The Potential of Outdoor Mathematics in a Digital Context

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Abstract: *In this paper, we present the MathCityMap system which aims at bringing the idea of mathematics trails into an educational context. To facilitate the preparation and conduct from both, teacher's and students' perspective, the system provides two components: a web portal to create outdoor tasks and a smartphone app to guide students to discover mathematics outdoors. In this paper, we focus on the benefits of the math trail idea for educational purposes and, in particular, its enrichment through digital tools in the context of mobile learning. By taking learning theories and empirical findings into consideration, the potentials of the system are analyzed and evaluated. Hereby, the focus is on different aspects of math trails and, in particular, math trails with MathCityMap that differ from "classic" mathematics teaching, i.e. the autonomous learning, the potential for modelling tasks, the use of mobile learning, the role of embodiment and their empirical impact on students' performance and motivation.*

1. Theoretical Background of Math Trails

Learning and the learning of mathematics can take place in different learning environments. For sustainable learning, [8] describe that a learning environment should offer enough space for own discoveries and still introduce new discoveries stepwise. In addition, it should communicate the required knowledge directly when needed and the task difficulty should not be too high.

In such a learning environment, mathematics does not have to take place exclusively in the classroom. Due to the demand for relating mathematics education to reality, i.e. contextualized tasks with relevance to the living world, a current trend emerges to do mathematics outside – often found under the catchword "outdoor mathematics" [10]. So-called mathematical trails (also known as "math trails") offer a possibility to meet this demand and to consciously perceive and apply mathematics in the environment. When running a math trail, objects in the environment become the centre of mathematical tasks through appropriate questions. In this way, it is possible to transfer tasks that are known from the textbook to everyday objects, places and concrete situations that are familiar to the students. A math trail is thus a mathematical path along a map with several fixed places where mathematics can be experienced. At these places, there are tasks that can only be solved on site because to solve the problem, the own data have to be collect exactly at these places. In order to decide which data must be collected, the task solver must have a mathematical model for this situation [15]. This can be, for example, the model of direct proportionality when counting paving stones in a limited area, the gradient triangle when calculating the slope of a wheelchair ramp, or a suitable geometric body when determining the weight of a stone.

The idea of the math trail is not new, but already several decades old. However, its original intention was not focused on a school setting but was to popularize mathematics in society. The math trail created by [4] in Melbourne in the 1980s was intended as a vacation activity for the whole family. Accordingly, the tasks were set in such a way that simple basic arithmetic was sufficient to discover or just discuss mathematical relationships and phenomena in the environment.

Over the past several decades, there have been scattered reports of positive experiences using math trails in schools [17]. Hereby, using math trails in school seems legitimate in many respects. Emotions, interest and personal significance have been shown to correlate positively with (mathematical) performance [18].

Already at first glance, math trails differ from the “classic” mathematics teaching and learning inside the classroom. When walking a math trail, students work in small groups on various tasks. The groups are independent of each other and also quite independent of the teacher. Because of this open form of learning, math trails are reminiscent of the “learning at station” method, in which students work on tasks independently. [3] emphasizes, in particular, the autonomous learning at one's own pace and the observing, rather passive role of the teacher during the processing as advantages of this method. It is true that the observing role of the teacher in math trail can only be realized to a limited extent due to the spatial distance of the tasks – in contrast to learning at station in the classroom. The feature of the Digital Classroom of MathCityMap (see section 2), which allows the teacher to observe the students' entries, walking paths and hint retrievals and to analyze them after they have been completed, provides the teacher with diagnostic possibilities. In addition, students can chat with the teacher if necessary and ask for individual hints. Nevertheless, this request for assistance and, in particular, the development of the solution lies with the students, whereby autonomous learning takes place.

Not only methodically, but also in terms of content, the math trails require independent decisions from the students. Unlike in the textbook, the tasks do not specify data about the object and students have to decide independently which data should be collected based on a mathematical model and which data can be collected at all [10; 12]. Through a math trail, an introduction to mathematical modelling is usefully possible.

Another benefit of working on math trail tasks is the own (physical) activity on site. Studies show that one's own physical activity has a positive effect on cognitive learning [19], which is understood in terms of "embodied mathematics" as an important basis for grasping mathematical concepts [16]. The embodiment view emphasizes that mathematical concepts and terms cannot be meaningfully thought about and grasped without corresponding bodily experiences. For example, the circumference and diameter of a cylinder can be internalized in a contrasted manner if these quantities have been actively measured and thus the difference is also evident in the active measurement. Especially the connection between these enactive actions on the one hand and iconic representations (e.g. a sketch of the object) and the symbolic representation (e.g. formula of an object’s volume) on the other hand, is valuable for learning mathematics [15].

Nevertheless, these theoretical benefits have not led to a widespread use of math trails in mathematics education. The main reason for this is probably the high effort required to create a math trail. In particular, the development of the tasks and the compilation of the map that contains the tasks should be mentioned here. More recently, there has been increased experimentation with electronic maps (e.g. Google Maps) and QR codes for task delivery to cell phones. But even this did not lead to the desired minimization of the workload in creating math trails. By combining a web-based database and a mobile app, the MathCityMap system has brought the idea of math trails into a digital and didactic context [15].

2. The Digital Components of the MathCityMap System

The MathCityMap system consists of two main technical components. First, there is a web portal (www.mathcitymap.eu; Figure 2.1 left) that acts as an international database and community portal. On the other hand, there is a corresponding smartphone and tablet app (Figure 2.1 right), which loads a selected math trail from the portal and makes it available to users (e.g. students) while they are walking along the trail.

