# Three Aspects of Maple™ for Education

William C. Bauldry*
Professor Emeritus & Adjunct Research Professor
Appalachian State University

**Abstract**

*We discuss three aspects of utilizing the computer algebra system Maple to enhance learning. First, we consider Maple as platform to develop both static and dynamic images and displays for classes and for students for their studies. Second, we consider Maple as a student exploration tool under the* Action-Consequence-Reflection Principle *where a student* acts *on a mathematical object, observes the* consequences *of their action, and* reflects *on the mathematics of the situation. Third, we look at Maple as a tool to support mathematical exposition via linking to documents, posting worksheets in the MapleCloud, and by creating packages of routines and programs to support a document.*

## Introduction

Computer algebra systems (CAS) have become ubiquitous tools in didactics. Introduced to educational use in the 1980's, CAS's developed impressive capabilities and user interfaces designed for ease-of-use. From Axiom to Xcas, systems were aimed at both research computations and student explorations. Modern systems run on mainframe computers, smart phones, and everything in between. Several calculators, such as the TI's Nspires and Casio's ClassPads, have CAS capabilities. In this session, we will explore three aspects of pedagogical use: CAS for Classroom Displays, CAS for Student Exploration, and CAS for Document Support. To focus exclusively on the didactic aspects, and not CAS differences, we will use Maple[1] exclusively, one of the first systems to offer extensive graphical capabilities.

## 1 Maple For Classroom Demonstrations

Often a classroom demonstration will help students understand a concept. These visualizations can be either *static* or *dynamic* depending on the needs of the concept. A static display shows information as an aggregate letting students compare specific values/objects. Static displays and graphs are the typical entry for an instructor to begin using a CAS in the classroom. A dynamic display lets students observe interwoven behaviours in action.

Maple is a computer algebra system that has a very powerful mathematics engine capable of research-level mathematical computations and programming, and has excellent graphics. Figure 1 shows Maple 2020's Startup Screen. Even though incredibly powerful, Maple can also be an entry-level platform for student's mathematical explorations. There are many resources for using Maple for instruction, starting with those on Maplesoft's website (www.maplesoft.com). A search with Google for "Maple CAS in the classroom" returns over three quarters of a million results in under half a second. A free trial of Maple can be downloaded from Maplesoft. There is also a free tool, Maple Player, that allows students to use pre-made worksheets, but not enter their own calculations.

*Email: bauldrywc@appstate.edu
[1]Maple is a registered trademark of Waterloo Maple, Inc.

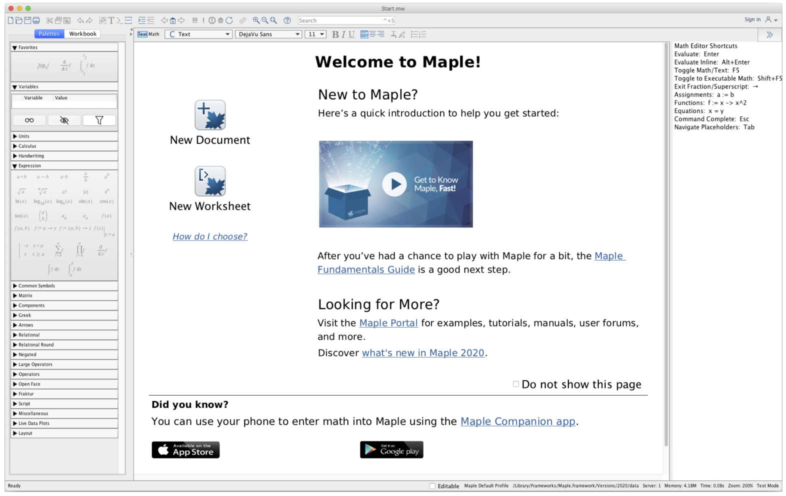



Figure 1: Maple 2020 Startup Screen

## 1.1 Static Sums Display

Learning about convergence properties of series is quite difficult. The Comparison Test is one the more easily understood tools. A nice example that illustrates both the test's strengths and weaknesses at the same time is the set of series $\sum 1/k^n$. This set of series is also a fundamental convergence-checking tool for students to have "in their back pocket."

Our first example of a static display is of the sums $\sum 1/k^n$ as $n$ increases. There are two questions to students:

- What does the Comparison Test tell us about this set of series?
- Is there a pattern in $S(n) = \sum_{k=1}^{\infty} \frac{1}{k^n}$ as $n$ changes?

