Leveraging Technology for Effective Teaching and Learning

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Abstract: *During the COVID-19 pandemic, online teaching has become a major teaching method, which must involve the application of various technologies. From initial passive usage to ongoing exploration, our teaching and research team has found that teaching integrated with technology can help students better master and apply mathematical concepts to solve problems compared with traditional teaching, and technology has revolutionized mathematics education by offering innovative tools and resources that can promote active learning and enhance a motivating practice of math. In the realm of integration, technology can play a vital role in helping students visualize and comprehend fundamental concepts and applications. Firstly, the essay introduces powerful technological tools to dynamically enhance the teaching approach of complex mathematical concepts in the classroom through the utilization of TI-Nspire Software. Subsequently, it explores how technology can be integrated into curriculum design.*

1. Introduction

It is widely acknowledged that some calculus concepts are abstract and complex for students which has made teaching and learning challenging and even exasperating at times. Indeed, some researchers have shown that a large number of students have difficulties learning some key concepts of calculus because traditional calculus courses tend to focus more on algebraic drills and practice on calculus problems without understanding the underlying concepts [1], which would cause students to regurgitate what they have been taught and duplicate it in examinations. However, the problem is that over-emphasis on algebraic drill and practice methods will produce students who are only able to regurgitate what has been taught and duplicate it in examinations. The study conducted by Gordon reveals that algebraic solutions to problems and lengthy derivations of formulas are commonly expected among students in calculus. Gordon also pointed out that some calculus teachers felt that they were teaching algebra rather than the concepts of calculus. This phenomenon is also noted by Axtell[2] who feels that conventional teaching of calculus fails to produce students who are able to understand the fundamentals of calculus. Teachers seem to focus more on calculation techniques rather than understanding of the concepts among the students in teaching calculus. As a result, students solve differentiation or integration problems by simply applying the steps they have memorized without having a good grasp of the calculus concepts.
Understandably, Gordon and Axtell advocated that the calculus curriculum should be reformed by putting more emphasis on conceptual understanding of the fundamentals of calculus and complementing the use of graphical, numerical, algebraic, and verbal representation in the teaching and learning of calculus. Gordon (2004) further suggested that students must learn to select the right tools such as graphing calculators to help them learn calculus and use a variety of algebraic, numeric, and graphical approaches, to solve calculus problems. The purposes of the essay were therefore to examine the role of the TI-Nspire Software in teaching and learning calculus and provide some class examples in conceptual understanding and comprehensive practice based on TI-Nspire Software according to my teaching and exploration for years.

2. The Role of the TI-Nspire Software in Calculus

The TI-Nspire Software was used as a pedagogical tool to complement the conventional teaching approach. During observation within recent several years, the students' learning has often gone through such a process with TI-Nspire Software. TI-Nspire Software examines the setting of a given problem, formulate conjectures or hypothesis, examine the hypothesis, recheck the solutions, and finally generate a result or solution to the given problem. To facilitate exploratory learning and guided-discovery, worksheets were designed to guide the students in investigating and exploring mathematical concepts with the help of TI-Nspire Software, finally, students could understand the whole process of concepts gradually and visually at the end of the exploration. Therefore, TI-Nspire Software was also used as a confirmatory tool to verify their answers to the given problems or conjectures for a particular calculus concept.

Now, we observed that students also tried to use various methods of TI-Nspire Software to find the solutions when they were given problems in the exercise. In addition, we also found that students have a better understanding of the calculus concepts when they can use various approaches to solve problems. For instance, in the lesson on definite integral, we found that students who use graphical methods to solve stereotyped problems tended to develop a more complete understanding of the concepts of definite integral and area under the curve.

On the other hand, in solving problems involving maximum or minimum, we found that some students tended to approach the problems from a conceptual perspective using the graphical method or graphical tracing function of the TI-Nspire Software, which shows that the availability of TI-Nspire Software seemed to encourage a shift from a rigid to a more flexible technique of problem-solving among the students which is a consequence of students achieving conceptual understanding.

Through classroom observations, analysis of students' TI-Nspire Software documents, and regular reflections, we attempted to know what functions students have learned, what they would like to try, and what they would like to do with TI-Nspire Software. It was found that students used the TI-Nspire Software as a tool in several ways. For instance, students used the TI-Nspire Software as a visualization tool to better understand the behavior of graphs, new concepts taught, or problem situations. They also learned how to use the TI-Nspire Software as a confirmatory tool to verify the correctness of their answers. Table 1 summarizes the ways in which the TI-Nspire Software was actually used during the intervention program either as a pedagogical tool in teaching or as a learning tool by the students.

