Construction of six heptahedra each line-symmetric to its dual

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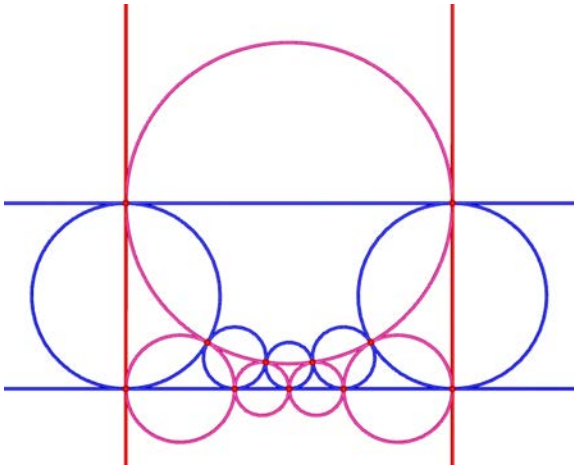
Extended Abstract (updated 2023-8-19)

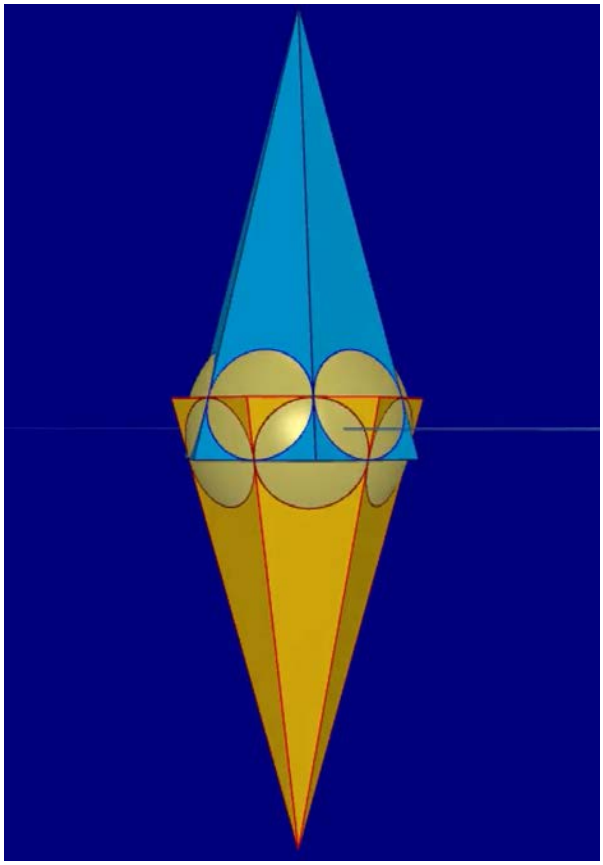
It is known that there are 34 topologically distinct convex heptahedra. Among them only six are self-dual. With the technology furnished by Cabri 3D and WolframAlpha, this paper presents a concrete construction of each of the 6 self-dual pairs together with the associated midsphere and the line of symmetry.

Main Results

1) (6,3,3,3,3,3,3)-(6,3,3,3,3,3,3) (one hexagonal and six triangular faces)



RSF (6,3,3,3,3,3,3)-(6,3,3,3,3,3,3)
(96KB) Animated gif: 5 frames
Construction: cg3 (77 KB)
Animated gif: 5 frames (318K)
Mov: (16.4 MB) (29,1 MB)



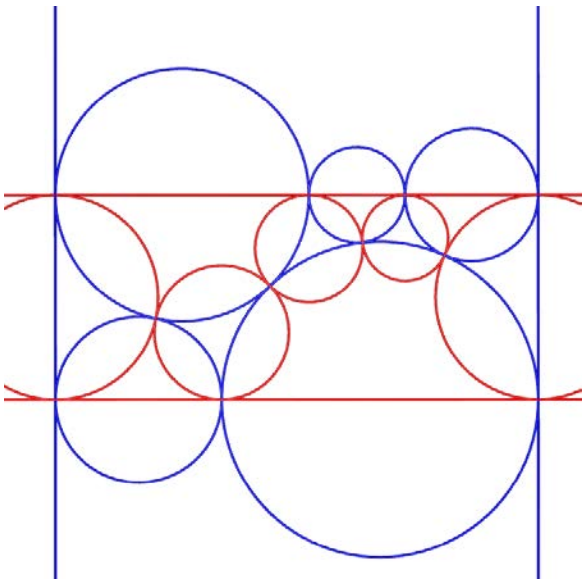
Line symmetric Heptahedra self-dual pair
(6,3,3,3,3,3,3)-(6,3,3,3,3,3,3).mov

