Secondary Level Mathematics Teachers' Critical Reflections on the Use of GeoGebra for Teaching Trigonometry

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Abstract

Technology-integrated pedagogy creates an engaged learning environment that supports conceptual, relational and procedural understanding. This study explores the roles of the GeoGebra Application (GA) in teaching trigonometry. The data were collected after and before the seven-day online training programs on using GA in teaching trigonometry through reflective experiences. We used critical reflective practice, cognitive theory and social constructivism to interpret and make meaning. This study revealed that teachers had disempowering and negative images towards trigonometry and its teaching before training. They believed trigonometry is an interpretation-free discipline and teaching as preparation for the final examination. Likewise, they lacked conceptual and relation understanding of the trigonometric knowledge and concepts that severely affected their teaching-learning activities. After the training program, the participant teachers reported that GA is a handy technological tool that helps visualize abstract trigonometrical concepts, supports to develop positive images toward trigonometry, and fosters an engaged learning environment. Finally, this study signified that integrating GA in trigonometric teaching can positively affect respective teachers' thinking, knowing and doing.

1. Introduction

Student disengagement in mathematics learning has become one of the significant problems for the ensuing quality of mathematics education in Nepal [1]. Researchers have explored that mathematics teaching-learning activities focus on imparting abstract mathematical formulae, concepts and theorems without considering conceptual and relational understanding [2-4]. The pouring and banking approaches of pedagogy [5] suffer mathematics education practices in schools and universities, resulting in negative images [6] and attitudes and beliefs [3]. It instigates mathematical anxiety among the learners that adversely affects mathematics education and creates the vicious circle of disliking, disengagement, underachieving, failure and dropout [7-8] and supports to spread of negative images of mathematics in society as abstract, difficult, mysterious, cold and dry, masculine and elite [33]. The pedagogical approach is one of the responsible factors for producing these images [2]. To bring a paradigm shift in mathematics pedagogical practices, we must break the boundary of conventional informing and banking pedagogy by integrating modern technology into mathematics teaching-learning activities [9].

Technology has become integral to various aspects of human life, including education. Many research studies have explored that teachers need technological knowledge to enrich the quality of mathematics education [10][13]. Technology covers a wide range of software and hardware designed for specific mathematical purposes, such as GeoGebra, Geometer's Sketchpad, Cabri Geometry, Graphing Calculators, Mathematica, MATLAB, etc., that are applicable in different devices and platforms (tablet PC, spreadsheets, virtual blackboards, PowerPoint, Java Applets, etc.). One of the technologies that is widely used in education and has a significant potential to enhance conceptual and relational understanding is the interactive GeoGebra application/software (GA) [11-12].

GA is a technological tool that supports mathematical thinking and learning. It enables practitioners to create, visualize, and demonstrate mathematical objects, formulae, concepts, and theorems and establish relationships creatively and intuitively. Interactive and dynamic features of GA significantly support and encourage learners to engage in mathematical activities independently and/or collaboratively [13]. Likewise, GA helps bring variability in teaching-learning activities by demonstrating a single concept dynamically. It is one of the best ways to boost the pupils' cognitive capabilities by incorporating alternative thinking. It signifies that interactive GA is a learning environment that scaffolds the learners with the dynamic interplay of seemingly static and rigid mathematical ideas.

Despite having such potential of GA for enhancing mathematical learning, it has not been reached in school mathematics classrooms in the context of Nepal. Until now, most of the mathematics teaching-learning activities in Nepali schools have incorporated chalk and talk and learning by doing without reflection, resulting in the rote memorization, recitation and reproduction of mathematics mathematical facts, formulae, concepts and theorem [16]. It un/intentionally creates a barrier to engaging in conceptual and relational understanding through critical reflective practices. Against this backdrop, this study explores the participants' reflections on integrating GA in trigonometry teaching at the secondary level.

2. Purpose and Research Questions of the Study

The purpose of the study is to explore the secondary-level teacher's critical reflections on the use of GA in trigonometry teaching before and after participating in seven-day training on "GeoGebra for Teaching Trigonometry at the Secondary Level." To address the purpose and navigate our research, we formulated the following research questions:

i) What practices and presumptions did secondary-level teachers have about teaching trigonometry before training?
ii) How did secondary-level teachers reflect on their experiences integrating GA after the training?