Use of the TI-Nspire Software	Description
Exploratory tool	The TI-Nspire Software was used to explore and understand concepts of differentiation and integration.
Confirmatory tool	Students used the TI-Nspire Software to verify their answers to the questions in the exercises.
Problem-solving tool	using the graphing functions of the TI-Nspire Software. Students used the TI-Nspire Software to try algebraic, graphical and numerical approaches to solving a given calculus problem.
Calculation tool	Students used the TI-Nspire Software to calculate values or evaluate complex expressions.
Graphing tool	Students used the TI-Nspire Software to graph functions and to solve a given problem graphically.

3. Class Design Examples

Here's a practical exploration course about the definition of definite integral and its application, and it would show how calculator works in the teaching from conceptual teaching and comprehensive practice.

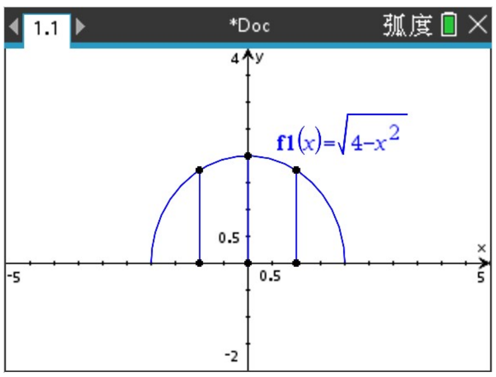
Case 1—Definition of definite integrals

Activity I: Introduce the background.

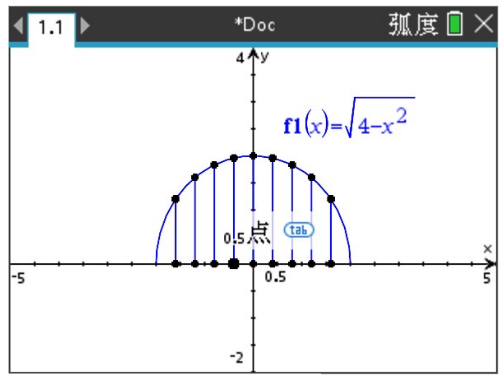
We are already aware that the area of a semicircle with a radius of 2 is equal to 2π, however, it is important to understand the mathematical reasoning behind this result.

Activity II: Explore the problem step by step with TI-Nspire

Step 1-Repartition the area of the curved trapezoid into smaller subdivisions.

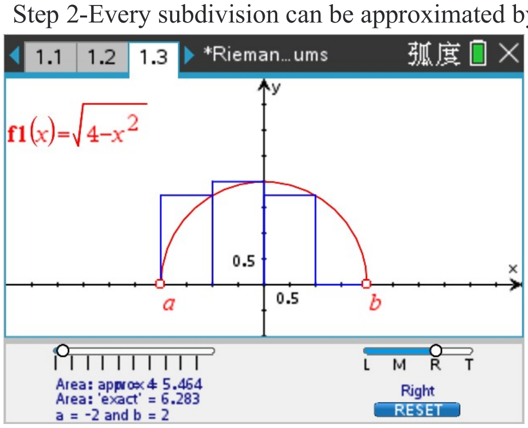


Case 1.1-Partition 1

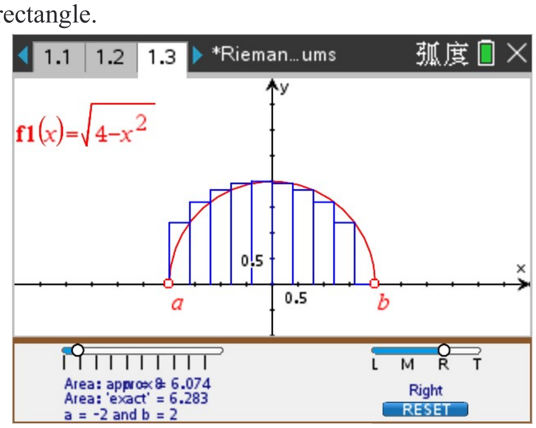


Case 1.2-Partition 1

Step 2-Every subdivision can be approximated by the area of a rectangle.