Figure 2.1 The MathCityMap Web Portal (left) and Smartphone App (right)

Task and Trail Creation **in the Web Portal.** The web portal offers the possibility to view tasks to get ideas and create your own tasks. When creating tasks, the portal allows to position the task pin on the map by mouse click and to upload a photo of the task's object. If geotagging is activated when the picture is taken, the system automatically takes over the positioning of the task pin. During the task creation, it is possible to choose from different answer formats. As solution format, the system allows exact values, intervals, multiple choice, fill-in-the-blanks, set, vector as well as GPS coordinates as answer. Thus, combinatorial tasks with an exact solution as well as measurement and modelling tasks, where small deviations should not lead to a wrong result, can be equally realized. In addition, stepped hints and one possible sample solution must be provided for each task.

To make it easier for the user to create tasks, there is a catalogue of so-called "generic tasks" – selected tasks that can be created as if by magic with the help of templates (Task Wizard) with just a few clicks. The intention here is to make frequently found task objects, such as the slope of a ramp or the speed of an escalator, transferable to new locations with as little effort as possible.

Users can combine their self-created tasks with public tasks to form a math trail. Once the trail has been created, it is assigned a unique code. The trail can now be downloaded using the MathCityMap app via code. Afterwards it is available for the actual math trail walk.

The MathCityMap App. In the app's section "Add Trails", students can add a private trail by entering the trail code from the web portal. Once the download of a trail is complete, no further internet connection is required to run a trail with the basic functions described in this section. With the help of a map and your own location (Figure 2.2 first row left), the app guides students along the tasks of the trail. By clicking on the respective task pin, the task text and the picture of the object are displayed (Figure 2.2 first row middle).

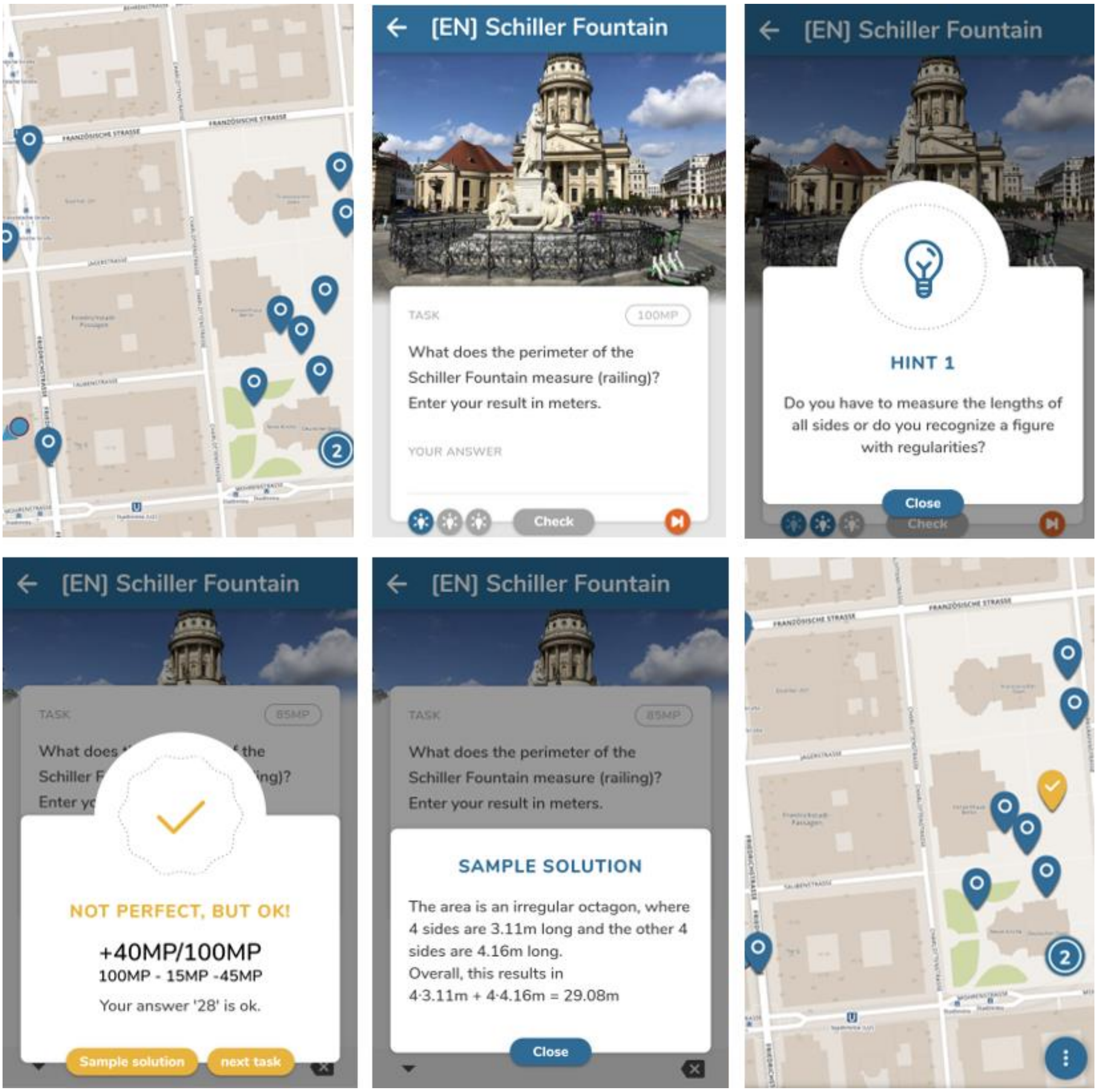
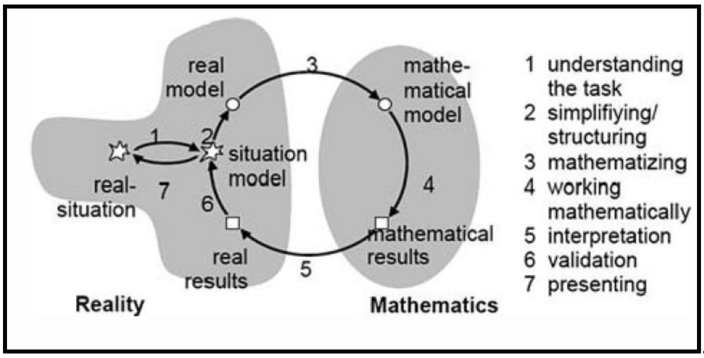