3. Theoretical Referents

To explore the experiences of secondary-level mathematics teachers from multiple perspectives and make meaning accordingly, we used critical reflective practices, cognitive learning theory, and the theory of social constructivism as theoretical referents. We used critical reflective practice as a way of writing to help us make holistic meaning by interconnecting three levels of experiences: reflection-on-action, reflection-in-action, and reflection-for-action [15]. The first level of reflection offers ways to analyze the participants' stories and experiences, what they did, the meaning perspectives of these activities, who benefited, and whose voices were heard [16]. Secondly, the reflection-in-action provides the lens for exploring contemporary practices. Finally, the third stage of critical reflection focuses on imagining and envisioning future tasks by improvising their past and present actions for the betterment of the overall programs.

Likewise, Brookfield also provided four procedures of critical reflection [17]. First, researchers and practitioners deeply engage in contemporary theory and research to understand the present discourses. In the second stage, they critically reflect these theoretical and philosophical underpinnings. In the third stage, practitioners offer the peer-group reflection on their ways of knowing and doing to improve their actions, which is used mainly in our training session to improve methods of constructing, presenting and discussing the trigonometrical concepts. Finally, critical reflective practitioners offer students reflections on their actions to improve classroom practices.

Cognitive learning theory focuses on developing the cognitive capabilities of learners through diverse perspectives. It uses the immediate environment as the source of information that underpins rational reasoning and intuitive thinking. Internal and external factors influence learning, but the rational and intuitive power of the learners has played a significant role in cognitive development [18]. The new environment and interaction create disequilibrium with their prior knowledge, and then learners comprehend the situation by rational and intuitive reasoning to make sense. After comprehending it, learners store new knowledge in their cognitive structures for future application [19]. From this perspective, the GA learning environment provides ample opportunity for the learners in which they interactively engage in learning mathematical concepts utilizing dynamic visual presentation, variability of representations, and their relations with other concepts, indeed evoking the learners to revive their prior knowledge by integrating the new ideas and concepts, that contribute to cognitive development [20].

Social constructivism denies an approach to learning as accumulating mathematical facts, concepts, skills and muted symbols, but it is a process of social and cultural interactions. Cognitive capability is not only the individual endeavour but also the product of their culture and social interactions. Social constructivist learning theory rejects that learners can learn mathematical ideas after cognitive development but believes that cognitive development occurs during learning [21]. From this perspective, learning is shifting the lower boundary of the zone of proximal development (ZPD) through the scaffolding process [21-22]. The GA learning environment provides alternative ways of performing, demonstrating and interpreting mathematical concepts and ideas, which certainly contribute to shifting the ability to be independently involved in the inquiry process to enrich their professional development.

4. Methods of Data Generation and Interpretation

The required data were generated during the training sessions on using GeoGebra in teaching trigonometry at the secondary level, organized by the Council for Mathematics Education (CME) Chitwan, Nepal, during COVID-19. The government of Nepal declared a lockdown in the country to prevent the rapid spreading of COVID-19. CME Chitwan Branch decided to organize training for the respective teachers and appointed the first author as a program coordinator. The second and third authors helped develop the detailed training session plan. Through the Google form, we offered the proposal of interest from teachers teaching trigonometry at the secondary level and taking online classes during COVID-19. We received approximately 60 applicants' forms. We selected only 21 teachers on a first come, first served basis because selecting large numbers may create difficulty in handling and possibly disrupt making the class more interactive.