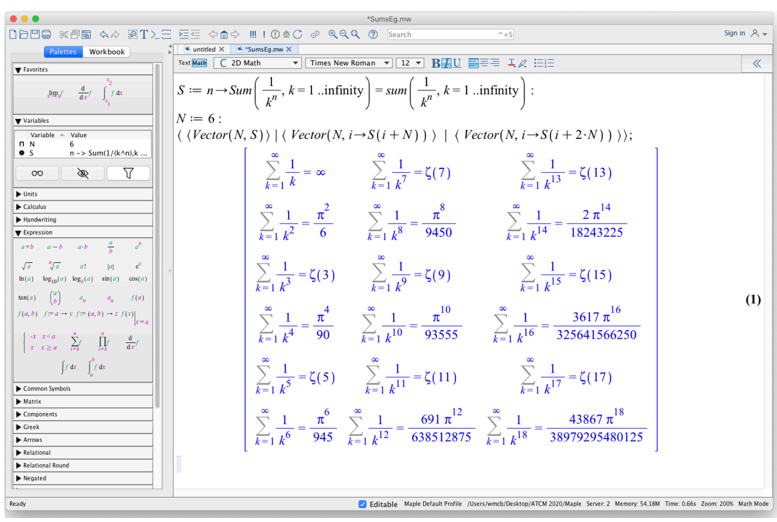



Figure 2: Static Display of Sums

They quickly observe that the even power, $n$, sums are all rational multiples of $\pi^n$, and the odd power sums involve the $\zeta$ function which most students have never seen. There are many fun directions to go. For example, students will quickly conjecture that the odd powers should also be rational multiples of the odd power of $\pi$ — this is an open question!

### 1.2 Static Approximate Integral Display

Many beginning calculus students see the Fundamental Theorem of Calculus as the definition of the definite integral. One way to help fix this misunderstanding is to investigate the definition in relation to numerical approximations. Since the set of functions with elementary antiderivatives has measure zero, it is important for them to use and understand numerical approximations. This understanding will help them construct better intuition for the definition of integral. Maple's function *ApproximateInt* produces excellent images for static classroom displays to help with this task.

Here we integrate $f(x) = \tan(\sin(x))$, a functions with no elementary antiderivative, from 0 to $\pi$. The question to students is: What will improve the approximation.

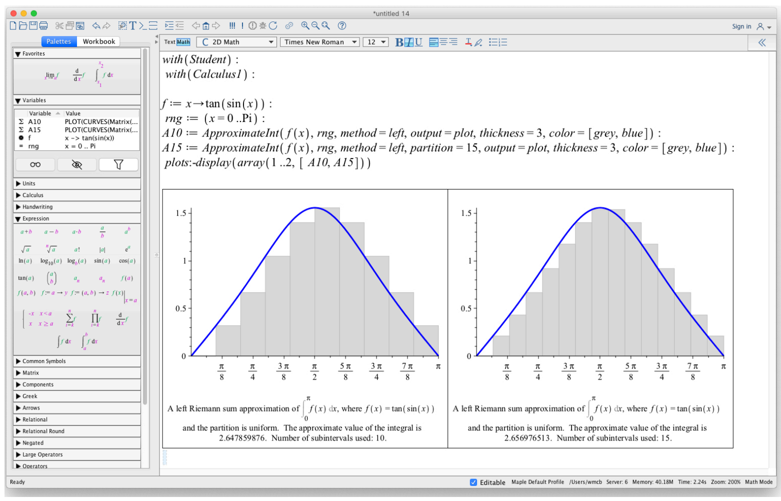



Figure 3: Static Display of Approximate Integration

Students' geometric intuition leads them to increase the number of rectangles, but the image also leads them to explore changing the approximator shape from rectangle to triangles — the first step on the way to general methods. One easy change in Maple's *ApproximateInt* statement, from *method* = *left* to *method* = *trapezoid*, facilitates this adjustment. Actually, *method* can take any of *lower, upper, left, midpoint, right, trapezoid, simpson, simpson[3/8], boole, newtoncotes[posint], or random.* (NB: *random* instantiates a Riemann sum.)

For eager or advanced students, a fascinating exercise is to look at the results of

> $infolevel[int] := 3;$
> $int(f(x), x = 0..\text{Pi});$

where Maple shows all its attempts to find an antiderivative.

The *ApproximateInt* function can also produce animations, but that is a topic for later section.

### 1.3 Static Logistic Population Data Fit Display

More and more, mathematicians recognize that it is very important for our current students to develop ability to analyze data mathematically. Students are inundated with data in the news, often with incorrect or unsubstantiated conclusions. They must be able to make their own analysis.

Here we consider logistic fits to the U.S. decennial population counts from the first census of 1790 to the most recent 2010.[2]

[2]The US Census data is available at http://mathsci2.appstate.edu/~wmcb/ATCM2020/Maple/US Population Data.txt.

Fitting a logistic function to data is a nontrivial exercise — logistics are highly nonlinear. We use a technique based on linearizing the logistic differential equation, $\tilde{p} = p'/p = \alpha - \beta p$, and using divided differences, $p'(t_k) \approx (p_{k+1} - p_{k-1})/(t_{k+1} - t_{k-1})$ to approximate the derivative.

