Different types of test questions and test systems for student assessment in Applied mathematics for Informatics

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Abstract: Appropriately chosen teaching and testing methods enable students to adequately understand the taught matter through direct contact teaching in classrooms and distance e-learning and self-study at home. When teaching the subject of Applied mathematics for Informatics, students must not only be able to recognize which mathematical model to apply to solve a given problem but also to analyze and interpret their obtained results. In this paper, we demonstrate our approach to teaching and testing Applied mathematics for Informatics using the e-learning system Moodle for students of the Bachelor's degree at the Czech University of Life Sciences. Theoretical lectures and seminars take place in the Applied Mathematics for Informatics course. In seminars, students solve typical practical problems. In these seminars, the students solve specific problem situations from practice. They learn to recognize which mathematical models to apply to solve the problem and are encouraged to work independently and think critically. Students demonstrate this knowledge in the final exams, which are an "open book" standardized examination. Students can use their notes, books, and all resources available in Moodle. The tests contain a management story in which students, as managers, have to solve many decision problems using different mathematical models. The chosen method of teaching and testing knowledge helps increase the student's ability to understand the problems taught in Applied mathematics for Informatics, recognize and apply the appropriate mathematical model to solve a real decision problem and analyze and interpret the results.

1. Introduction

Teaching mathematics and mathematical subjects plays an essential role in today's era of modern technology. Quality and effective teaching still require personal contact with the teacher and the collective of students, especially in full-time study. Nevertheless, e-learning as a form of online learning also plays an increasingly important role. During the spread of the COVID-19 pandemic, e-learning played an essential role. Many educators have been forced to change their face-to-face learning activities and adapt to online teaching methods. E-learning is developing and changing as fast as information technologies and responds to current societal events. It is not only new technologies and information systems that play an essential role in e-learning but also the lecturers, tutors, and materials available to students. Of course, the method of teaching is also essential. According to Yang [13], teaching focused only on passing a test cannot foster creative thinking in the classroom. Early introduction of appropriate technological tools into teaching, learning, and research can never be wrong. The design of mathematics education in online environments is also discussed in more detail by Hollebrands, Anderson and Oliver [6]. They summarize the ideas presented by instructional designers and mathematics educators that relate to learning mathematics in online environments.

Nevertheless, the role of e-learning information systems is becoming increasingly important. One of the systems used to support e-learning is Moodle. Moodle is a free and open-source learning management system for creating learning materials and e-courses on the Internet [7]. It is a constantly evolving project based on a social constructivist approach to education. The word Moodle was originally an acronym for Modular Object-Oriented Dynamic Learning Environment. Moodle is a
freely available and popular e-learning system that can be used as a support tool for face-to-face learning and as a full-fledged software for building a distance learning portal, or possibly a combination of both. It allows easy publish teaching and learning materials online for the public and/or a closed group of users (students). The approach to learning depends entirely on the course creator. It can be designed as automated for large numbers of students, as individual with a high degree of feedback, or as a combination of both approaches.

When presenting teaching materials, their quality and processing are crucial for the understanding of the taught subject matter. Especially when teaching Applied mathematics for Informatics (AMIT), students must be not only able to recognize what mathematical model can solve a given problem but also be able to analyze and interpret the results obtained. The students are often unfamiliar with context-based mathematical questions ([10]). Well-prepared materials, based on practical examples taking into account the realities of a given country, can increase students' understanding of the problem, as investigated in their study Bhatti and Miranda [1].

In addition to the presentation of learning materials, Moodle also allows feedback through testing. Within the framework of student testing, the teacher can determine what level of knowledge the students have achieved. In Moodle, each attempt to pass a test is automatically graded. The teacher can set the number of attempts allowed, give feedback, display correct answers, shuffle the problems or randomly select from a bank of test questions. A time limit for testing can be set. Testing in Moodle allows the creation and upload the tests consisting of many types of questions. Appropriate selection and setup of test questions and tests with providing feedback and preparing students to take tests can contribute to an overall improvement in understanding of the taught matter, as shown by Rydval and Brozova [9] and Brozova and Rydval [3] in the subject AMIT.