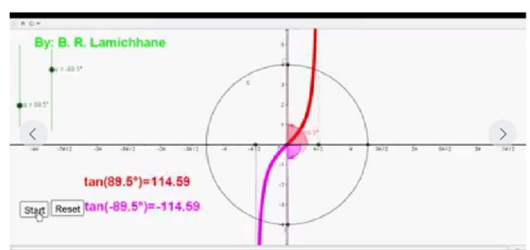

Fig. 1. Screenshot of tracing of tan curve in positive and negative direction

To generate the required data, we asked the participants to critically reflect on their experiences of teaching-learning activities throughout their academic and professional journey. They seemed hesitant to express their experiential experiences. Then, I (the first author) told my story. I learnt the trigonometrical relation by rote memorization through the techniques given by my mathematics teacher, such as "*Pandit Badri Prasad Hara Hara Bole*", which helped me to remember trigonometrical relations without making any sense and meaning. The techniques of remembering were that we have to take the first alphabet of the above mantra as P, B, P, H, H, B, and the first three alphabets constitute the numerators and the remaining three work as denominators of the trigonometrical relations. Then they look like $sin\theta = \frac{p}{h}, cos\theta = \frac{b}{h}$ and $tan\theta = \frac{p}{b}$. And for the other relations, we reverse these relations. It is one example of how the first author learned trigonometry relations from the first trigonometric class. After that, participant teachers freely shared their experiences, which helped us to capture the essence of their presumptions and practices.

We have used the online classroom discussions in a training session, participant teachers' stories and their lived experiences [14] in trigonometry teaching as data sources. The participants were divided into three groups to systematize the discussion and sharing session. The team leaders are assigned to collect the queries of the other participants through the chat box. The three-hour online session was conducted on a one-day alternative basis to provide enough time for the participants to engage deeply in GA environments. At the beginning of every session, approximately 20-30 minutes were provided for the participants to share their experiences. The team leaders shared the experiences of the teaching-learning activities, and each participant was also allowed to share their experiences if any, distinct experiences from the shared ones. We recorded all the activities of the training sessions facilitated by the first authors by obtaining consent from the participants, assuring them that the data would be used only for research purposes, and strictly maintaining confidentiality. Grounded in the theoretical referents, we have generated the themes and interpreted the data with the help of second and third authors.

5. Discussion and Interpretations

This section briefly discussed the participants' views on trigonometry and its teaching before and after the online training sessions. We have discussed the overall findings in two parts—before and after training.

5.1. Reflection of Teachers before Training

To explore the participant teachers' experiences, the first author opened the discussion by asking questions.

Me: *How do you demonstrate the value of* $tan(90^o) = \infty$ *in your classroom?*

Participants*: Sir, we can easily show it. All participants argued in the same fashion. The value of* $tan(90^o) = \frac{Sin(90^0)}{cos(90^0} = \frac{1}{0} = \infty.$

Me: *Do the students understand the value of* $\frac{1}{0} = \infty$*? How do they represent* $tan(90^o)$ *Geometrically? Can you give some examples that visualize the value of* $tan(90^o)$*?*

Participants: *Sir, we do not think from this perspective. Our curriculum does not demand such types of skills and concepts. We believe students do not have difficulty remembering the value of trigonometric functions at the given range.*

Me: *I respect your argument. Let's forget students for a while. Can you visually present the value of* $tan(90^o)$*? We break the session into three rooms for discussion and ask them to come to a common understanding of the group.*

Participants: *Three groups share their understanding. However, there is no variability in interpretation and presentations. They repeated the above arguments. One of the groups presents it with a graph automatically generated by the GA.*

Me: *How do you respond to the students, if they ask, how* $tan(90^o) = \infty$**.**

Participants: *As we have already said, it is a mathematical formula and needs no further discussion. We have never raised such questions during our academy journey and have never faced such types of questions. Please don't waste our time discussing such a topic.*

Me: *Ok, it is fine. However, I have one more query for you, and then we start our main session. Do you realize pedagogical practices in mathematics education are responsible for generating students' popular negative images or attitudes towards mathematics/trigonometry?*

Participants: *We are somehow responsible. Curriculum experts/designers, the Ministry of Education, local governments, educational authorities and other line agencies are more responsible than teachers. Teachers are innocent, powerless and obedient; they have fulfilled their responsibilities.*

From the above discussion, we have captured the teacher's conceptions and teaching experiences before training, which are presented in the following subsections.