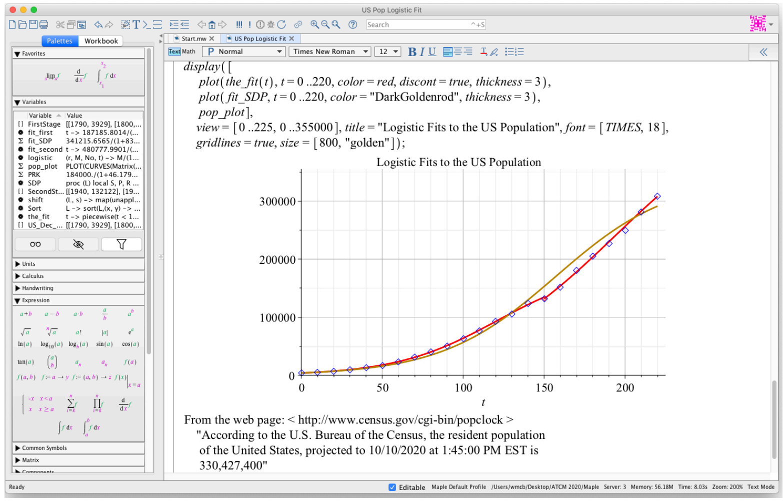
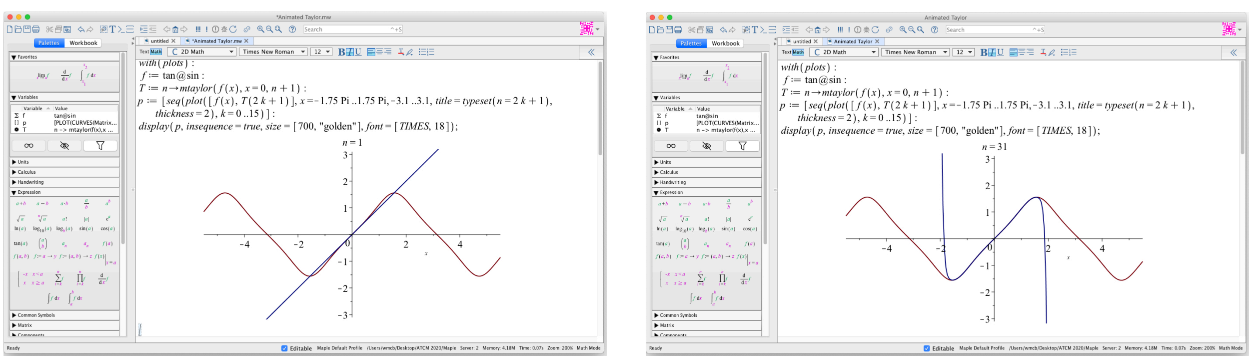



Figure 4: Static Display of Logistic Data Fits

## 1.4 Dynamic Taylor Approximation Display

Taylor polynomials, usually first encountered in elementary calculus, form the basis for many numerical analysis concepts and algorithms. So understanding these polynomials is crucial to learning computational mathematics. The approximating polynomials are an easy, natural choice for a dynamic display. We can watch how, or if, convergence progresses. A very nice animation shows plots of a function and its Taylor approximators.

Return to the "simple" function $f(x) = \tan(\sin(x))$, now on the interval $[-1.75\pi, 1.75\pi]$. Using naive reasoning leads us far astray: The sine is bounded by 1, so the argument of tangent is "far" from the nearest pole of the tangent, $\pi/2$. The Taylor series for sine converges everywhere. These observations lead us to believe that the Taylor polynomials for $f$ should get better and better over larger and larger intervals. Time for the animation!




(a) The First Frame, $n = 1$

(b) The Last Frame, $n = 31$

Figure 5: Dynamic Taylor Polynomial Display

For $n = 1$, Figure 5a shows no surprises. But as we watch the animation as $n$ increases to $n = 31$ in Figure 5b, we see the interval of convergence is not widening. What's happening?[3]

[3]See the animation at http://mathsci2.appstate.edu/~wmcb/ATCM2020/Animations/

A little digression into the complex domain answers the question. The nearest singularity of $f$ to $x = 0$ is easy to find with Maple. Solve the equation $\sin(z) = \pi/2$ to find $z_k = \pm\pi/2 \pm i \ln\left(\pi/2 + \sqrt{\pi^2/4 - 1}\right)$. We see that $f$ has singularities at $z_k$. Since $|z_k| \approx 1.87$, our interval of convergence is approximately $[-1.87, 1.87]$. A 3D graph of $|f(x + iy)|$ shows quite nicely what's happening. The real answer to the convergence question required complex variables, so it is rarely encountered in elementary calculus classes. We'll revisit this topic soon.

## 1.5 Dynamic Transition Curve Display

A *transition curve* is a short spline that merges two curves matching 2D-derivatives at joints. These curves are used for railways and roadbeds. A 90° turn through a circular arc imparts a sudden unpleasant force on passengers. With a transition curve, the force smoothly grows and decays. Parametrically, an ideal transition curve, called a clothoid, is given by the Fresnel integrals

$$C(t) = \left[\frac{1}{a}\int_0^t \cos(\pi t^2/2)\,dt, \frac{1}{a}\int_0^t \sin(\pi t^2/2)\,dt\right].$$

Clothoids are also known as Cornu spirals or Euler spirals. These curves were studied by Euler, but only used for designing railways and roadbeds since the 1970s. In practice, civil engineers usually use cubic approximations to the transition curve $C(t)$.

Question to Students: How do you describe the apparent force $(-\vec{a})$ as the train moves through the different curves.