Figure 2.2 Screenshots in the MathCityMap App
from left to right - First Row: Map (left), Task Formulation (middle), Hints (Right) – Second Row: Answer Validation (left), Sample Solution (middle), Map with Validation (right)

The app upgrades the paper version of a math trail with a stepped help system by displaying hints (Figure 2.2 first row right), as well as automated and direct feedback after solutions have been entered (Figure 2.2 second row left). Task creators can add points as gamification elements when creating a trail. Then students receive up to 100 points per task. From the second wrong solution entry, 15 points are deducted to prevent guessing. The use of hints, on the other hand, does not lead to a point deduction. With the "interval" answer format, further points may be deducted, depending on the quality of the solution (in the example of Figure 2.2 the deviation is about 45 points). When setting the solution interval, the collected measured values and possible deviations are taken into account. This results in a green interval for very good solutions, a yellow interval for acceptable solutions and a red interval for wrong solutions. After entering a correct or acceptable solution or giving up on the task, it is possible to view the sample solution (Figure 2.2 second row middle). Depending on the range of the solution interval in which a task was solved, the app gives corresponding feedback and displays the tasks that have already been solved on the overview map (Figure 2.2 second row right). Blue pins represent tasks that have not yet been worked on. Grey pins represent tasks that have been skipped and can be worked on again later.

MathCityMap sample tasks. After presenting the basic technical components, this paragraph presents sample MathCityMap tasks get an idea about the actual use of outdoor mathematics. One of the main goals of outdoor mathematics is to foster the application of mathematics and mathematical modelling competencies.

Hereby, we understand modeling as the ability to work out relevant questions from the environment, to transfer them to mathematics, to work on them mathematically and finally to validate and interpret them on the basis of the given real situation. Furthermore, the ability to choose from different models and to evaluate them is also part of the modeling ability [5]. In particular, the aspect that different models can be chosen to address a real-world problem is emphasized by MathCityMap tasks. At this point, it should be noted that MathCityMap tasks are mainly questions that are particularly suitable for providing an introduction to modeling and should therefore be distinguished from complex and extensive modeling tasks. The fact that a math trail is usually composed of ten different tasks and that each task should be completed in a time frame of up to 15 minutes makes it clear that MathCityMap tasks cannot require every single modeling step in its full complexity. Therefore, in the following we deal with individual steps in the modeling cycle and present suitable examples from the MathCityMap project. Most of the MathCityMap tasks focus on simplifying and mathematizing the real situation into an adequate mathematical model, which corresponds to steps 2 and 3 of the seven-step modeling cycle (Figure 2.3 left) according to [5]. In simplifying, important information is separated from unimportant information taken from the real situation. Mathematization involves the translation of the simplified real situation into mathematical models. This is illustrated by the sample task (see Figure 2.3 right).



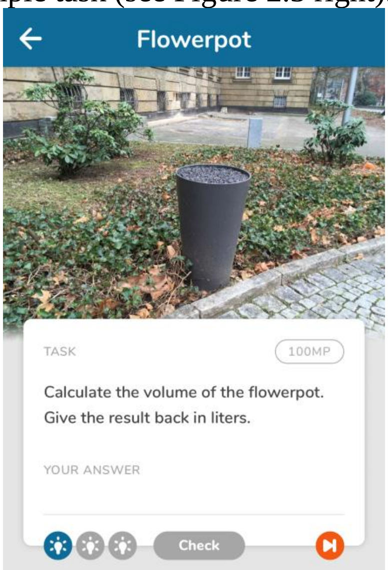


Figure 2.3 The Modelling Cycle according to [5] (left) and the Sample Task “Flowerpot” (right)

For the flower pot, which can be described fairly accurately by a truncated cone, three different geometric approaches could be observed among 9th grade students solving the task outdoors. This can probably be attributed to the fact that the formula of the volume of a truncated cone is not familiar or present to every group. Despite students not being able to solve the task, the following modelling processes were performed (see Figure 2.4).