5.1.1. Trigonometry as an Interpretation-free Discipline

All participants have the same presumptions that trigonometry is a branch of mathematics with already established formulae, concepts and theorem. Trigonometric knowledge is like other mathematical knowledge that has existed and remained somewhere in the universe [23]. Teachers' tasks are to bring such mysterious knowledge, formulae, and concepts into a classroom as it is and transmit them to the students. Likewise, they view teaching as reproducing the given trigonometrical knowledge [35] that does not need further interpretation. The already established knowledge, concepts and ideas do not demand further interpretation. To do so is a waste of time because trigonometrical knowledge is universal.

5.1.2. Teaching as/for Preparation of Examination

Most teachers agreed that teaching is the preparation for the upcoming final examination. They think teaching-learning activities need not go beyond the narrowly framed textbooks. Teaching means solving the textbooks' questions by following the rigid algorithm. Its mechanistic way of finding the solution to bookish questions restrains creative, critical and imaginative thinking in mathematics, and learners become muted followers. They never raised questions regarding its genesis, values and relation to the other disciplines. Students become passive receivers of knowledge and teachers as knowledge depositors [5]. Through the long-run practices of informing pedagogy, it appeared to be prevalent in school trigonometric teaching. Teachers frequently reject the necessity of alternative ways of presentation and discussion to prepare the students for the final examination.

5.1.3. Lack of Conceptual and Relational Understanding

One of the research participants reflects on his/her experience. *As a mathematics student, I read trigonometry for the first time in grade IX. I did not make sense of the trigonometric functions and their values. Teacher entered the room and drew a right-angled triangle on a backboard. He denoted the sides of the triangle by p, h, and b, wrote the formulae of sin, cos, tan, cosec, sec, and cot, and urged us to remember them. Likewise, a few days later, the teacher made a matrix of values of standard angles of the trigonometric functions and gave some tricks to memorize them. Then, we unquestionably adopt the same learning and teaching procedures.*

It indicates that teachers were rarely orients to create alternative discourses in teaching trigonometry. They hardly realized the importance and roles of variability principles in teaching trigonometry for cognitive development [20]. Due to the lack of teachers' conceptual and relation understanding of trigonometrical concepts, most teachers could not relate the trigonometric concepts to real-world problems. From the other perspective, our teachers have been deskilled by the education system of the country in general and university in particular so that teachers can only perform the files duties [24]. From the beginning of the modern education system in Nepal, teaching-learning activities tied within external recognition, such as success and failure in content domains, were determined through externally executed paper-pencil tests. Consequently, our future teachers become the disseminators of bookish knowledge.

5.2 Reflection of Teachers After Training

After listening to the teachers' presumptions and experiences, we started our session from the beginning of the trigonometric functions and their values. It is impossible to present all the materials developed during the training sessions. So, we select some exemplary cases that could evoke the participant teachers to think alternatively and become energized to be creative, critical, imaginative and collaborative learners.