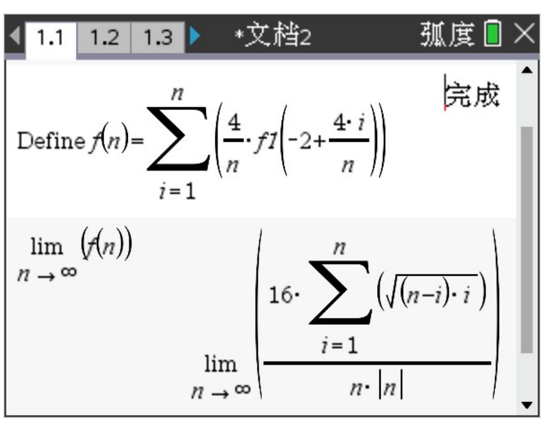


Case 1-3 Approximation



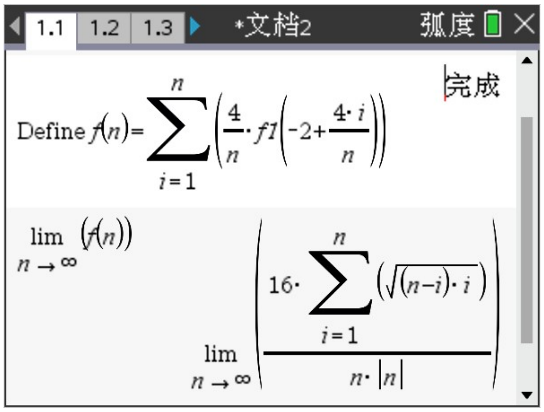
Case 1-4 Approximation

Step 3-Compute the total area of all rectangles.

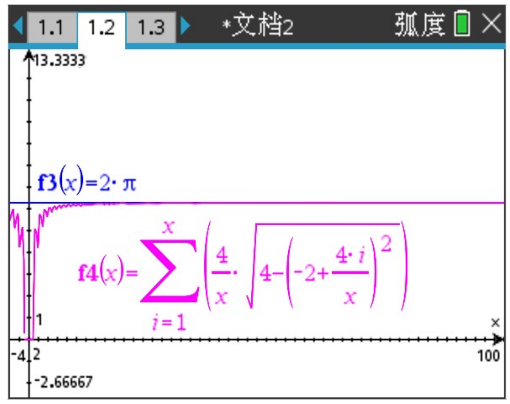


Case 1-5 Summation

Step 4- find the limit of sum in step 3 as the subdivision becomes infinity.



Case 1-6 Limit

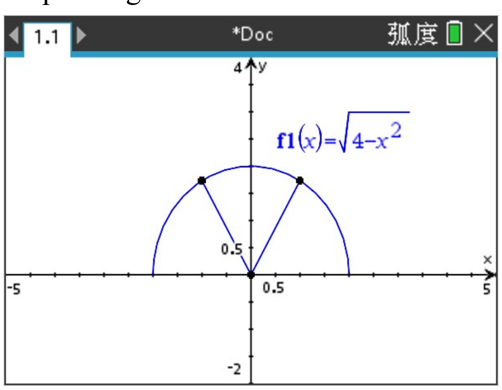


Case 1-7 Graph

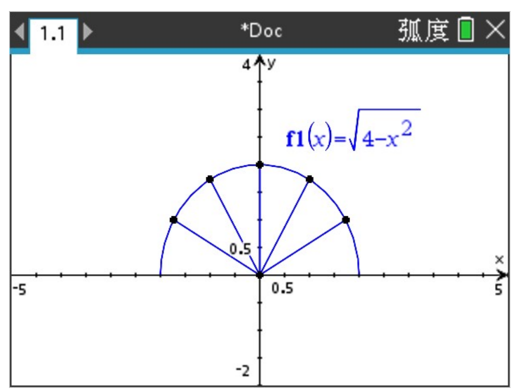
This class explores the implementation of TI-Nspire-based problem-solving techniques in the design of a comprehensive course for mathematics education. The objective is to empower students to break down definite integration into manageable steps: segmentation, approximation, summation, and limit-taking. Additionally, the study leverages the drawing and visualization capabilities of calculators to facilitate a deeper understanding of integrals from both algebraic and geometric perspectives. Notably, the limitations of calculators come into play, as direct calculation of sequence limits $(f(n))$ is not feasible, necessitating a geometric approach. However, these constraints serve as catalysts for enhancing students' critical thinking abilities. After class, students employ by TI-Nspire-based concept of definite integration to address the identified challenges and expand their mathematical proficiency.