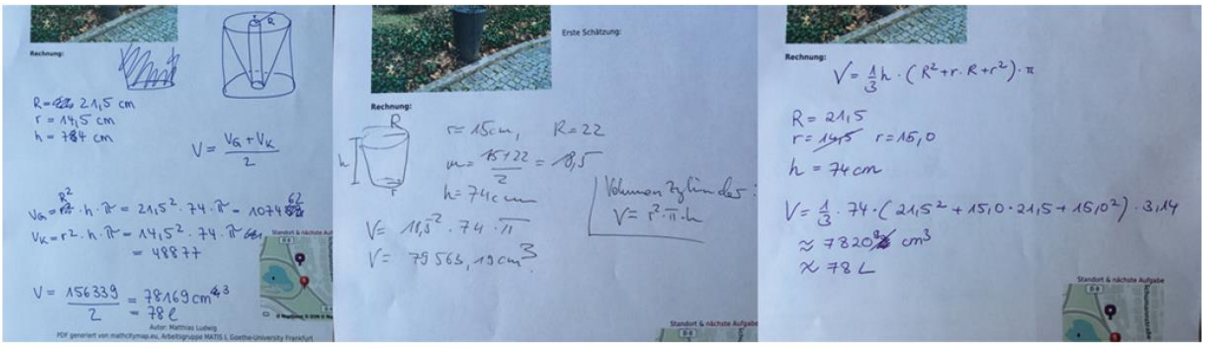


Figure 2.4 Three different Student Solutions on the Sample Task "Flowerpot"

Here, each solution emphasizes a different mathematical model and the students have worked with it mathematically in different ways. The first solution variant (Figure 2.4 left) approximates the result by the mean value of the volume of a cylinder with the large radius (R) and a cylinder with the small radius (r), thus:

$$V = \frac{R^2 + r^2}{2} \cdot h \cdot \pi$$

The second possible solution (Figure 2.4 middle) approximates the result by a middle cylinder. For this, the students take the mean value of the small (r) and large radius (R) as the radius, resulting in the following mathematical model:

$$V = \left(\frac{R + r}{2}\right)^2 \cdot h \cdot \pi$$

The third student solution (Figure 2.4 right) is based on knowledge of the formula of the volume of a truncated cone with:

$$V = \frac{R^2 + Rr + r^2}{3} \cdot h \cdot \pi$$

It seems particularly interesting here that the real results of all three approaches hardly differ. On the one hand, this can be explained by the carefully collected measured values of all three groups of students. On the other hand, the shape of the truncated cone shown here does not differ that much from an exact cylinder, so that the models used only lead to minor deviations.

We conclude: Each solution deals with different mathematical models. For each approach, the students have to create their own real model and then they mathematize it by adding variables they have to measure. The students hereby have to think about which data they have to measure – a difference to modelling tasks in the classroom.

Other typical sample task in the MathCityMap system – also with respect to the idea of generic tasks at frequently found objects – are the determination of the slope of a handrail, the weight of a stone or the age of a tree (see Figure 2.5).