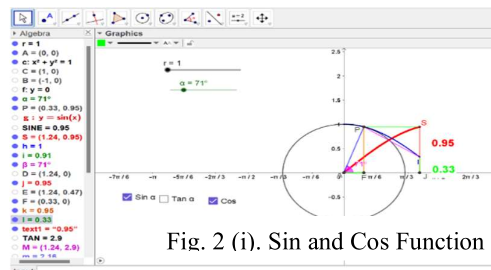

Fig. 2 (i). Sin and Cos Function

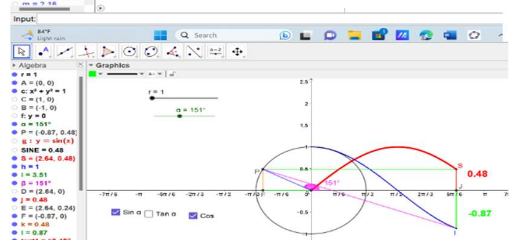

Fig. 2 (ii). Sin and Cos Function in 2nd Quadrant

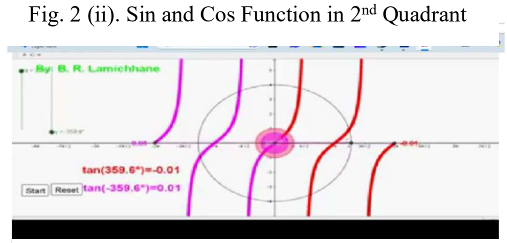

Fig. 2 (iii). Tangent Function

Based on the participant's interest, we chose the case to demonstrate trigonometric values. In doing so, I provided them with a construction protocol and requested them to follow the procedures. First, we construct the dynamic graph of the trigonometric functions of sine, cosine, and tangent that demonstrate the values of the trigonometric function in different ways (see Fig. 2 (i), 2 (ii) and 2 (iii)) and ask them to prepare the dynamic graph collaboratively in their groups that can differently visualize the values of the trigonometric function. After completing the construction, we discussed its pedagogical implications. To illustrate the value of the $\tan(90^0) = \infty$*, we create dynamic graphs (see Fig. 3(i), 3 (ii), 3 (iii)) and discuss them.*

Me: We have just prepared the graph to demonstrate the value of $\tan(90^0) = \infty$ *on the standard unit circle. Look at figure 3(i); the tangent line at point P on the circle extended to the X and Y axes, which meet axes at points E and F, respectively. In this figure, we can see the values of* $\tan(45^0)$ *and* $\cot(45^0)$*. What are the values?*

Participants: *Both values are equal.*

Me: *Which parts of the graph represent the values of* $tan(45^0)$ *and* $cot(45^0)$*?*

Participants: *The values of* $tan(45^0)$ *and* $cot(45^0)$ *are represented by the line segments PE and PF, respectively.*

Me: *What happens when we increase the value of the angle from* 45^o *to* 85^o *(see Fig. 3(ii))?*

Participants: *The values of tan increased from 1 to 11.43, and the value of cot decreased from 1 to 0. 087.*

Me: *Please increase the value of the angle from* 85^o *to* 89.99^o *and note the values of tan and cot (see Fig. 3(iii)).*

Participants: *Sir, the value of tan rapidly increases, but the value of the cot tends to zero. When the angle value increases gradually, the tangent line tends to parallel to the X-axis. It does not meet the X-axis. Oh! It's astonishing! Similarly, when the value of the angle is* 90^o*, the value of the cot becomes zero. That is, the distance between PF (cotangent) becomes zero.*

Me: Yes, you can't see point E in the figure, and the line becomes asymptote to a parallel line to the X-axis at point P (it seems parallel in figure 3 (iii)). Do these line PE and X-axis meet at the X-axis as in figures 3(i) and 3(ii)? In this case, what would be the values of PE? Can you determine it?

Participants: No sir, we cannot determine the point E on the X axis, i.e. we cannot find the value of PE.

Me: That is why it is infinity. You can see the visualization of the value of $\tan(90^o) = \infty$. *Again, drag a slider and construct a matrix of the values of the sin, cos and tan, and observe the trends of values of these functions.*

From this, you can also observe that sine and cosine values tend to be one and zero. When one is divided by the small number between one and zero, it becomes infinitely large. It is also other ways of understanding the values of $\tan(90^o) = \infty$ *(see Fig. 4). Other tools, such as Desmos, Mathematica, MATLAB, etc., help you demonstrate the value of trigonometric functions. In these ways, we can visualize the value of trigonometric functions differently (See Fig. 1, 2(i-iii), 3(i-iii) and 4). Can you draw a graph to illustrate the values of these trigonometric functions differently? They designed the graph differently in a dynamic version and presented it in a group (see Fig. 5). They described the nature of the sine function, its periodical values, and negative and positive angles with curve tracing.*