Moodle allows the creation of dichotomous problems, comparison problems, short-answer, multiple-choice questions, and many more. Multiple-choice questions are very often used for student assessment. These questions can be modified in various ways in Moodle but also by using other tools; for example, Nagasaka [8] used the Python programming language to generate different answers within multiple-choice questions in Moodle in the Linear Algebra and Fundamental Calculus.

The system of how students answer tests plays an important role. A strictly standardized testing or examination approach allows students to use only their knowledge or MS Excel software for calculations. The open-book system allows the students to use their notes, books, and materials available in Moodle when taking the test. However, open-book test conditions imply that the questions must be reformulated so that students apply their subject understanding during the exam, rather than knowledge or the Internet to answer the question. Well-designed questions for an open-book test should emphasize the student's critical thinking and independence in discussing the subject matter [11]. However, students are not only affected by the presented study materials and tests during online learning, but they may also encounter various difficulties in online learning. The students must be competent computer and Internet users ([12]). Borges and Costa [2] found that the common difficulties encountered by students were: lack of space at home, Internet connection outages, and concerns about clarifying doubts in context.

The disadvantage to online assessment is the lack of instructor control over assessment conditions, leading to a breach of academic integrity ([14]), to cheating, impersonators, and outside help. Therefore randomizing questions is recommended as a useful technique. The advantage is easy management of tests, automated assessment, and collection of results on computers.

Our paper aims to show the use of different types of questions in the Moodle tests and the open-book testing system and their impact on student achievement in the subject AMIT.
2. Different types of tests and examining systems in AMIT

The subject AMIT has been in the Informatics program curriculum in regular and distance study in Czech and English in the Faculty of Economics and Management, Czech University of Life Sciences, Prague, since 2011. This subject followed the former subject "Methods of Operations Research" (MOR), with very similar topics. The lectures and seminars cover the main topics of the subject, definitions and steps of algorithms are highlighted during the teaching. The main topics of the subjects are:

- Introduction to Linear Algebra and Systems of Linear Equations
- Linear optimization model
- Transportation model
- Vehicle routing problems
- Multiple attribute decision-making models
- Matrix Games and Decision models

This subject builds on previous mathematics courses, which include calculus, mathematical logic, combinatorics, and graph theory.

The successful completion of AMIT means receiving the credit and passing the exam. For the credit, the students must pass the six Self-tests and, from the year 2019, one Big test. The exam consists of a written Exam test with an oral examination for the final grade. (The terms Self-test, Big test, and Exam test are used in our syllabus to distinguish different steps and forms of examination.)

The Self-tests consist of multiple-choice test questions, and students work on them at home. The total possible score of each Self-test is 100 points. Students have five attempts to complete each Self-test within a certain period. The best score is included in the final score. The maximum score from all Self-tests is 600 points. The students have to receive at least 360 points (60%). Each test covers one topic of the subject listed above.

The Big test is a shorter version of the Exam test and would help students prepare better for the Exam test. We prepare the Big tests because the results of this subject have not reached a satisfactory level. The Big test is always planned in the middle of the semester and covers the first four subject topics. The questions are more complex and consist of model construction, solving, and solution description. The total score is again 100 points. We have planned that the students have to receive at least 50% of the total points. In reality, we must decrease the limit to 30% in all three years of Big test application.

The examination until 2018/2019 had two parts: a written test and an oral examination. During the last lecture, the subject content is recapitulated, and the Exam test structure (Figure 2.1) is described together with the scoring and grading system.