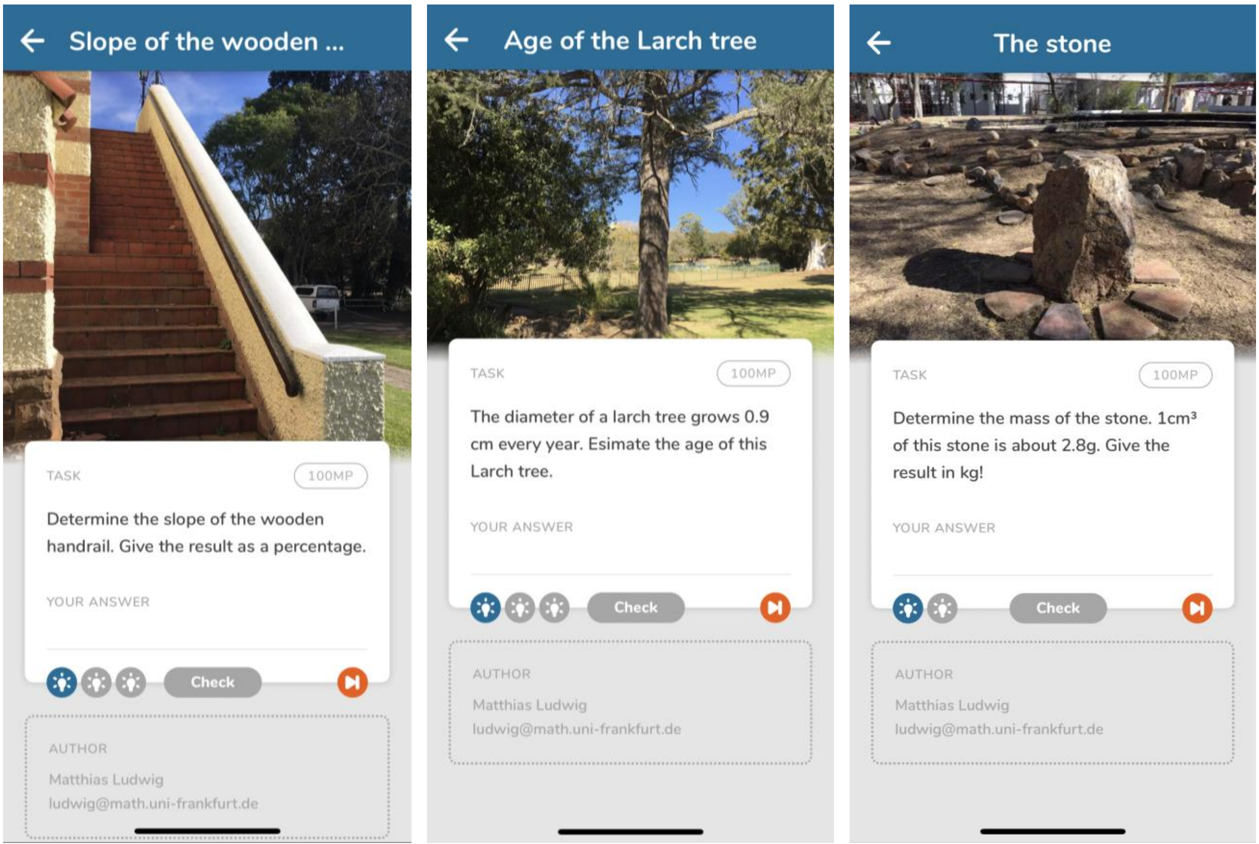


Figure 2.5 Three Generic Tasks in the MathCityMap App

The Digital Classroom. A challenge when conducting a MathCityMap math trail with a school class is to keep track of what is happening. Similar to the learning at station method, when starting a MathCityMap math trail, students are sensibly divided into small groups to work at different stations in the area of the math trail. Subsequently, they work on task after task. This makes it impossible for the teacher to keep an eye on all the learners and, if necessary, to provide support. This is where the digital classroom of MathCityMap comes into play. The digital classroom is a temporary pedagogical digital environment that allows the teacher to communicate with the learners via smartphone during a math trail, while at the same time tracking their learning status and determining their position.

To create a digital classroom, the teacher goes to the MathCityMap web portal and selects the desired trail, he/she would like to use in the digital classroom. Since the digital classroom is intended to be used only temporarily, a start and end time must be defined. The system now generates an access code for the digital classroom that has just been created. This code is used to download the trail onto the smartphone. The participating students are informed about the conditions of use and have to enter a "player name" and the group members. While the digital classroom is active, the teacher has access to a special interface in the web portal. In this interface, three main functions for class management and diagnosis are available during the math trail.

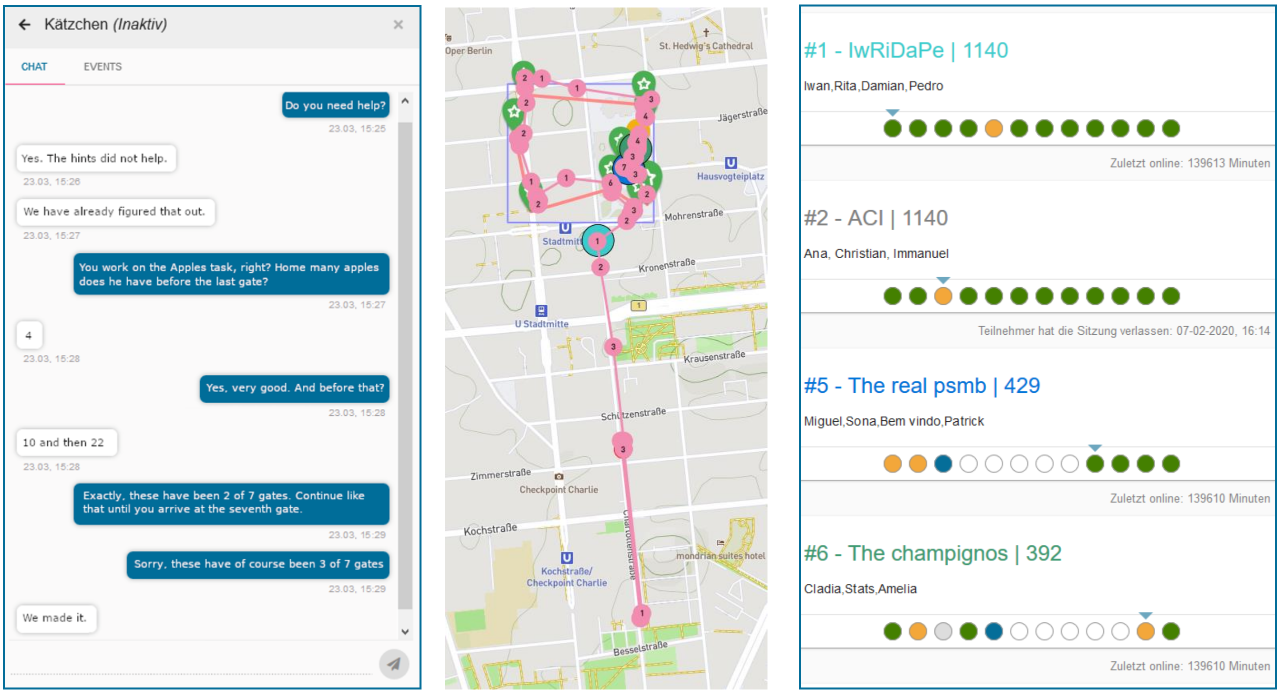


Figure 2.6 Chat (left), Walking Paths (middle) and E-Portfolio (right)

The chat (see Figure 2.6 left) as a communication channel allows the teacher to give instructions or differentiated help to all or selected students digitally in real time. In addition, students can request help when problems arise or have part of their solution (e.g. measured values) validated by the teacher.

The walking path tool (see Figure 2.6 middle) shows the teacher on which tasks the learners are currently active as well as their previous walked path during the math trail. For example, if several small groups are jammed at one station, the teacher can recognize this without being there and react to it via a broadcast message. In this way, teachers retain pedagogical control over their learning group without being physically present at all stations.

