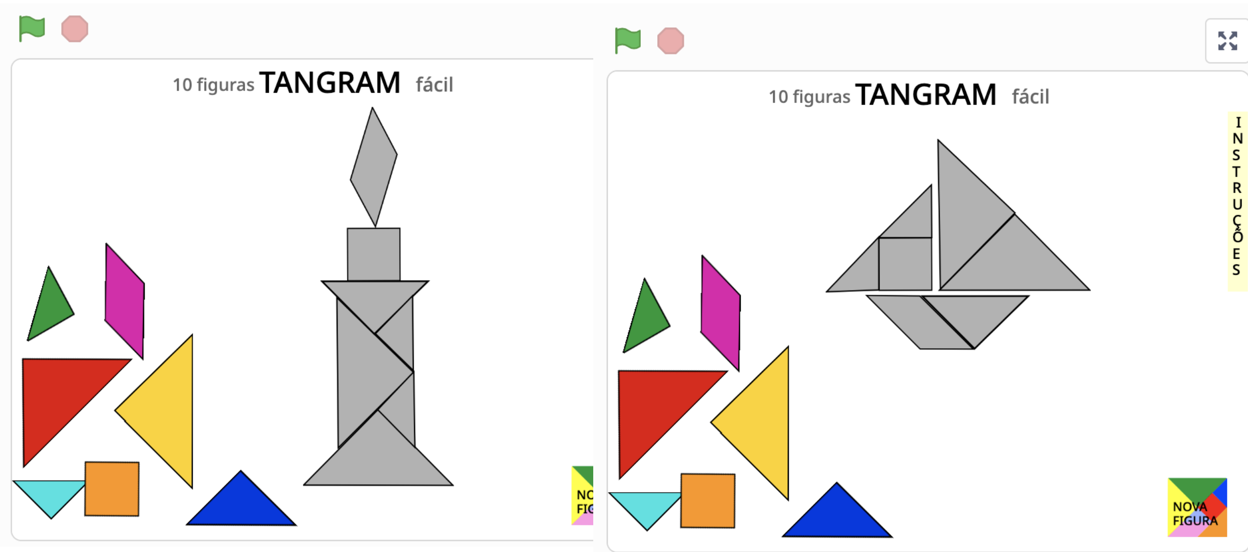

Figure 9: Scratch Tangram, two diferent constructions to do

Group D, prepare an activity (Figures 10) for the 2nd grade, that aims to classify plane figures based on their characteristics (such as straight lines, the number of sides, the number of vertices, and the equality of sides) [7]. The activity involves presenting and explaining their ideas, as well as justifying the congruence between plane figures using sliding, rotating, and turning movements. Additionally, the activity encourages students to present and explain their reasoning.

The transversal skills to be developed include problem-solving, where students apply their mathematical knowledge in various contexts and develop appropriate strategies to obtain valid solutions. Computational thinking is also emphasized, involving practices such as abstraction, decomposition, pattern recognition, analysis, and the definition of algorithms, along with developing habits of debugging and process optimization. Finally, the activity fosters creativity, encouraging the originality of creating new figures.

Notably, the activity plan did not specify its duration, leaving flexibility for its implementation.