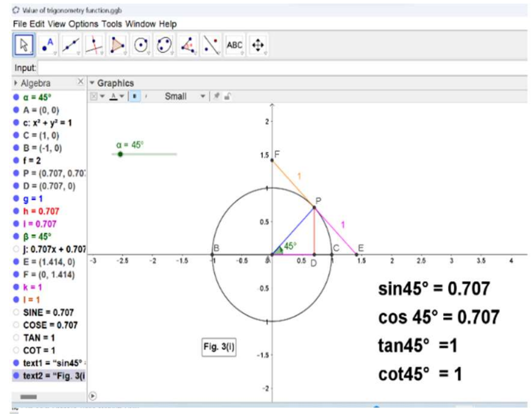

Fig. 3 (i) Value of Sin, Cos and Tan, at 45^0

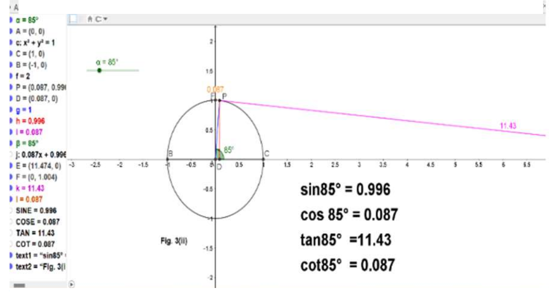

Fig. 3 (ii). Value of Sin, Cos and Tan, at 85^0

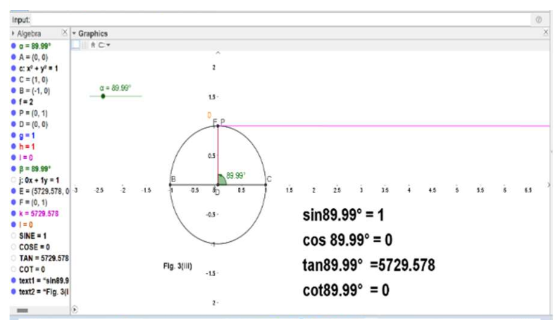

Fig. 3 (iii) Value of Sin, Cos and Tan, at 89.99^0

After the end of the sessions, we asked them to reflect critically on the impact of training that will help us design training in the coming days. From their reflections, we have captured the following themes.

5.2.1 GA helps Visualize Abstract Trigonometric Concepts

Most teachers view trigonometry as an abstract, vapid, decompensate and mysterious subject before engaging in the training session. Describing the roles of the GA in teaching trigonometry, one of the participants from group A narrated his/her experience. *We are really in an illusionary state. Our university and government institutions have not taught and trained us through technology during pre-service and in-service teacher training programs. We also do not try to enhance our content knowledge and technological skills independently. We forcefully compelled our students to memorize the trigonometric formulae, concepts and algorithms for the recitation in the upcoming final examination. I realize, we have only learned to imitate and taught the students to do so. This training is really fruitful for us. We have learnt to visualize the abstract trigonometric formulae like the value* *of tan 90^0, relations between and among the trigonometric concepts, double angle formula (sin2A) and others.*

This reflective experience explores that the most challenging tasks for teachers and students are visualizing seemingly abstract trigonometrical concepts. The GA helps demonstrate abstract concepts in a dynamic visual mode effectively [34]. Likewise, incorporating the GA in teaching mathematics supports enhancing conceptual, relational and procedural understanding [25] and the development of positive images of practitioners toward the subject.

Angle Measure	sin	cos	tan
30°	0.5	0.87	0.58
60°	0.87	0.5	1.17
86°	0.998	0.07	14.301
89.51°	1	0.009	116.927
89.99°	1	0.00017	5729.57789
90°	1	0	∞

Fig. 4: Value of sin, cos and tan at different degrees generated by GeoGebra

5.2.2 GA Develops Positive Images toward Trigonometric

Participant teachers' reflective narratives before training indicated that most teachers have negative images towards mathematics in general and trigonometry in particular. Many researchers [26-27] have also explored that images of mathematics significantly affect their classroom teaching-learning activities. There is a high possibility of transmitting negative images, anxiety, and beliefs from teachers to their students that adversely affect the teaching-learning activities, resulting in low achievement in mathematics [7].