With the help of a wireless interactive classroom in TI-Nspire Software, we found another method in partition and approximation as below:

Step 1: Segment with small sectors

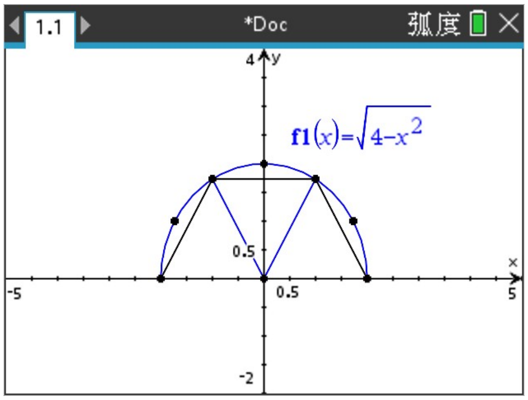


Case 1-8 Partition 2

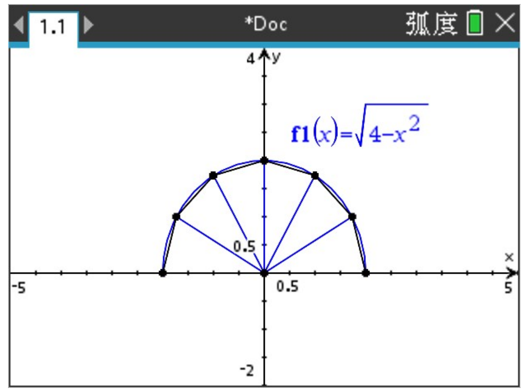


Case 1-9 Partition 2

Step 2: approximate sectors by triangles

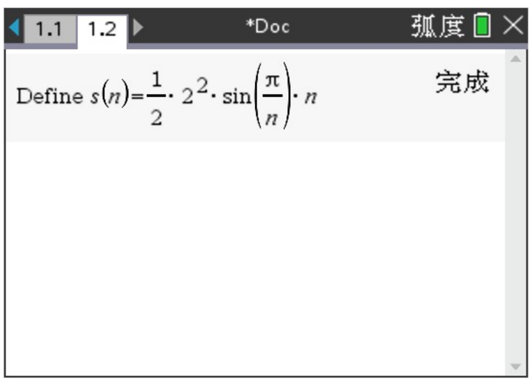


Case 1-10 Approximation



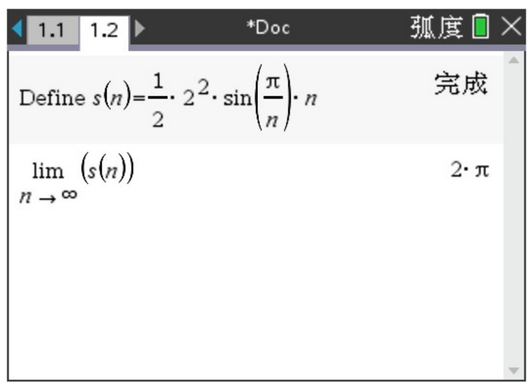
Case 1-11 Approximation

Step 3: sum all the areas of triangles



Case 1-12 Summation

Step 4: limitation



Case 1-13 Limit

The above process fully also explains how students master the concept of definite integral from graphically and numerically, and it also shows that an idea that partition and approximation is not unique. In addition, the second partition will also encourage students to solve the area of curve in polar coordinates later. Teachers could focus on each student's practice process and thinking methods with the function of wireless interactive classroom, which also increases the active participation and devotion of students in class.

Case 2—Comprehensive Practice of Application of integration 1

Activity I: Introduce the background

The instructor presents a series of phenomena and prompts students to identify their underlying causes, subsequently synthesizing these explanations into a common theme.

1) Why is it that when a coin is tossed, it predominantly lands on either its obverse or reverse face rather than its edge?

2) When 20 coins are thrown in a vertical arrangement, it is observed that they predominantly land on their edges rather than on their faces or backsides.