Variations

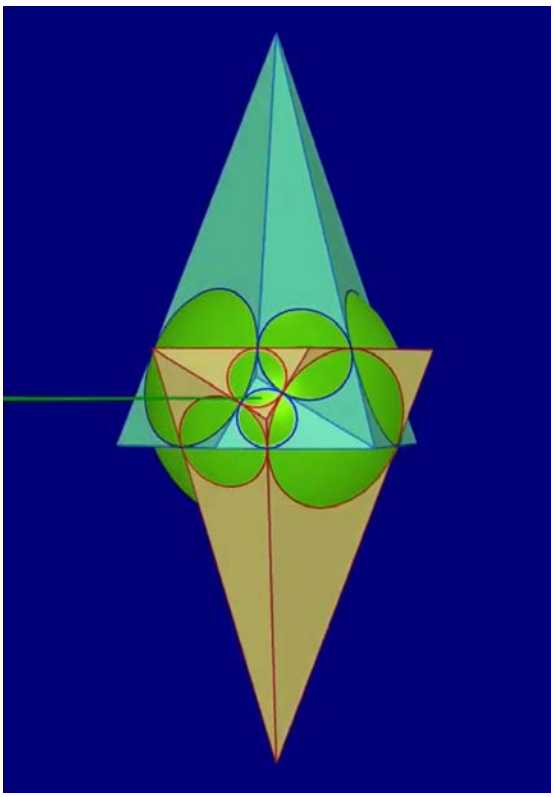
RSF of (6,3,3,3,3,3,3)-(6,3,3,3,3,3,3).pdf, RSF of (6,3,3,3,3,3,3)-(6,3,3,3,3,3,3).cg3

Related: 6T1H-6T1H.mov, Sangaku figure of (6,3,3,3,3,3,3)-(6,3,3,3,3,3,3).png

2) (5,4,3,3,3,3,3,3)-(5,4,3,3,3,3,3) one pentagonal, one quadrilateral and five triangular faces

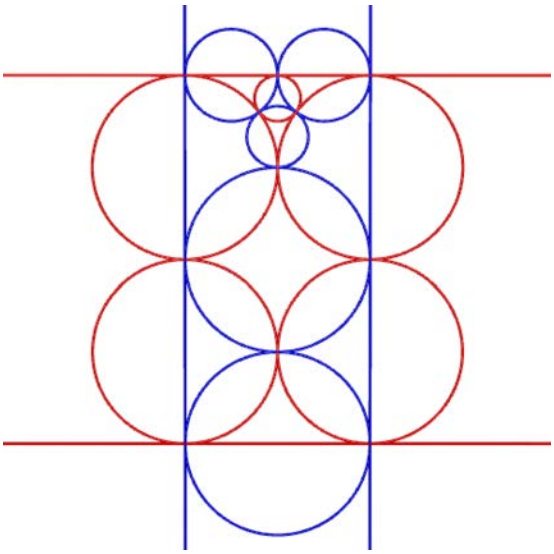


RSF(5,4,3,3,3,3,3;2)-(5,4,3,3,3,3,3;2)
RSF(5,4,3,3,3,3,3)-(5,4,3,3,3,3,3) based on 1.23486379761349