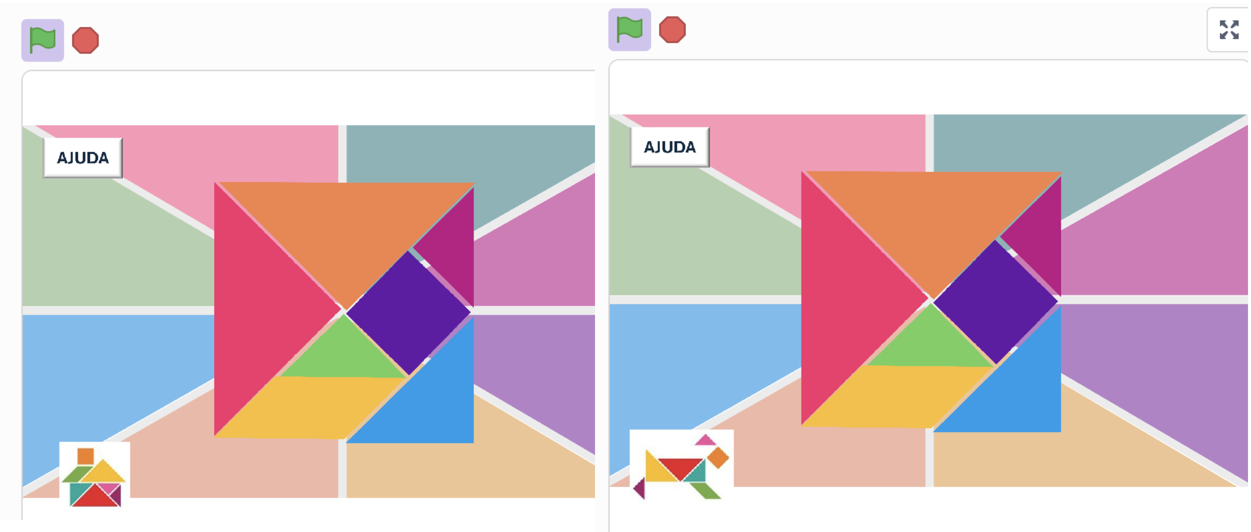

Figure 10: Scratch Tangram, two diferent constructions to do

4.2 Analysis and Discussion

Students are encouraged to reflect on the content, objectives, and target age group when designing activities, incorporating both physical and digital components. Creating an activity goes beyond simply aligning it with the curriculum, it involves understanding the underlying goals of the task, requiring students not only to design but also to solve the activity themselves. This process deepens their engagement with the material and fosters a more comprehensive understanding of how the task fits into broader educational objectives.

In terms of practical implementation, students found the first activity relatively easy and encountered few difficulties. However, the second activity, which required the use of Scratch, demanded more effort and posed greater challenges. Many students struggled to effectively utilise the digital tool to achieve the desired outcomes, revealing the complexities of incorporating programming into their educational toolkit.

The mastery of computer tools that are appropriated for exploring mathematical concepts, as well as the ability to demonstrate autonomy in completing these tasks, proved to be a significant challenge. Students found that developing proficiency in digital tools requires more than just basic familiarity, it demands sustained practice and a deeper understanding of the connection between technology and mathematics instruction.

This advancement will equip them with the capability to craft impactful and engaging mathematics learning experiences that are both well-informed and enriched through the strategic application of technology. The aim is to empower them to seamlessly integrate computer technologies into their instructional practices, thereby fostering an environment where learning is not only more interactive and dynamic but also tailored to the needs of contemporary education. By leveraging these tools effectively, they will be able to enhance student comprehension and engagement, making the learning process more responsive to the evolving landscape of educational demands and technological innovations. Ultimately, this approach seeks to prepare students to meet the challenges of modern teaching, where the integration of digital resources is essential for creating meaningful and effective learning experiences.

5 Conclusion

The Tangram puzzle, with its rich history and educational value, continues to be a relevant and powerful tool in modern education. By embracing innovation and technology, educators can harness the full potential of Tangrams to enhance learning outcomes. Whether through traditional methods or cutting-edge digital platforms, the Tangram remains a timeless resource that bridges the past and the future, fostering creativity, critical thinking, and a joy for learning.

Throughout the task, manipulating this resource provided opportunities to develop and articulate various geometry concepts. Notably, two key aspects emerged: the way students integrated Tangrams into their activity proposals and the incorporation of Tangrams into a digital context. Instead of starting with pre-made sets, having students create their own activities encouraged them to focus on the mathematical relationships embedded in the pieces. This approach effectively redirected their attention from the game itself to the mathematical concepts they aimed to study [1].