3) Why do organisms' transition from crawling on the ground to quadrupedal locomotion and ultimately to bipedalism?

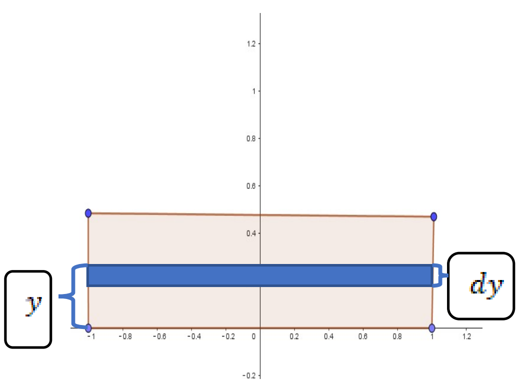
4) Look at curtain, why does it hang vertically instead of floating up?
Discussion:
These problems exhibit two primary characteristics: firstly, some are taken as given and obvious situations; secondly, they do not appear to share any common traits. However, delving beneath the surface to explore the essence of things is a crucial aspect of scientific inquiry. With this in mind, we will use coins as an example to uncover the underlying similarities behind these occurrences.

Activity II: Mathematical modeling for the coin

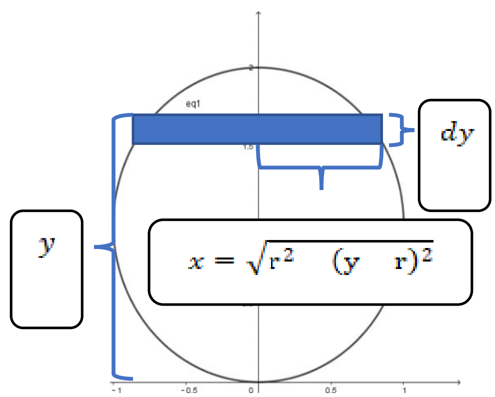
Assumption:

- A coin can be described as a right circular cylinder with a base of radius $\mathbf{r}$
- The coin exhibits a thickness of $\mathbf{d}$
- The density of the coin is ρ
- Acceleration of gravity is $\mathbf{g}$

According to the fundamental principles of calculus and with the aid of TI-Nspire Software, students can readily compute the gravitational potential energy of the coin W_1, W_2



Case 2-1 Partition in small piece



Case 2-2 Partition in the whole

Geopotential potential energy landing on the front or back:	Gravitational potential energy landing on the thin side:
$W_1 = \int_0^d y \cdot g \cdot \rho \cdot \pi \cdot r^2 \, dy = \pi\rho g \cdot \frac{d^2 \cdot r^2}{2}$	$W_2 = \int_0^{2r} \rho \cdot g \cdot 2y\sqrt{r^2 - (y-r)^2} \cdot d = \pi\rho g \cdot dr^3$

Therefore, if $d = 2r$, then $W_2 = W_1$. If $d < 2r$, then $W_2 > W_1$. If $d > 2r$, then $W_2 < W_1$. For a coin, d is significantly smaller than $2r$, thus prompting the coin to settle in a state of lower geopotential potential energy, either heads or tails. However, when considering 20 coins arranged in a column by tape where d exceeds $2r$, the water bottle also gravitates towards a state of lower energy upon landing on its side. This principle can be applied to other phenomena as well.

Activity III: Summary and Extension

Thus, it can be inferred that objects tend to exist in their lowest energy state. In this regard, the most natural state for humans would be lying down or walking on all fours, so that human standing upright is a form of resistance against nature. However, ultimately people cannot escape the fate predetermined by nature; much like a tossed coin will almost always land either heads or tails.

Extensional questions from the knowledge aspect:

1. The string assumes a singular configuration when suspended from two nails that are shorter than its own length. Is this curve representative of a quadratic function? What is the equation describing this particular shape?
2. Despite my efforts to toss them in a controlled manner, the handful of coins I hold inevitably scatter upon release rather than coalescing into a more compact formation. Why?

Extensional thought from students' growth perspective:

The act of human standing itself represents a form of resistance against nature, yet ultimately we cannot evade the predetermined fate bestowed upon us by nature. Similar to how a tossed coin will inevitably land on either heads or tails, humans too are subject to their own ultimate destiny. Therefore, how should we approach our existence?