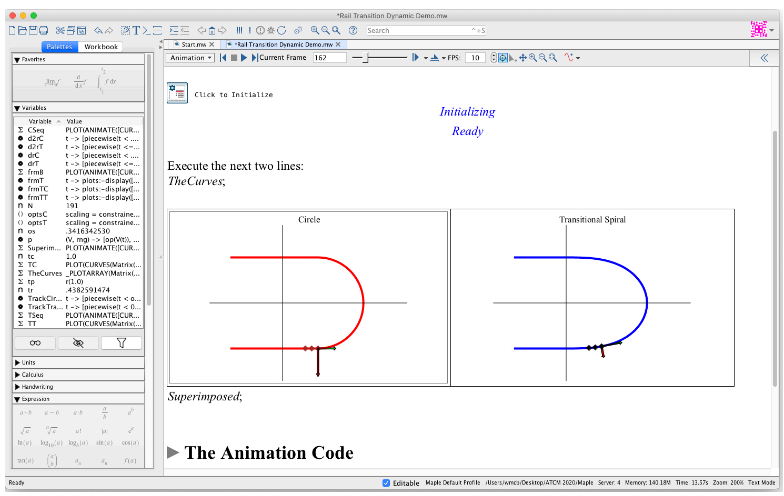



Figure 6: Dynamic Transition Curve Display

We see the apparent force on the circle connector track instantaneously jumps to its maximum throwing the passengers against the side. The apparent force on the clothoid connector track is smooth.[4] An interesting exercise for calculus students is to develop the cubic curves approximating a clothoid.

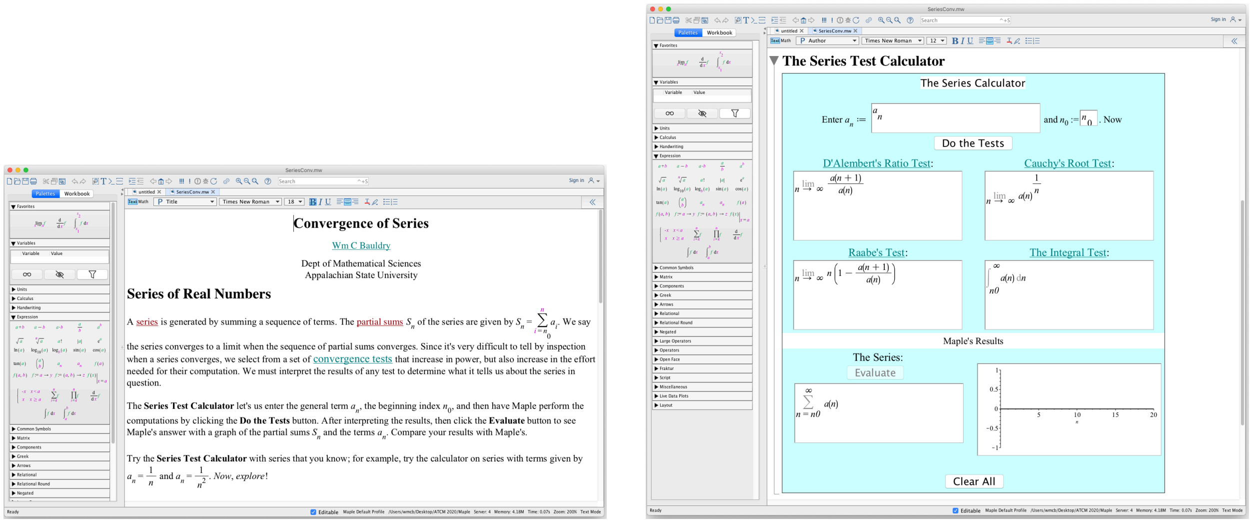
## 1.6 Dynamic Series Convergence Test Display

The last half of the second semester of elementary calculus usually delves into series and convergence. Series convergence is a very deep concept and underlies much of advanced mathematics. Among the standard tools to be learned are the ratio, root, and integral tests. Students typically focus on the computations required for conducting the tests and lose sight of their significance. Since the focus should be on the test results, and not the algebra of computation, putting these tests all together in a dynamic document helps students to gain understanding.

[4]See the animation at http://mathsci2.appstate.edu/~wmcb/ATCM2020/Animations/

We can use Maple to create a document that performs the tests letting students focus interpreting the results. Along with sandboxing the calculations, we can add definitions and explanatory text at the beginning, and add pointers to references and further help at the end. With these additions, the Series Calculator has become a stand-alone document for students to use to study the basic convergence tests.