The paper Exam test (used until 2018/2019) consisted of two theoretical questions, one small and one large example (Figure 2.1). The total possible score of the Exam test is 100 points. The grading system uses three grades: 60 – 73 points is a "good" grade, 74 – 86 points is a "very good" grade and 87 – 100 points is an "excellent" grade. During oral examinations, students must confirm their knowledge and fix their grades. The results level was constantly decreasing from 80% of passed students to 42% ([3], [9]). The reasons probably are that it is a mathematical subject, and students see mathematics as something difficult and removed from life and are unable to imagine its practical application.

Due to the COVID-19 pandemic, the testing process was moved to an online space from the summer 2020 exam session. The Exam tests have been reworked into Moodle tests. The Exam test consists of Open questions (the students have to paste calculations or describe the solution obtained), Calculated questions (partial results of calculations), Multiple choice questions (theoretical
questions), True/false questions, and so on. Example of the Moodle test is in the pictures Figure 2.2, Figure 2.3, and Figure 2.4. This test is based on a hypothetical practical story. It presents concrete mathematical problems from a real environment, where students demonstrate that they understand the problem and can interpret the results. The last three questions (not described in the Figures) are the multiple-choice questions "Elements of the models," "Solving methods of the models," and "Results of the models." The majority of questions are automatically corrected. The Open questions must be graded manually by a teacher. Even with the necessity of it, Moodle system greatly simplified the marking of the tests. Therefore, we were constantly looking for new and better test questions in the Moodle test system.

The students wrote these tests home during mandatory mass dates and without oral examination. It was a non-standardized examination procedure. The final grade was set only according to the number of points (minimum 60%). The students' results were much better than before (Table 2.1), undoubtedly due to the possibility of using all materials, but mainly due to cooperation with colleagues, i.e., "friend on the phone."

From 2021/2022, the examination has its original form again - a written part (Moodle test) and an oral part. We also returned to the system of many exam dates with tests written in standardized conditions, but we used an open-book system. The students' results improved to 2018/2019 (Table 2.1) but, surprisingly, got worse to 2019/2020 and 2020/2021. It turns out that students assume that if they have the advantage of using books, Moodle materials, and notes from lectures and seminars, they will be able to look up answers and not need to study for the exam.

<table>
<thead>
<tr>
<th>Year</th>
<th>Credit</th>
<th>Exam</th>
<th>No of students</th>
<th>Passed</th>
<th>Failed</th>
</tr>
</thead>
<tbody>
<tr>
<td>2018-2019</td>
<td>Self-tests</td>
<td>Paper test (on-site)</td>
<td>183</td>
<td>76</td>
<td>107</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>41.53%</td>
<td>58.47%</td>
</tr>
<tr>
<td>2019-2020</td>
<td>Self-test + Big test</td>
<td>Paper test (later online Moodle test)</td>
<td>216</td>
<td>143</td>
<td>73</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>66.20%</td>
<td>33.80%</td>
</tr>
<tr>
<td>2020-2021</td>
<td>Self-test + Big test</td>
<td>Moodle test (online)</td>
<td>280</td>
<td>191</td>
<td>89</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>68.21%</td>
<td>31.79%</td>
</tr>
<tr>
<td>2021-2022</td>
<td>Self-test + Big test</td>
<td>Moodle test (open book, on-site)</td>
<td>298</td>
<td>168</td>
<td>130</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>56.38%</td>
<td>43.62%</td>
</tr>
</tbody>
</table>

Table 2.1 The final students' results

From 2021/2022, the examination has its original form again - a written part (Moodle test) and an oral part. We also returned to the system of many exam dates with tests written in standardized conditions, but we used an open-book system. The students' results improved to 2018/2019 (Table 2.1) but, surprisingly, got worse to 2019/2020 and 2020/2021. It turns out that students assume that if they have the advantage of using books, Moodle materials, and notes from lectures and seminars, they will be able to look up answers and not need to study for the exam.