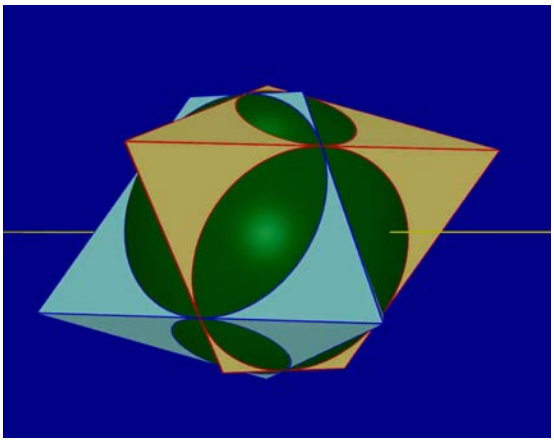


Variations (5,4,3,3,3,3,3)-(5,4,3,3,3,3,3) 1 min 7 sec.mov (88.8 MB)
1x.mov (11.7MB), 2x.mov (5.5 MB), 4x.mov (32 MB), Animated gif (6 frames 684 KB)
Self-dual (5,4,3,3,3,3,3)-(5,4,3,3,3,3,3) 2 min 12 sec.mov (65.9MB)
Self-dual heptahedra (5,4,3,3,3,3,3)-(5,4,3,3,3,3,3) with a pair of parallel faces.mov (26.6MB)
Concentric Sangaku for (5,4,3,3,3,3,3)-(5,4,3,3,3,3,3).m4v (10.6 MB)
Coaxial (5,4,3,3,3,3,3)-(5,4,3,3,3,3,3).mov (122.6MB)

3) (4,4,4,3,3,3,3;1)-(4,4,4,3,3,3,3;1), three mutually adjacent triangular faces



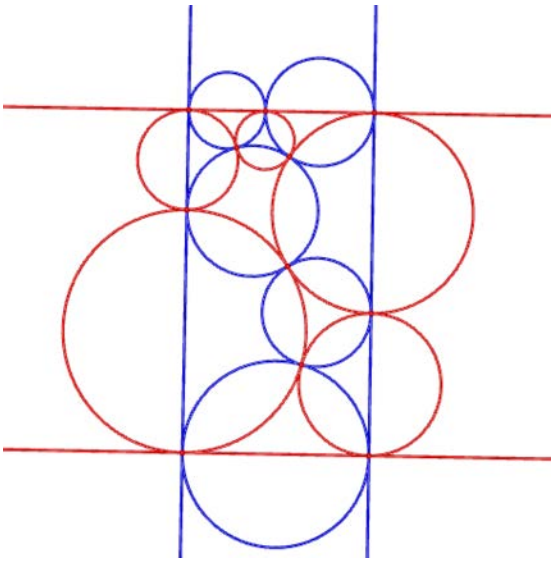
RSF(4,4,4,3,3,3,3;1)-(4,4,4,3,3,3,3;1)



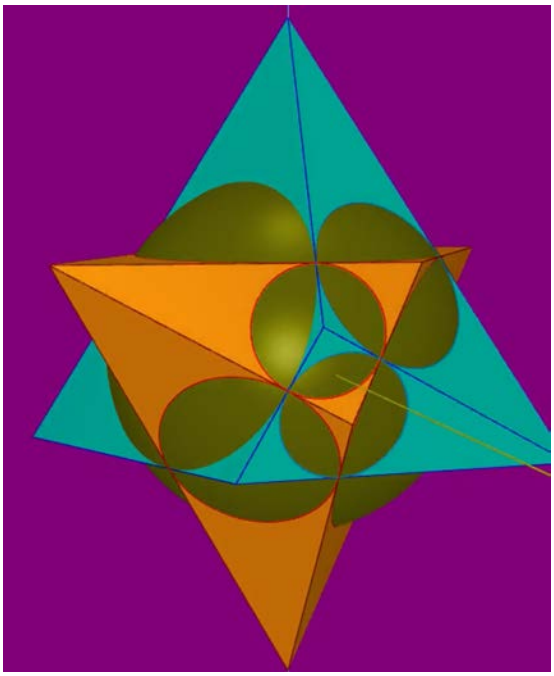
Variations of (4,4,4,3,3,3,3;1)-(4,4,4,3,3,3,3;1) 1 min 18 sec (68.1 MB)

Related: Circles of the same axis (4,4,4,3,3,3,3)-(4,4,4,3,3,3,3) 20 sec.mov (13.5MB)

4) (4,4,4,3,3,3,3;2)-(4,4,4,3,3,3,3;2), two pairs of adjacent triangular faces



RSF(4,4,4,3,3,3,3;2)-(4,4,4,3,3,3,3;2)



Variations of (4,4,4,3,3,3,3;2)-(4,4,4,3,3,3,3;2) 1 min 14 sec.mov (63.4 MB)