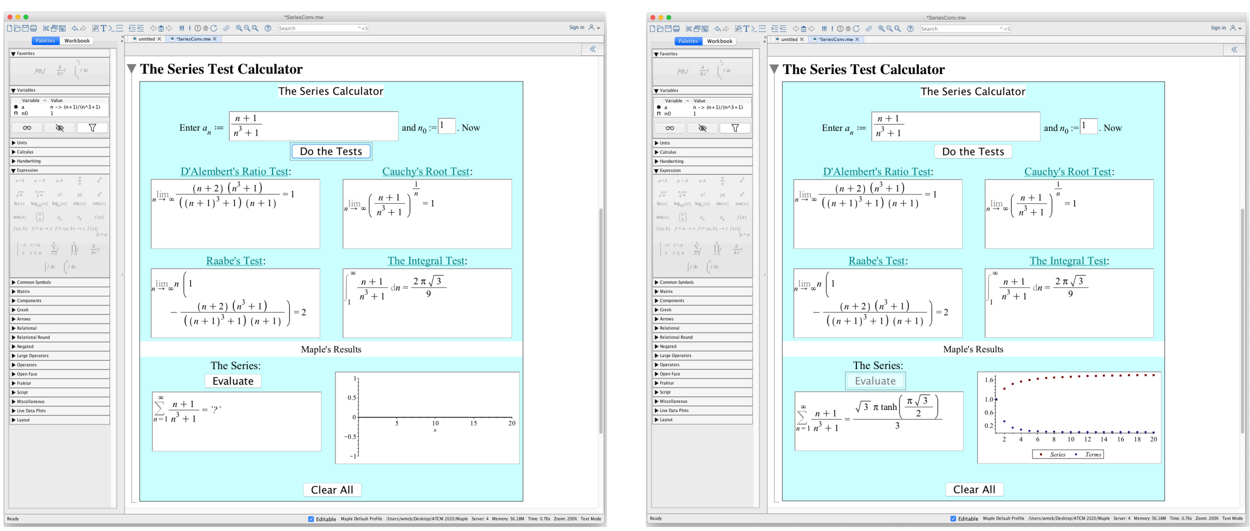
Question to students: Does the series $\sum \frac{n+1}{n^3+1}$ converge?




(a) Series Definitions (b) Initial State

Figure 7: The Dynamic Series Calculator

Enter the series in terms of $n$ and the starting value $n_0$ in the "edit boxes" to define the series $\sum_{n=n_0}^{\infty} a_n$. Now click the "Do the Tests" button. The results of the four convergence tests are shown in Figure 8a. To verify the results, see if we have a closed form sum, and check the graphs, click the "Evaluate" button. Maple's result and a graph are shown in Figure 8b.




(a) Doing the Convergence Tests (b) Maple's Results

Figure 8: The Dynamic Series Calculator at Work

The Series Calculator also works in Maplesoft's free Maple Player.

# 2 Maple For Student Explorations

Maple provides an excellent environment for student explorations and investigations. These activities can be sandboxed with carefully constructed worksheets. Explorations can be guided by sets of leading questions, even by quite simple "What happens when. . . " questions, or lead to significant student-research projects. There are a large number of manuals, books, and online resources for student projects. Books from today and back to the 1990's, like "Calculus Laboratories with Maple," [8], are available. These texts offer carefully defined projects that are classroom-ready and tested with students. (Maplesoft has a list of textbooks by topic.[5])

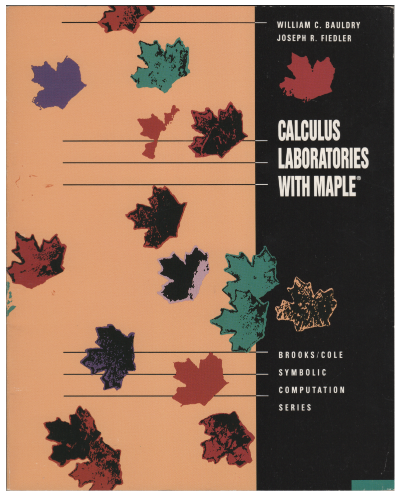



Figure 9: Text with Student Explorations from 1990

## 2.1 The *Action-Consequence-Reflection* Principle

The *Action-Consequence-Reflection paradigm* is a research-based pedagogical approach that provides students with a microworld in which to perform a mathematical *action*, observe the *consequences* of their action, and *reflect* on the behaviour they observed in order to construct mathematical understanding.

Students' inquiry-based explorations are led by a set of carefully crafted questions, guiding their experiments in computer algebra dynamic microworlds.

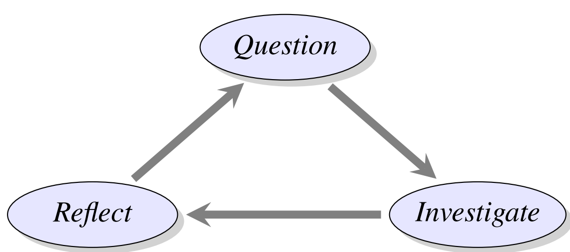



Figure 10: The *Action-Consequence-Reflection* Cycle

## 2.2 An *ACR* Activity for Data Analysis

Students typically have great difficulties understanding linearity in data. The traditional coefficient of determination $R^2$ from regression can be quite misleading and very difficult for students to interpret correctly. An *ACR* document

[5] *Writing a book? Visit Maplesoft's Author support.*

introducing a new linearity measure, $Q^2$, helps students develop better intuition [16]. This new measure is defined in terms of the orthogonal and perpendicular regressions:

> $Q^2$ is the percentage reduction in the sum of squared orthogonal distances using the orthogonal regression line as opposed to using the line perpendicular to the orthogonal regression line through the centroid of the data. [16]

The "Rotate Data Points about their Centroid" tool lets student reflect on what happens as they vary the angle of rotation. The tool presents the random data with its graph, the regression lines and equations, and the $R^2$ and $Q^2$ values with each change.