![Image of Exam test](Figure 2.1)

Figure 2.1 The paper form of the Exam test (closed-book written test)
Figure 2.2 Test questions 1-3 Small linear optimization model (open-book written Moodle test)

The HRV company decided to expand its product range because it found a supplier of the necessary plastic types for special ergonomic and design tools for working with a computer. Now it is necessary to decide which of the 3 products would be the most suitable to produce. It must produce at least 101 pieces.

The first type of plastic can be ordered up to 900 g, consumption for three new products is 5.6 and 1 g, respectively.

The second type of plastic can be ordered up to 888 g, consumption for three new products is 7.8 and 5 g, respectively.

The company assumes that he will achieve the following costs in CZK. The cost of producing the first product will be 289 CZK. The cost of the second product will be 391 CZK and the third 394 CZK.

What are the minimum production costs for an optimal production structure?

Answer: 

Choose... 9

Figure 2.3 Test questions 4-7 Transportation problem (open-book written Moodle test)

The HRV company supplies its stores in three different cities from its three warehouses.

The distances of warehouses and cities are in the following table:

<table>
<thead>
<tr>
<th>City 1</th>
<th>City 2</th>
<th>City 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warehouse 1</td>
<td>21</td>
<td>35</td>
</tr>
<tr>
<td>Warehouse 2</td>
<td>23</td>
<td>30</td>
</tr>
<tr>
<td>Warehouse 3</td>
<td>35</td>
<td>20</td>
</tr>
</tbody>
</table>

The capacity of individual warehouses is 100.

Ordered for stores in each city are 446, 438 and 425, respectively.

Is this transport system balanced?

Write down the size of the imbalance - how much it is unbalanced.

Answer: 

Transportation model:

Select one:
- always has at least one feasible solution
- may or may not have an feasible solution
- never has an feasible solution
- always has just one feasible solution
- always has an infinite number of feasible solutions

Please enter a calculation of the supply problem.

It is evaluated, is mandatory.

Answer: 

What are the transport costs for the optimal supply of shops in cities?

Answer: 

The company's new HW product is in demand, the demand is growing, and therefore the company is considering a change in production volume.

It considers three possible variants: basic production volume (100%), larger production volume (180%) and maximum production volume (300%).

They assess their impact on the basis of the expected size of demand in new products. Values mean maintaining (1), reducing or increasing profits with respect to unfulfilled and missing production.

At the same time in the table you will find the degree of expectations of individual states of nature expressed by points.

<table>
<thead>
<tr>
<th>Production volume</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 100%</td>
<td>1</td>
</tr>
<tr>
<td>2 - 180%</td>
<td>1.3</td>
</tr>
<tr>
<td>3 - 300%</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Determine the probability of the same demand in the future.

Hint: Determine the probability of states of nature as you determine the weights from the point evaluation using normalization.

If we assume that the demand for products would be the biggest in the future, what volume of production would be most appropriate?

In this case, it is best to choose:

Select one:
- 100%
- 180%
- 300%

Use the Laplace criterion for the decision support.

Then the decision of the 180% volume of production will be evaluated by

Answer:

Use the EMV criterion for the decision support.

Then the decision of the 180% volume of production will be evaluated by

Answer:

Analyze the dominance of individual possible decisions.

It is an evaluated answer, i.e. mandatory.

Indicate to the HW company the most suitable production volume.

Second answer, i.e. mandatory.

Describe and analyze the results obtained.

Decision alternatives are actually irrational player strategies.
3. Formulas question for the Simplex algorithm

Because we are trying to prepare tests in which we will also be able to evaluate the understanding of the problem, we are looking for new Moodle question types. The main requirements for question-type properties are the possibility:

- Easily write the necessary formulas to calculate the (un)correct answers;
- Randomly generate a sufficient number of numerically different question variants;
- Combine numerical and word answers;
- Choose different sets of sub-questions or questions;
- Contain algorithms for solving basic operational research models, primarily linear optimization models and transportation problems, vehicle routing problems, and others.