In class, the challenges of using a programming tool, even a basic one like Scratch, were evident among students with no prior programming knowledge. Despite the difficulties, students reported enjoying the tool and feeling motivated throughout the process. This enthusiasm highlights the potential of Scratch to engage learners, even those with no background in programming, and suggests that incorporating such tools in higher education contexts can enhance the learning experience.

The use of Scratch, specifically, not only makes the teaching-learning process more interactive but also serves as an accessible introduction to computational thinking. According to Resnick [18], programming significantly broadens the scope of what students can learn, extending beyond mere technical skills to foster problem-solving strategies and creative design across various domains. Engaging with programming encourages students to think critically and develop solutions in ways that transcend traditional subject boundaries. Resnick further emphasizes that programming also offers a unique opportunity for self-reflection. As students work through challenges, they begin to reflect on their own processes, gaining insights into their problem-solving approaches and overall learning strategies [18].

The Tangram remains a timeless resource that continues to inspire and educate. Its journey from a simple Chinese puzzle to a multifaceted educational tool illustrates its enduring appeal and versatility. By embracing both traditional methods and cutting-edge digital platforms, educators can harness the full potential of Tangrams to enhance learning outcomes.

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References

[1] Adler, J. (2000). Conceptualizing resources as a theme for teacher education. *Journal of Mathematics Teacher Education*, 3, 205-224.

[2] Amat, A. Z., Zhao, H., Swanson, A., Weitlauf, A. S., Warren, Z., & Sarkar, N. (2021) Design of an Interactive Virtual Reality System, InViRS, for Joint Attention Practice in Autistic Children. IEEE Transactions on Neural Systems and Rehabilitation Engineering, 29, pp. 1866-1876. https://doi.org/10.1109/TNSRE.2021.3108351

[3] Antrilli, N. K., & Wang, S. H. (2023). Tangible and digital materials for spatial play: Exploring the effects on parental talk and children's spatial reasoning. British Journal of Educational Technology, 54(2), 642-661. https://doi.org/10.1111/bjet.13269

[4] Barile, M. (2024) Tangram Paradox. From MathWorld–A Wolfram Web Resource, created by Eric W. Weisstein. https://mathworld.wolfram.com/TangramParadox.html

[5] Catenazzi, N., Sommaruga, L., & Locatelli,C. (2024) *Mixed Reality Affordances for Skill Development*. In 2In 2024 IEEE Global Engineering Education Conference (EDUCON) (pp. 1-9). IEEE., 10.1109/EDUCON60312.2024.10578579

[6] Direção-Geral da Educação (DGE). (2022). Novas aprendizagens essenciais de matemática [New essential learning of mathematic]. Direção-Geral da Educação. https://www.dge.mec.pt/sites/default/files/Curriculo/Aprendizagens_Essenciais/1_ciclo/ae_mat_1.o_ano.pdf

[7] Direção-Geral da Educação (DGE). (2023). Novas aprendizagens essenciais de matemática [New essential learning of mathematic]. Direção-Geral da Educação. http://www.dge.mec.pt/sites/default/files/Curriculo/Aprendizagens_Essenciais/1_ciclo/ae_mat_2.o_ano.pdf

[8] Direção-Geral da Educação (DGE). (2023). Novas aprendizagens essenciais de matemática [New essential learning of mathematic]. Direção-Geral da Educação. http://www.dge.mec.pt/sites/default/files/Curriculo/Aprendizagens_Essenciais/2_ciclo/ae_mat_6.o_ano.pdf

[9] Felleisen, M., & Krishnamurthi, S. (2009). ViewpointWhy computer science doesn't matter. Communications ofthe ACM, 52(7), 37-40.

[10] Ji, A., Kojima, N., Rush, N., Suhr, A., Vong, W. K., Hawkins, R. D., & Artzi, Y. (2022). Abstract visual reasoning with tangram shapes. arXiv preprint arXiv:2211.16492.