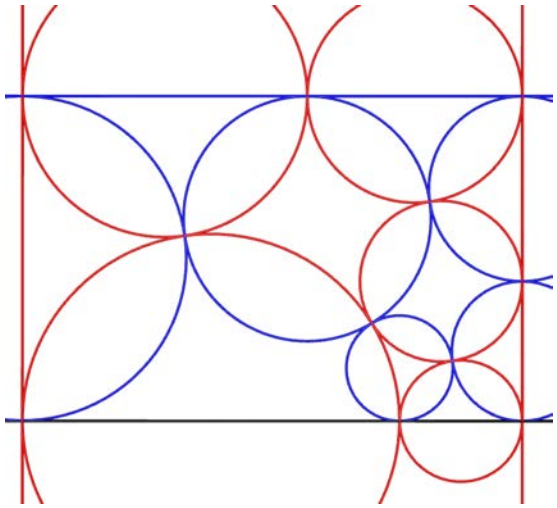
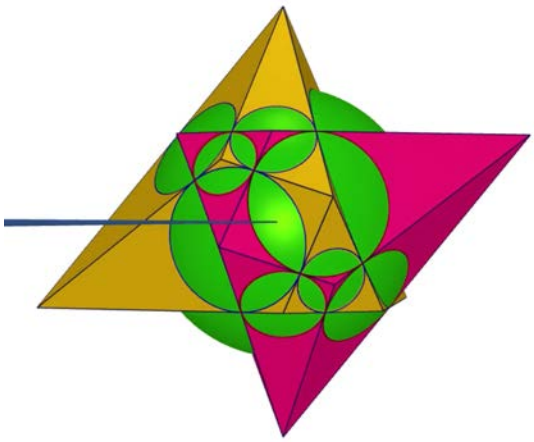
Related: (4,4,4,3,3,3,3) with 2 pairs of adjacent triangles.mov (30.9 MB)

Related: Concentric (4,4,4,3,3,3,3) with 2 pairs of adjacent triangles

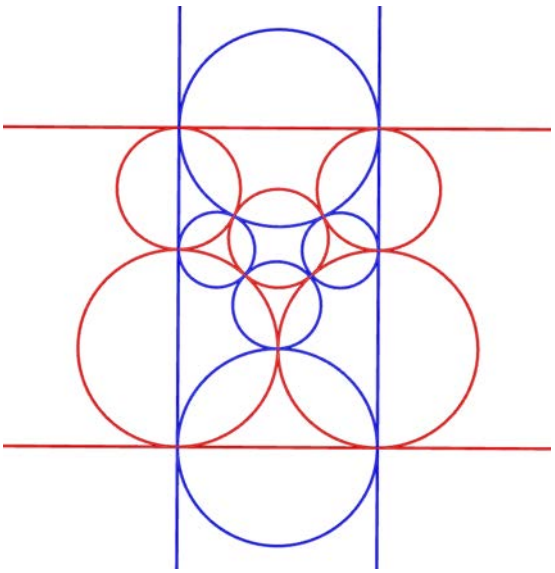
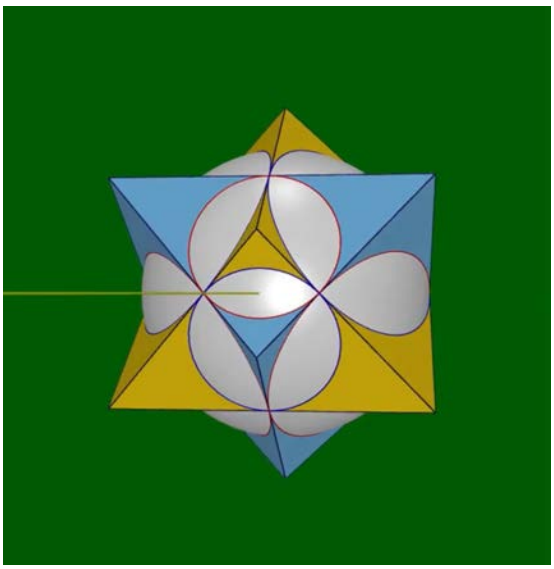
RSF of (4,4,4,3,3,3,3)-(4,4,4,3,3,3,3) having two pairs of adjacent triangles.cg3