Since there is no particular Moodle question type for Operations Research methods, we choose the Formulas question type, which meets the first four requirements. From the fifth requirement, we mainly focus on the Linear optimization model. The test questions about this model are tough to implement in some more general versions. Therefore, we decided to implement the Simplex algorithm as our new plug-in that adds this algorithm to the existing Formulas question, preserving the original question functionality. Our Simplex plug-in is written in PHP and uses the functions that the Formulas question type supports [5]. An example of the Formulas question type with the Simplex plug-in is in Figure 3.1.

Figure 3.1 Example of Formulas question with Simplex plug-in

This Formulas question focuses on solving a real problem where students must select the proper model, construct it, solve it, and interpret the results. The students receive only a story describing the problem and necessary data and have to demonstrate their knowledge and ability to choose an appropriate mathematical model to solve the problem. With the open book form of the test, students are encouraged to work independently and think critically.

The following tables (Table 3.1 and Table 3.2) show how to set up all necessary data in the Formulas question with the Simplex plug-in from Figure 3.1.
Random variables

- \( r = \text{shuffle([5,6,7,8,9])}; \) # random values of risk points
- \( c = \text{shuffle([1,2,3,4,5,6,7,8,9])}; \) # random values of return
- \( l = \text{shuffle([1,2,3,4])}; \) # random values of liquidity points
- \( P = [7:20]; \) # random values of total budget
- \( \text{structural} = [1,2,3]; \) # random index of structural variables
- \( \text{slack} = [4,5,6]; \) # random index of slack variables

Global variables

- \( \text{Variable} = ["Stocks","Bonds","Share certificates"]; \)
- \( \text{Svariable} = ["Uninvested money","Risk slack","Liquidity surplus"]; \)
- \( \text{all} = \text{concat(Variable,Svariable)}; \)
- \( r1 = r[0]; r2 = r[1]; r3 = r[2]; R = (\text{max}(r_1,r_2,r_3)-1)*P; \)
- \( l1 = l[0]; l2 = l[1]; l3 = l[2]; L = (\text{min}(l_1,l_2,l_3)+1)*P; \)
- \( c1 = c[0]; c2 = c[1]; c3 = c[2]; C = [c_1,c_2,c_3,0,0,0,-100]; \)
- \( A1 = [1,1,1,0,0,0,P]; A2 = [r_1,r_2,r_3,0,1,0,0,R]; A3 = [l_1,l_2,l_3,0,0,-1,1,L]; \)
- \( \text{Zopt} = [3,8,1,100]; \)
- \( \text{XS} = [3,8,1,\text{structural}]; \)
- \( \text{Var} = \text{Variable}[_{\text{structural}-1}]; \)
- \( \text{XSD} = [3,8,1,10*\text{structural}]; \)
- \( \text{XD} = [3,8,1,\text{slack}]; \)
- \( \text{VarS} = \text{Svariable}[_{\text{slack}-4}]; \)
- \( \text{XDD} = [3,8,1,10*\text{slack}]; \)

### Answers+Questions

**Question 1 (5 points)**

\[
\text{simplex(Zopt,C,A1,A2,A3)}
\]

Under the given conditions and requirements of the client, the maximum achievable return of the portfolio is \(_0\) million CZK.

**Question 2 (5 points)**

\[
[\text{simplex(XS,C,A1,A2,A3)},-1*\text{simplex(XSD,C,A1,A2,A3)}]
\]

\(\{\text{Var}\} \) is optimal to buy for \(_0\) million CZK. If they are not to be bought (if the variable is not in the base), their purchase would change the yield by \(_1\) million CZK. If the yield increases, write a positive number; if it decreases, a negative number; if the variable is in the base, write 0.

**Question 3 (5 points)**

\[
[\text{simplex(XD,C,A1,A2,A3)},-1*\text{simplex(XDD,C,A1,A2,A3)}]
\]

\(\{\text{VarS}\} \) equals \(_0\) million points. Changing this value (if the variable is not in the base) would change the yield by \(_1\) million CZK. If the yield increases, write a positive number; if it decreases, a negative number; if the variable is in the base, write 0.