In this regard, the reflective experience narrated by one of the participants from group B is worth including. *I have experienced during my academic journey of mathematics education that mathematics is a mental game played by some elite people. In my opinion, trigonometry is nothing more than the accumulation of formulae, symbols, axioms, and theorems that remain far from our lives. After participating in this training, I regret my previously held beliefs and images of mathematics and trigonometry that hinder my teaching-learning activities and possibly discourage my students from adapting mathematics as their future career subject. I am astonished that GA makes abstract trigonometrical knowledge and concepts dynamically visualize and establish the relations between and among the concepts. It amazingly explores the beauty of mathematics. This training supports eliminating the stereotypical views of mathematics and teaches us that many alternatives exist to solve or represent the same trigonometrical concept.*

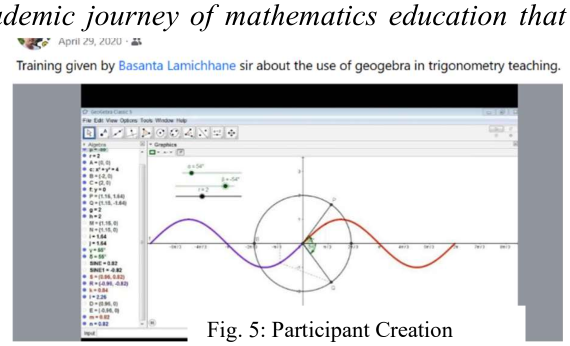

Fig. 5: Participant Creation

This representative narrative of teachers has unearthed many untold realities of trigonometric teaching-learning. Due to the lack of technological knowledge and its intertwined pedagogical skills, teaching-learning activities are incarcerated within the transmissionist, machinist and disempowerment realms [28]. In this regard, GA helps bring the variability in teaching-learning activities, fosters the collaborative learning environment, promotes the learner-centred pedagogical approaches and disrupts the notion of learners as passive receivers of knowledge [29]. Arrive at this point of inquiry, we have realized that GA has the potential to create an engaged learning environment.

5.2.3 GA Promotes Engaged Learning

The reflection of one of the participants from group C highlights the key issue of engaged learning enhanced by the training. *Now, I have realized probably all of my colleagues also agree with my views (no one disagreed with him) that trigonometry is one of the most useable branches of mathematics, but we teachers make it difficult, abstract, obscure, and mysterious. We victimize students because we cannot enable them to be creative, critical, imaginative, independent, and collaborative learners, but we compel them to imitate the disempowering notion of learning. I did not feel bored and tedious in such a long three-and-a-half-hour session, and the 7-days passed like a blink. I thoroughly enjoyed it. It helps us widen our cognitive thinking, develop the skills to work in a group, enhance our conceptual and relational understanding and create a learning environment in which we are emotionally engaged without external motivation.*

Learners' engagement in learning is a significant attribute for enhancing and promoting the learners' academic achievement [30] by addressing the issues of disliking, anxiety, dropping out, and boredom in teaching-learning. To promote an engaging learning environment, we must focus on the learners' academic, cognitive, social and affective dimensions [30]. The GA-integrated learning environment enhances cognitive development because it scaffolds the learners to deploy contrast and variation approaches and dynamic visualization. Likewise, the GA environment encourages the learners to work collaboratively by cooperating, feedbacking, commenting and encouraging each other to bring the desired outcomes. Finally, GA supports addressing the affective dimensions by enriching the emotional attachment of learners to the learning process and self-reporting learning values [31].

6 Conclusion and Implication

The disengagement in learning mathematics, in general, and trigonometry, in particular, is one of the major problems in the context of Nepal [32]. This study has indicated that factors behind the disengagement in learning trigonometry are teachers' negative images and beliefs towards trigonometry, presumptions of teaching as the transmission of an already existing body of knowledge to the passive minds of students, and lack of technological expertise and its intertwined pedagogical skills. This study further explored how integrating GA in teaching trigonometric functions helped address the disempowering notions above. The GA-integrated approach supports visualizing abstract trigonometrical concepts, resulting in conceptual, relational and procedural understanding. It also played a significant role in developing positive images toward trigonometry, possibly preventing the vicious circle of disliking, underachievement, anxiety and dropping out. Finally, the GA environment underpins engaged learning by incorporating the most significant aspects of learning: academic, cognitive, social and affective. We concluded that integrating technological tools in trigonometric teaching is necessary in the 21st-century complex era. It enhances the learning environment that explicitly contributes to developing positive attitudes, beliefs and images towards the subject, consequently contributing to the meaningful learning of trigonometry rather than imparting the discrete knowledge to prepare for the final examination.

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