Tagore's *Stray Birds*:

Death's stamp gives value to the coin of life; making it possible to buy with life what is truly precious.

Case 3—Comprehensive Practice of Application of integration 2

Activity I: Introduce the background

In Calculus BC, after students have learned application of integration including arc length of curve, and area bounded in polar coordinates, some practical problems make students motivated in deep learning. In the video or science museum, we have found not only circular wheels can roll smoothly, but also square wheels if provided a perfect road. Out of curiosity, they started to explore the perfect road for square wheels.

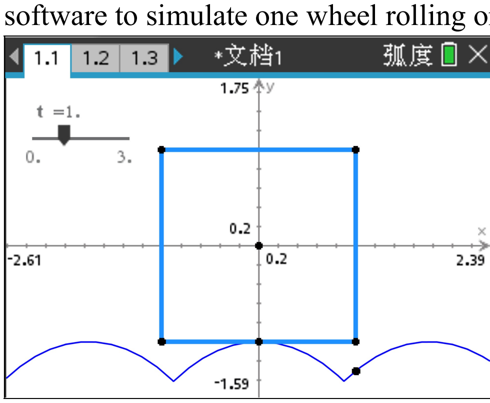
Activity II: Transform into mathematical problems

According to the video, teacher asked questions as followed:

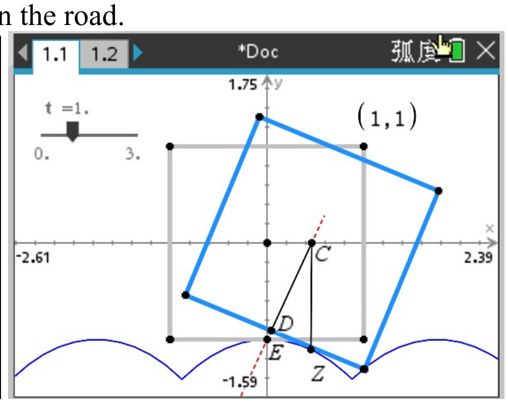
1) How to ensure smooth rolling of square wheels?

2) What is the relation of the contact point of the square wheel between the track and the axle (i.e. the center of the square)? Why?

In order to transform the practical problem into a mathematical problem, we used TI-Nspire software to simulate one wheel rolling on the road.



Case 3-1: Start Condition



Case 3-2: Process Condition

Discussion of student in groups:

Two key points to solve the problems are:

1) Specially designed track to ensure that the axle always moves in a straight horizontal line.

2) The contact point of the square wheel and the track is on a straight line perpendicular to the horizontal ground with the axle. At equilibrium, the force of gravity on the wheel is equal to and opposite to the force the track is exerting on the wheel.

Activity III: Solve the mathematical model

Due to the symmetry and periodicity of the wheel motion, it may be helpful to study the track equation of the wheel from the initial horizontal state to the first diagonal horizontal state. With the help of TI-Nspire Software demonstration for many times, continue the following questions:

1) Work in groups to discuss the position and geometry relationships in motion.

2) How to set up the equations?

3) How to solve?

Answer and Discussion:

1) Arc length of EZ=Line segment of DZ.

2) Let the contact point (x, y), so that C $(x, 0)$.

$$\int_0^x \sqrt{1+(y')^2}dt = \sqrt{y^2-1}$$

Differentiate with x on both sides,

$$\sqrt{1+(y')^2} = \frac{1}{2\sqrt{y^2-1}} \cdot 2yy'$$

Therefore, $(y')^2 = y^2 - 1$, $y' = -\sqrt{y^2-1}$ ($y' < 0$ for $x > 0$)

Solve

$$\int \frac{1}{\sqrt{y^2-1}}dy = \int -1\, dx$$

Method 1: (Formula)

$$\int \frac{1}{\sqrt{y^2-1}}dy = \cosh^{-1}(-y) + C, (y < 0)$$

So the solution of the differential equation: $\cosh^{-1}(-y) = -x + C$

With initial condition: $x = 0,\ y = -1$, so that $C = 0$.

$\cosh^{-1}(-y) = -x$

Therefore, $\boldsymbol{y = -\cosh(x)}$ (even function)

Method 2: (Trigonometric Substitution)

Let $y = \sec t$.

$$\int \frac{1}{\sqrt{y^2-1}}dy$$

As $y = \sec t < 0$.