### Table 3.2 ID codes of results

<table>
<thead>
<tr>
<th>ID</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>The optimal value of the objective function</td>
</tr>
<tr>
<td>10, 20,...,90</td>
<td>Shadow costs of variables 1, 2,...,9</td>
</tr>
<tr>
<td>11-19,21-29,...,91-99</td>
<td>Values from the optimal simplex table (first number = index of column, second = index of row)</td>
</tr>
<tr>
<td>1,2,...,9</td>
<td>Optimal values of variables</td>
</tr>
<tr>
<td>-1,-2,...,-9</td>
<td>The maximal possible value of variables according to the optimal solution and constraints</td>
</tr>
<tr>
<td>-11,-12,...,-19</td>
<td>Indexes of basic variables</td>
</tr>
<tr>
<td>-21,-22,...,-29</td>
<td>Pivot values of each step of the JEM calculation</td>
</tr>
</tbody>
</table>

The **Simplex plug-in** supposes the canonical form of a model with slack and artificial variables also, e.g., equation form of constraints, canonical basis, and non-negative right-hand side values. The maximal number of all variables is 9, and of constraints also 9. The Simplex method is calculated by calling \(\text{simplex}(q,c,a1,a2,...,am)\). The letter \(c\) is a vector of cost values, \(a1, a2,..., am\) are vectors of coefficients of each constraint, including the right-hand side value. The letter \(q\) represents the question as a quadruple \(q=[n,m+1,1/0,ID]\); where the \(n\) is the number of all variables, \(m\) is the number of all constraints, \(1/0\) means maximization/minimization, and \(ID\) is a code of needed result, for instance, 100 means the optimal objective function value (see Table 3.2).
This plug-in meets our requirements. We will use it in the Big and Exam tests in 2022/2023. We also anticipate creating other complex algorithms for Operations Research models.

4. Conclusions

The importance of e-learning has been growing recently. Its importance was demonstrated mainly during the previous years of the COVID-19 pandemic. In the teaching and learning of mathematics, the Moodle system seems to be very suitable, as it allows the creation and presentation of learning materials that, when based on real situations of managerial decision-making, encourage students to understand the material discussed and work independently and think critically. It is also very positive that the Moodle system allows to set up of testing students and allows the choice of many test questions. Proper selection of questions helps test students' knowledge and ability to understand the problem and interpret the results.

Within the subject AMIT at the Czech University of Life sciences Prague, in the Czech Republic, we use the following questions in the Moodle system for testing students: calculated questions, true/false questions, multiple-choice, and open questions. The form of student testing is an open book, allowing students to use their notes and thus encouraging them to work more independently. During the COVID-19 pandemic, students took the online test at home and performed better than before the pandemic, but with a different testing form. We suppose that their better success was not only due to the change in test form (from written test to e-test) but also due to the effect of the home environment and the opportunity to cheat.

After the end of distance learning, we have continued to keep the electronic form of the test, but students take it in a supervised classroom. Moreover, we have retained the format of the test questions and open book system. Although the current student scores are worse than during the COVID-19 pandemic, they are better than before the COVID-19 pandemic. We suppose this is because the new form of learning materials that better reflect real-world decision-making situations and the new form of test questions and open book testing system forced students to better prepare for the test and thus perform better.

Furthermore, we suppose the introduction of Formulas questions with the implemented Simplex plug-in in Moodle tests very positive for applying a linear optimization model using the simplex algorithm. This test question has allowed us to modify the Moodle testing system to meet our requirements better. We anticipate that the questions based on practical stories will force students to prepare better and learn the subject AMIT more effectively because this question will be more exciting and pleasant for them. Our next step now is to prepare the plug-in for the Transportation model.

References