The e-portfolio (see Figure 2.6 right) is an additional evaluation and diagnostic tool. It contains information about the progress along the math trail for each group. This includes, for example, the number of tasks completed so far, the hints used, and the answers entered. In addition, learners can transmit their answers and calculation methods to the teacher in the form of text, images and voice message, using the smartphone's camera and microphone. The information obtained through the e-portfolio can be used for diagnosis and incorporated into further lesson planning.

MathCityMap in the context of mobile learning. As presented in the introductory section, running a math trail with MathCityMap is supported by the use of digital media. From the student's perspective, this is done in particular through the use of the smartphone app. The use of mobile technologies in an educational context is called "mobile learning" (m-learning). Mobile learning describes learning that is enabled by mobile technologies at any time and any place, or learning that is supported by the use of mobile technologies in a profitable way [15]. Especially the second aspect is considered by connecting math trails with mobile technologies and mobile learning is implemented with MathCityMap. The added value of using a smartphone app compared to math trails with paper and pencil is seen - from a student perspective - in the following aspects:

1. Navigation support: The app shows your own location and the task location. In order to train the use of maps, the app does not provide a leading navigation, it is only supported by the position display. After completing a task, the app automatically shows the position of the next task in the trail and thus organizes the trail sequence.

2. Retrieval of hints: Unlike an analog math trail, the app not only presents the task and task object, but also holds hints entered by the author or teacher if needed. These are graded, i.e. up to three hints are stored, which increase in specificity.

3. Validation of the solution: The solution is validated automatically and immediately after input. The students thus learn directly on the spot whether their result is correct, acceptable or incorrect and can revise incorrect considerations again if necessary. Furthermore, the sample solution is available to them after correct answer or after giving up the task. By additional playful elements, like points or group sequences, math trails can be embedded further into an optional competition character.

The Math Trail Community. One reason why the math trail idea has not spread as much as one could have expected in the last century was that no (international) community could develop in which tasks could be exchanged or experiences could be shared. The previous math trail projects (e.g. Niagara Falls Math Trail and Canadian Math Trail by E. Muller) were therefore isolated solutions for individual areas or regions, or relatively old-fashioned and unknown websites that were not interactive and therefore did not contribute to the exchange among users [20]. It was important from the beginning of the MathCityMap project to create the possibility that on the one hand the web portal can be offered in different languages and that on the other hand the contact among the users can be established.

In the context of these demands, the MathCityMap project defines different roles for the users with different tasks. There is the classic simple user who can create tasks and combine them to a private trail. The users have the possibility to contact other users directly via the community portal.

In order to publish tasks, so that every user can view and use them, the tasks have to be put into a review process. Over the years, quite a large number of reviewers have been established in various languages who review MathCityMap task voluntarily. Currently, the system has more than 27.000 tasks of which nearly the half are public. These tasks have been created by more than 8.700 users on all continents in more than 40 countries around the world.

In addition to the reviewers, there are also translators who are responsible for ensuring that the latest new developments are translated into the respective languages.

Each member of the community receives various awards for reaching goals in certain categories (see Figure 2.7).

- Taskmanager (number of tasks created)
- Pathfinder (number of trails created),
- Teacher 4.0 (number of Digital Classrooms conducted),
- Influencer (number of followers),
- Consultant (number of reviews conducted) and
- Apptastic (number of downloaded own trails).

Figure 2.7 The Badges the MathCityMap Community

We do not know exactly which of the factors (e.g. community website, the MathCityMap idea itself, outdoor learning, mobile learning) has led to the fact that the MathCityMap community has been developing almost exponentially for the last six years. Every year, the users create as many tasks as at large of all previous years. The only exception was 2020, the first year of Corona pandemic. For 2021, we expect an increase of more than 15.000 tasks and 4.000 users. It is encouraging that the users have created 10 % more tasks per user since the Corona crisis. Before Corona, we were averaging 2.76 tasks per user, now we are averaging 3.06. This development can be seen in Figure 2.8.