$= -\ln\left|y - \sqrt{y^2-1}\right| + C$

So the solution of the differential equation:

$\ln\left|y - \sqrt{y^2-1}\right| = x + C$

Initial condition: $x = 0,\ y = -1$, so that $C = 0$.

As $y < 0$, so that $y - \sqrt{y^2-1} < 0$

$\sqrt{y^2-1} - y = e^x$

Therefore, $\boldsymbol{y = -\frac{e^x+e^{-x}}{2} = -\cosh(x)}$

Activity IV: Reflection and Extension

1) Summarize the contents AP calculus BC in this class.

2) Compare the two methods to solve the trajectory equation for square wheel in motion, what are the key points and difficulties to solve the problem?

3) Give the wide application of catenary and continue to expand learning after class.

4) Exploration:

(i) What are the perfect trajectories for the smooth motion of triangles, hexagons, ellipses, parabolas, and other shapes?

(ii) Can you find an ideal wheel for a known zigzag or sine wave track?

Feedback from students

Student A: Months ago, I went through a phase of what Albert Camus referred to as "absurdity." If our destiny is ultimately death, then what we do or care about now may seem meaningless. I was extremely confused and turned to reading to find some answers. I learned from "The Myth of Sisyphus," I understood why one could consider Sisyphus happy. Even though we may be tiny coins in a vast universe, we are still able to follow our personal beliefs. Our flips in the air may not cause any disturbances, but we can still be content with our own achievements.

Student B: My harvest mainly comes from the aspect of thinking. The accumulation of knowledge and concepts is sometimes novel, but the perception of essential ideas can always be memorable. Although I may not be able to understand every concept in the class, every time I understand the essence of a concept, I will have a strong sense of harvest and achievement. This may be the charm of mathematics.

Student C: Calculus is so useful in scientific research such as physics, chemistry, engineering, and economy problems, and helps us to analyze some interesting questions in our daily life. Technology has helped me to understand mathematical ideas intuitively and motivated me to focus on the formation of concepts and the core of mathematical methods. In addition, I was amazed by some practical problems to witness the process and method of mathematical modeling, which encouraged me to think of questions as a mathematician.

Student D: Technology is so powerful and helpful, which helped me to deeply understand the definition and idea of calculus, rather than calculation only from my self-study. It's easy for me to find out the geometrical relation in some complicated kinematics simulation made by my teacher. In my class, lots of peers enjoyed high interaction with each other and teachers to complete a challenging question. And I think technology has helped us a lot, and we will make more use of it in research in the future.

4. Conclusion

Incorporating technology into mathematics education enhances visualization and conceptual understanding of calculus. Graphing tools, interactive applets, virtual manipulatives, and problem-solving platforms provide more opportunities to explore integration in dynamic and interactive ways for students by themselves. By leveraging these technological resources, educators can foster a deeper understanding of integration concepts, promote active learning, and connect calculus to real-world applications. In addition, technology empowers students to develop the ability to combine algebra and geometry and to improve mathematics abstract, mathematical modeling, and intuitive imagination abilities which play a significant in mathematics learning. Furthermore, educators need to pay attention to evaluation and feedback collection from students, which will help to keep track of students' thinking processes in order to ensure the quality and effect in class.

It would be useful for teachers to find mistakes or misunderstandings so as to adapt the teaching process, and it will also encourage students to reflect on their studies.

In general, TI-Nspire Software is a powerful tool that can significantly enhance students' understanding of differentiation and integration concepts in calculus. With the help of visualization capabilities, step-by-step calculations, real-world applications, interactive activities, simulation, and error analysis, students can develop a deeper conceptual understanding of calculus, and solve practical problems by abstracting into mathematical models. Integrating technology effectively with sound pedagogical approaches will empower students to grasp these fundamental concepts with greater clarity and confidence, paving the way for success in advanced mathematics and related fields. All in all, technology not only has provided huge support for educators in teaching mathematics classrooms but also makes it more challenging for educators to explore the best way to facilitate the development of students' problem-solving abilities and mathematical core competencies in order to improve the development of students and education.

References

[1] Gordon, S.P.(2004). Mathematics for the new millennium. The international journal of computer algebra in mathematics education, 11(2), 37-44.

[2] Axtell, M. (2006). A two-semester calculus sequence: a case study, Mathematics and computer education, 40(2),130-137.