Pseudo-Riemann's quartics in Finsler's geometry

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An extension of Riemmann's geometry into a direction dependent geometric structure is usually described by Finsler's geometry. Historically, this construction was motivated by the well-known Riemann's quartic length element example. Quite surprisingly, the same quartic expression emerges in solid-state electrodynamics as a basic dispersion relation—covariant Fresnel equation. Consequently, Riemann's quartic length expression can be interpreted as a mathematical model of a well-established physics phenomena. In this paper, we present various examples of Riemann's quartic that demonstrate that Finsler's geometry is too restrictive even in the case of a positive definite Euclidean signature space. In the case of the spaces endowed with an indefinite (Minkowski) signature, there are much more singular hypersurfaces where the strong axioms of Finsler's geometry are broken down. We propose a weaker definition of Finsler's structure that is required to be satisfied only on open subsets of the tangent bundle. We exhibit the characteristic singular hypersurfaces related to Riemann's quartic and briefly discuss their possible physical interpretation.