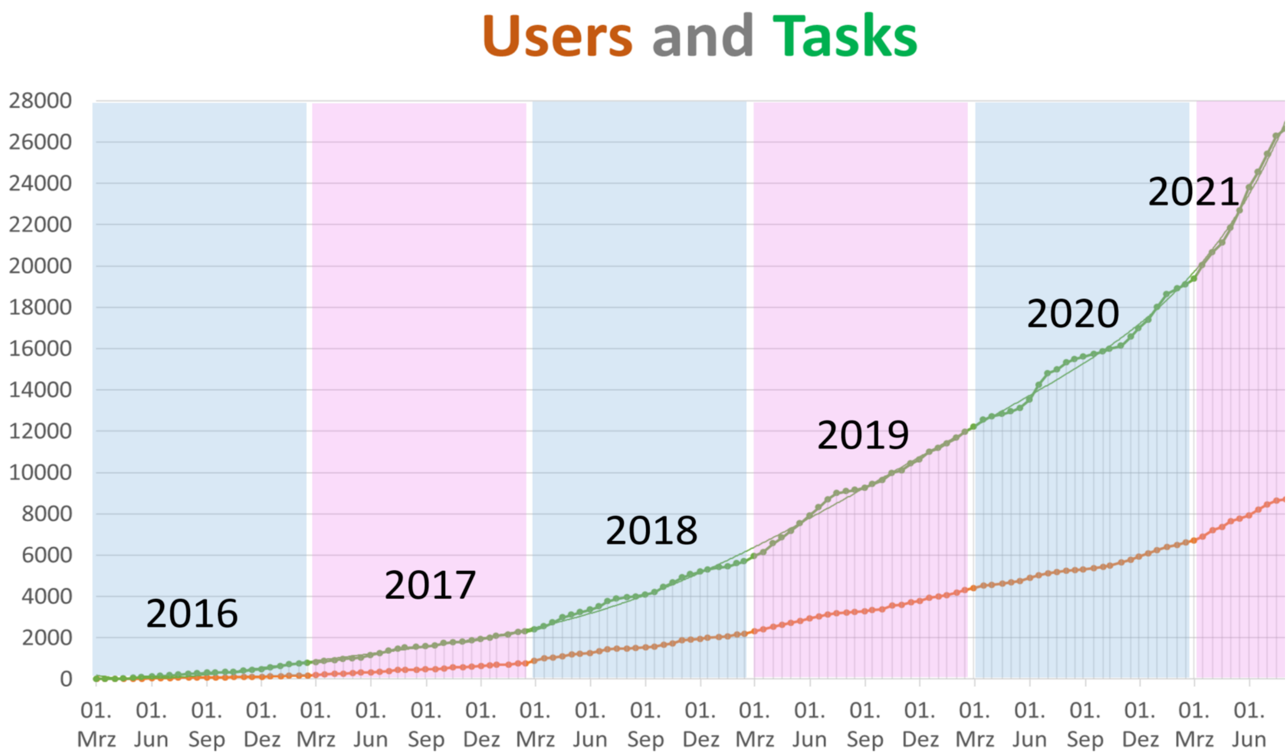


Figure 2.8 The Exponential Development of the Number of Tasks and Users

3. Research Interest on Mobile Math Trails

As stated in the introductory part, MathCityMap has been created for the use and digital enrichment of math trails in the educational context. In order to state the relevance of math trails in mathematics teaching and learning, considerations on both levels – theoretical and empirical – are necessary. Especially the digital enrichment of the math trail idea through the digital components of MathCityMap are of interest for the educational purpose. In order to legitimate the use of math trails on the one hand and the digital enrichment through MathCityMap on the other hand, we focus on the following research question:

In what way do math trails and, in particular, their digital enrichment support the teaching and learning of mathematics?

In order to answer the question, several research findings from the educational context are taken into consideration. Hereby, the focus is on different aspects of math trails and, in particular, math trails with MathCityMap that differ from "classic" mathematics teaching. More specifically, we narrow the research question to the aspects of performance, motivation, and mathematical skills using modeling as an example. Already at this point, it should be pointed out that this compilation of potential benefits certainly cannot claim to be exhaustive.

4. Empirical Findings on Math Trails

Math trails in the context of performance. [6] was the first to investigate the impact of the MathCityMap system on mathematical performance. For this, an empirical study was conducted in Indonesia with over 500 students. The experimental group tried math trails with the MathCityMap app, while the control group had normal math classes [21]. The Indonesian study found no significant difference between the control and experimental group regarding the pretest on mathematics ($p = .35$ for a two-sided t-test). However, the post-test has a significant difference between the two groups ($p < .000$, $d = 1.2$) [21].

Also [20] investigates the influence of math trails with MathCityMap on mathematical performance. For this purpose, an empirical study was conducted in 2017 with 235 German ninth grade high school students on the topic of cylinders. After a grouping test, treatment and control groups were formed. For the treatment group, math trails with cylinder tasks were created and run in two 90-minute sessions. In a comparison test on cylinder tasks, there was a strong significant difference between treatment and control group ($p < .01$) with a medium effect ($d=0.5$) in favor of the treatment group. This showed an increase in performance especially after the second run [20]. Notably, long-term learning through math trails was demonstrated in the context of the study with a smaller sample, which is consistent with the findings of previous studies on long-lasting memories of learning in outdoor situations [7].

Math trails in the context of motivation. In addition to the pre- and post-test, [6] used the Self-Determination Index in both experimental and control group to specify the influence of MathCityMap on the students' motivation. The values of the experimental group were significantly higher than those of the control group. The follow-up math trail activity and the survey one year later showed that increased motivation is also a long-term effect.

Additionally, the previously described German study examined the influence of gamification elements (game elements) on students' motivation while completing a math trail. In total, 196 ninth grade students were divided into three different gamification groups [9]:

- G0: No Gamification - The app only provides feedback on the correctness of the answer.
- G1: Points gamification - There are up to 100 points for each task. Each incorrect answer is penalized with a point deduction. The first incorrect entry has no consequences yet. Answers within the acceptable solution range are rewarded with partial points depending on their proximity to the correct solution.
- G2: Local Leaderboard Gamification - In addition to points, groups can see which group is ahead or behind them in the score.

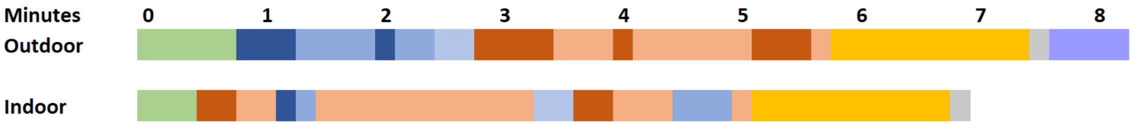
After completing a trail with one of the above gamification variations, students completed an intrinsic motivation questionnaire. Missed entries and the number of tasks solved were also recorded using the app.