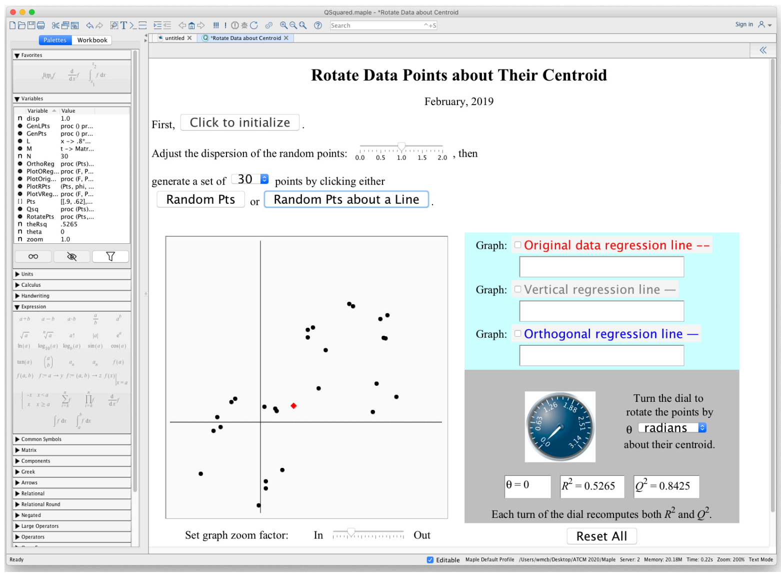
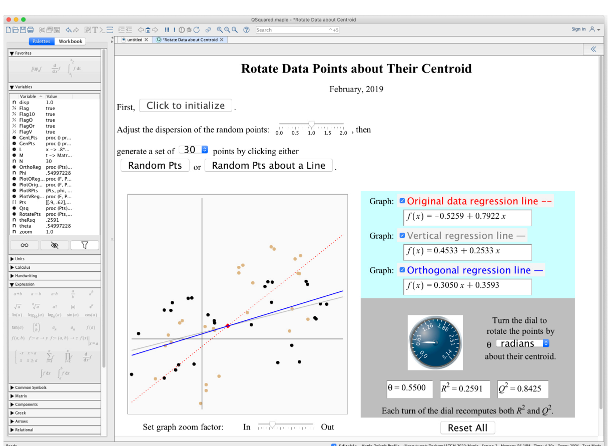


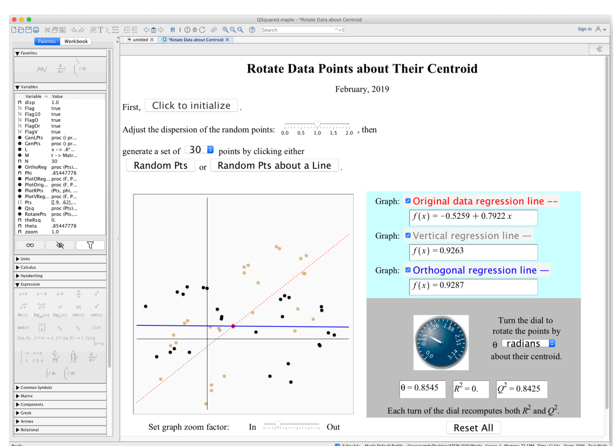

Figure 11: $Q^2$, a Measure of Linearity

Our original data has $R^2 = 0.5265$ and $Q^2 = 0.8425$. The data was randomly generated about a randomly chosen line.

Questions to students: Does rotating the data change the relationship among data points? (Figure 12a.) Can data always be rotated about its centroid to achieve $R^2 = 0$? (Figure 12b.)




(a) Starting Data




(b) Rotated Data

Figure 12: The Data Linearity ACR Exploration

The data has been rotated to have $R^2 = 0$, but $Q^2$ is still 0.8425. $Q^2$ is invariant under rotation about the centroid. Has the data's intrinsic linearity relationship changed?

## 2.3 The *Explore* Command

Maple's *Explore* command lets us quickly build microworlds for student investigations. Figure 13 shows

$$Explore(x^n - 1 = factor(x^n - 1), n = 1..9, echoexpression = false)$$

with the slider set at $n = 6$. The option $echoexpression = false$ tells *Explore* to only show the output, not the command giving it.

Question to students: Does the factorization show any patterns as $n$ changes?