[11] Kamalodeen, V.J., Ramsawak-Jodha, N., Figaro-Henry, S., Jaggernauth, S. J. & Dedovets, Z. (2021) Designing gamification for geometry in elementary schools: insights from the designers. Smart Learn. Environ. 8, 36. https://doi.org/10.1186/s40561-021-00181

[12] Khairiree, K. (2015). Creative thinking in mathematics with Tangrams and the geometer's sketchpad. In Proceedings of the 20th Asian Technology Conference in Mathematics (pp. 153-161).

[13] Kmetová, M., & Lehocká, Z. L. (2021). Using tangram as a manipulative tool for transition between 2D and 3D perception in geometry. Mathematics, 9(18), 2185.

[14] Lehocká, Z. N., Betak, N., Žitný, R. , & Szabó, T. (2021) Improving Spatial Ability of Primary School Students in Mathematics and Informatics during the Distance Learning, EDULEARN21 Proceedings, pp. 5896-5902.

[15] Lin, C. P., Shao, Y. J., Wong, L. H., Li, Y. J., & Niramitranon, J. (2011). The Impact of Using Synchronous Collaborative Virtual Tangram in Children's Geometric. Turkish Online Journal of Educational Technology-TOJET, 10(2), 250-258.

[16] Miller, C. D., Heeren, V. E., & Hornsby, J. (2011). Mathematical ideas. Pearson Higher Ed.

[17] Prensky, M. (2001). Digital natives, digital immigrants part 2: Do they really think differently?. On the horizon, 9(6), 1-6.

[18] Resnick, M., Maloney, J., Monroy-Hernández, A., Rusk, N., Eastmond, E., Brennan, K., Millner, A., Rosenbaum, E., Silver, J., Silverman, B., & Kafai, Y. (2009). Scratch: Programming for all. Communications of the ACM, 52(11), 60-67. https://doi.org/10.1145/1592761.1592779

[19] Santos, V., Vaz-Rebelo, P., Bidarra, G., Stettner, E., Guncaga, & J., Korenova, L. (2022). Teacher Training in the Fields of STEAM: From Physical to Digital Tools. In: Reis, A., Barroso, J., Martins, P., Jimoyiannis, A., Huang, R.YM., Henriques, R. (eds) Technology and Innovation in Learning, Teaching and Education. TECH-EDU 2022. Communications in Computer and Information Science, (1720). Springer, Cham. https://doi.org/10.1007/978-3-031-22918-3_13

[20] Siew, N. M., & Chong, C. L. (2014). Fostering Students' Creativity through Van Hiele's 5 Phase-Based Tangram Activities. Journal of Education and Learning, 3(2), 66-80.

[21] Sommaruga, L., Catenazzi, N. & Locatelli, C. (2023) The Tangram Study for Mixed Reality Affordance Comparison. EDULEARN23 Proceedings, pp. 3205-3212.

[22] Starikova, I. (2018). Aesthetic preferences in mathematics: A case study. Philosophia mathematica, 26(2), 161-183.

[23] Tian, X. (2012). The art and mathematics of tangrams. In Proceedings of Bridges 2012: Mathematics, Music, Art, Architecture, Culture (pp. 553-556).

[24] Vaz Rebelo, P., Bidarra, G., Costa, C., Evangelista, S. de A., Santos, V., & Thiel, O. (2022). Heads-on, hands-on, hearts-on: educators perceptions on vidukids contributions to integral education. Revista INFAD De Psicología. International Journal of Developmental and Educational Psychology., 2(1), 351–358. https://doi.org/10.17060/ijodaep.2022.n1.v2.2363

[25] Yamada, F.M., Batagelo, H.C., Gois, J.P., & Takahashi, H. (2024) Generative approaches for solving tangram puzzles. Discov Artif Intell 4, 12. https://doi.org/10.1007/s44163-024-00107-6

[26] Yamada, F. M., Gois, J. P., & Batagelo, H. C. (2019). Solving tangram puzzles using raster-based mathematical morphology. In 2019 32nd SIBGRAPI conference on graphics, patterns and images (SIBGRAPI) (pp. 116-123). IEEE.