The results show that gamification variants G1 and G2 increase motivation, but not significantly. Nevertheless, they significantly change the groups' solving behavior. On the one hand, the number of incorrect entries - and thus the guessing behavior in particular - was reduced. In particular, the leaderboard increased the number of solved tasks [9].

Math trails in the context of modelling. Also when it comes to mathematical skills, outdoor mathematics shows potential to enrich the teaching and learning of mathematics. As stated in section 2, through the necessity of collecting data from the real environment under consideration of simplification, a strong connection to mathematical modelling [5] can be assumed. In the context of a comparative case study [14], the differences in indoor and outdoor modelling settings were taken into consideration. Through the nature of outdoor mathematics, the modelling process was mainly limited to the “Structuring and Simplifying”, “Mathematizing” and “Validating” steps [5].

Two groups of students solved similar modelling tasks – one group being outside at the real object’s location and one group solving the task inside the classroom with a picture including an object of reference. Both group’s solving processes involve the “Structuring and Simplifying” and “Mathematizing” modelling steps. The basic chosen models are often similar in the indoor and outdoor context.

Still, the discussion which model should be chosen is more intense in the group of students being outdoors. While doing so, they take different perspectives and discuss which data and knowledge are needed. Through the possibility of measuring at the real object, the students try to be as precise as possible and do not take any estimations. Still, it is their aim to work effectively. The students inside the classroom have fewer possibilities to choose a model because of the limited data that can be collected. Through the picture, they only have one unchangeable perspective of the object. In contrast, the students indoors have more intense discussions on the assumptions and estimations that they have to make in order to collect the necessary data for the mathematical model [14]. These findings are exemplary presented in Figure 4.1 which gives an overview on the different modelling steps and the duration for the outdoor and the indoor group. The activities in “Simplifying/Structuring” and “Mathematizing” are divided into subcategories. Still the choice of colour (deep and light blue and red) should symbolize their relationship.



		Outdoor	Indoor
1	Understanding	10%	7%
2.1	Simplifying/Structuring: Choose Model	8%	2%
2.2	Simplifying/Structuring: Compensate Inaccurancies	12%	10%
2.3	Simplifyling/Structuring: Make Assumptions	4%	5%
3.1	Mathematizing: Choose Data	16%	9%
3.2	Mathematizing: Collect Data	20%	40%
4	Working Mathematically	20%	24%
5	Interpreting	2%	2%
6	Validating	8%	0%

Figure 4.1 Modelling Activity Diagram [1; 14]

5. Discussion and Limitations

On the one hand being an open form of learning, on the other hand through strongly emphasizing embodiment and enactive actions, we have assigned math trails in the introductory part a special role alongside or in mathematics learning and teaching. The results from the various empirical studies confirm the special significance of math trails hypothesized from the theory. In addition, the empirical results show that this special role indeed seems to have an impact on the learning of mathematics.

First, the impact on (initially short-term) learning performance could be shown. In both studies presented by [20] and [6], the experimental groups that were outside with MathCityMap performed better. With reference to the theoretical advantages of mobile math trails (own decisions, open-ended work, embodiment and enactivity, and digital enrichment), the theoretical and empirical considerations seem to fit together. Nonetheless, it remains unclear at this point which influences lead to this increase in performance – going outside, the smartphone, or even a mixture of both. This remains to be investigated on a qualitative level.

Furthermore, circumstantial evidence for long-term learning success emerges in [20]. Based on a subsample of his study, he hypothesizes that math trails with MathCityMap lead to long-lasting memories of what is learned. This hypothesis will be investigated by [2] in the quantitative longitudinal study “MEMORI”.

In terms of motivation, the MathCityMap system was found to be intrinsically motivating. As with performance, this may be due on the one hand to going outside, but also to the use of the smartphone. Especially the latter shows relevance in the study of [9]. The supplementary use of digital gamification elements shows that these playful elements have an additional influence on motivation and prevent undesired behavior such as guessing.

To confirm the quantitative results also on a qualitative level, we have listed the results on the actual solution process when modelling in the classroom and outside. Here we find different emphases regarding the steps structuring and mathematizing from the modeling cycle of [5]. Validating also has a different meaning in the context outside and is particularly stimulated by the digital component of the app. Modelling outside the home thus seems to be a profitable enrichment of "classical" mathematics instruction in the classroom. In the qualitative study "MAP - Modelling, Arguing and Problem Solving in Outdoor Mathematics" we will investigate the modelling competence in more detail and furthermore the competences arguing and problem solving in outdoor mathematics tasks [11].

The limitations and outlooks show that there is still much work to be done concerning the special learning form of math trails. Both, theoretically and empirically, math trails show advantages that can be used to enrich normal mathematics instruction. Thus, in addition to these research activities, our practice-oriented goal is to convince teachers to use the tool regularly. In the strategic partnership MaSCE³ (Math Trails in School, Curriculum and Educational Environments of Europe), we realize this by creating theme-based trails – trails that fit a specific topic from the curriculum (e.g., slope or statistics) and that can be created on objects that can be easily found in numerous locations (e.g., stair railings or city maps) [13]. Ultimately, through this curricular adaptation, we hope to see an even greater emphasis on the benefits of math trails – and an ever-growing international community of teachers and instructors willing to use them.

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