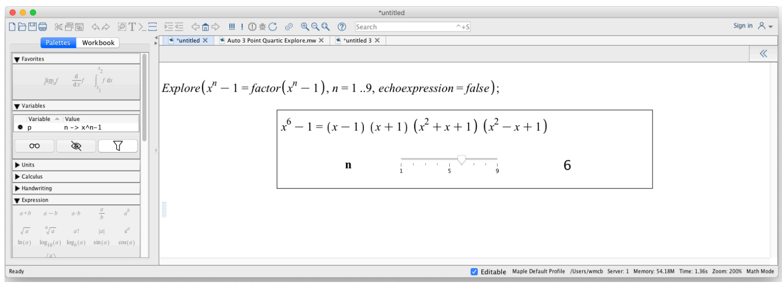
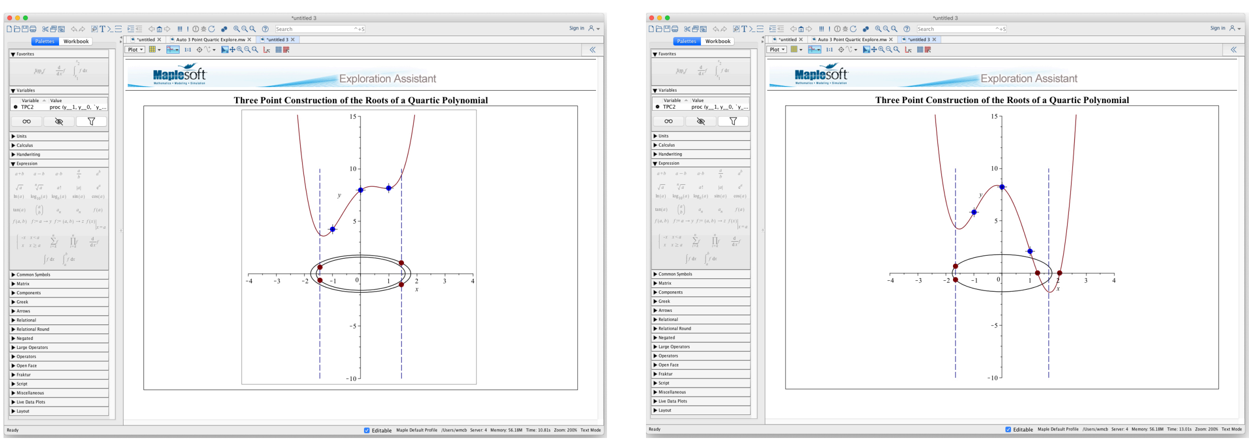



Figure 13: A Simple *Explore* Command

Students quickly pull out the factorization as special when $n$ is prime.

## 2.4 Roots of a Quartic

Students are surprised to learn that the roots of a monic, reduced quartic $q(x)$ can be determined from only three special values: $q(x_0)$, $q(0)$, and $q(-x_0)$ where $x_0 \neq 0$. Analogously, a monic, reduced quartic is determined by the three points, $q(0)$ and $q(\pm x_0)$ with $x_0 \neq 0$. Figure 14 shows that the *Explore* command makes a nice, easy, stand-alone tool for investigating this situation. Executing the command creates a new worksheet that contains only the dynamic tool.




(a) Roots Starting Position (b) Roots Adjusted

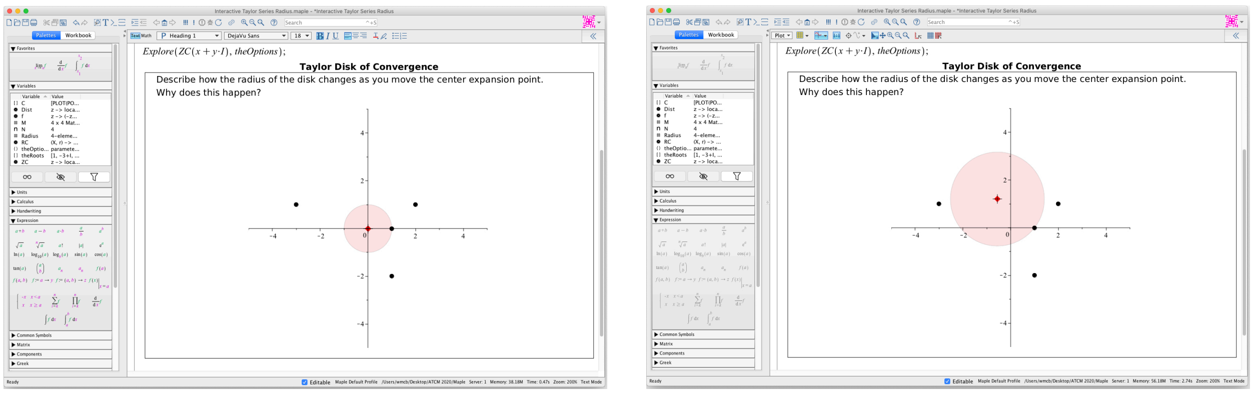
Figure 14: The Three-Point Theorem Investigation

Students use the mouse to move the points defining the quartic (blue points) and observe what happens to the quartic's roots (red points) and the circles they appear on. This tool can be used with Maple Player.

### 2.5 Taylor Series Circle of Convergence

The circle of convergence of a Taylor series for a function is a very pretty, geometrical topic for student investigations. They can quickly discover the theorem that gives the radius as the distance to the nearest singularity.[6]

The Taylor Series Convergence tool shown in Figure 15 generates a random function with singularities that are marked by black points. The red point marks the center of the circle of convergence. Students move the center with their mouse and observe what happens to the circle of convergence. Figure 15a shows a typical starting image, while Figure 15b illustrates the picture after the circle's center has been moved.