Related: Two circles of the same axis (4,4,4,3,3,3,3) with 2 pairs of adjacent triangles.mov (13.7MB)

5) (4,4,4,3,3,3,3;3)-(4,4,4,3,3,3,3;3), exactly one triangular face adjacent to two triangular faces

<table>
<tr><td> 
RSF(4,4,4,3,3,3,3;3)-(4,4,4,3,3,3,3;3)</td><td>  </td></tr>
<tr><td colspan="2">Magic number: 1.32471795724475</td></tr>
<tr><td colspan="2">Variations(4,4,4,3,3,3,3;3)-(4,4,4,3,3,3,3;3)1 min 25 sec.mov (62.4MB)</td></tr>
<tr><td colspan="2">Self-dual heptahedra (4,4,4,3,3,3,3)[3]-(4,4,4,3,3,3,3)[3] 1min 25 sec.mov (112.8MB)</td></tr>
</table>

6) (4,4,4,3,3,3,3;4)-(4,4,4,3,3,3,3;4), exactly one triangular face adjacent to three triangular faces

<table>
<tr><td>  </td><td>  </td></tr>
<tr><td colspan="2">Variations of (4,4,4,3,3,3,3,3)[4]-(4,4,4,3,3,3,3,3)[4] 1 min 37 sec.mov (53.3MB)</td></tr>
<tr><td colspan="2">Variations of (4,4,4,3,3,3,3)-(4,4,4,3,3,3,3).pdf</td></tr>
<tr><td colspan="2">Variations of (4,4,4,3,3,3,3)-(4,4,4,3,3,3,3) 42 sec.mov (33.4 MB)</td></tr>
<tr><td colspan="2">Self-dual Heptahedra pair (4,4,4,3,3,3,3,3)-(4,4,4,3,3,3,3) 46 sec.mov (45.3 MB)</td></tr>
<tr><td colspan="2">Construction of RSF(4,4,4,3,3,3,3)-(4,4,4,3,3,3,3).gif (9 frames, 128K)</td></tr>
<tr><td colspan="2">RSF(4,4,4,3,3,3,3)-(4,4,4,3,3,3,3).cg3</td></tr>
<tr><td colspan="2">Related: Sangaku Figure based on the Golden Ratio</td></tr>
<tr><td colspan="2">Related: Direct construction of Sangaku Circles for (4,4,4,3,3,3,3)-(4,4,4,3,3,3,3)</td></tr>
<tr><td colspan="2">Related: Sangaku Figure from (4,4,4,3,3,3,3)-(4,4,4,3,3,3,3) dual-pair.pdf</td></tr>
<tr><td colspan="2">Related: (4,4,4,3,3,3,3)-(4,4,4,3,3,3,3) starting from Icosahedron</td></tr>
</table>

All animated gif files of this paper can be linked from Index of gif files.pdf

In designing/constructing the models, we have consulted these ancient geometric wisdom:

1) The Arbelos (Shoemaker's knife), in Leon Bankoff, A Mere Coincidence, Mathematics Newsletter, Los Angeles City College, November 1954.
2) Apollonius construction (to construct all the circles that are tangent to three given circles). See "Special cases of Apollonius' problem" in Wikipedia.
3) Sangaku problems
4) Inversion

We are unable to construct (5,4,3,3,3,3,3)-(5,4,3,3,3,3,3) nor (4,4,4,3,3,3,3;3)-(4,4,4,3,3,3,3;3) using ruler-and-compass along. Our construction of these two heptahedra depends on high-precision numerical approximations with WolframAlpha queries:
solve (1+(x^2-1)^ (3/2)+(x^2-1))(×^2-1)=1 for (5,4,3,3,3,3,3)-(5,4,3,3,3,3,3)
solve(1+x^4=x^2+x^3) for (4,4,4,3,3,3,3)[3]-(4,4,4,3,3,3,3)[3]

The paper is of interest in 3D Visual Art Design, in Math Competition and in Science Fair Project.

The extended abstract together with the associated *.cg3, *.mov, *.gif files are located at:
https://drive.google.com/drive/folders/1O4tlGHw7Z5JsSkgRXcziaD6Z0dXH2KDo?usp=sharing