(a) Convergence Circle Starting Position (b) New Center, New Circle

Figure 15: Investigating Taylor Series Convergence with *Explore*

The tool of Figure 15 can also be used to introduce analytic continuation. Small modifications will let students run this exploration in Maple Player.

### 2.6 Using Maple for Assessment

Maple's *Grading* package lets us create many types of questions that can be algorithmically designed for practice and for assessment with any of the "styles:" math (math expression input for an answer), true/false, multiple choice, or multiple select. Figure 17 shows a practice test generator for an amateur radio license examination.

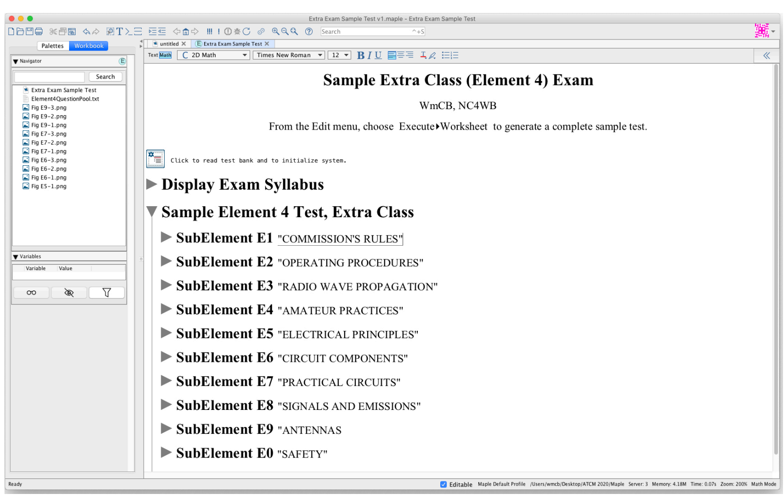
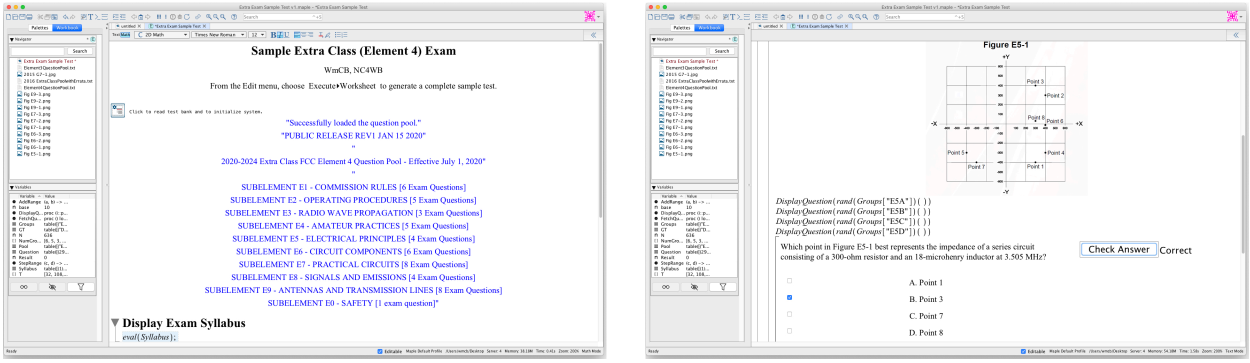



Figure 16: A Multiple Choice Test Generator

[6]Care must be taken when the function in question has a branch cut; the distance to the nearest singularity may then only give an upper bound to the largest radius of convergence.




(a) The Test Generator Initialized

(b) A Randomly Chosen Question

Figure 17: The Multiple Choice Test Generator at Work

The workbook has a "Code Edit Region", collapsed to a button. When the button is clicked, the testing system initializes, reads in the database of 500 questions, and prints the exam's syllabus for the 50 question test. Figure 17b shows a sample, randomly chosen question for the *Electrical Principles* topic along with a diagram relating to the question. The student selects the answer, then clicks the "Check Answer" button. It's correct! It is easy to code the "Check Answer" to deactivate to prevent changing the answer for a non-practice assessment. Clicking the !!! button in the Maple toolbar, executes the entire workbook creating a complete test.

## 3 Maple For Document Support

Maple contains extensive capabilities for producing sophisticated documents that can link to other items. We can also save worksheets or workbooks into the Maple Cloud making them easily and readily available to students. Collections of programs, functions, and data can also be combined into Maple packages for ease of use.

### 3.1 Hyperlinking In Maple

Hyperlinking is a powerful tool for creating extensive microworlds for student explorations. Maple hyperlinks use a text label or an image that acts in the same fashion as an html hyperlink tag.

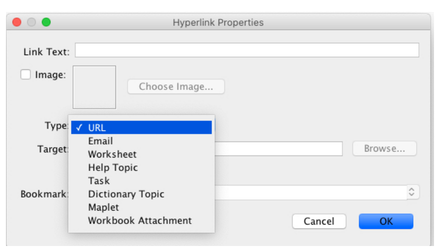
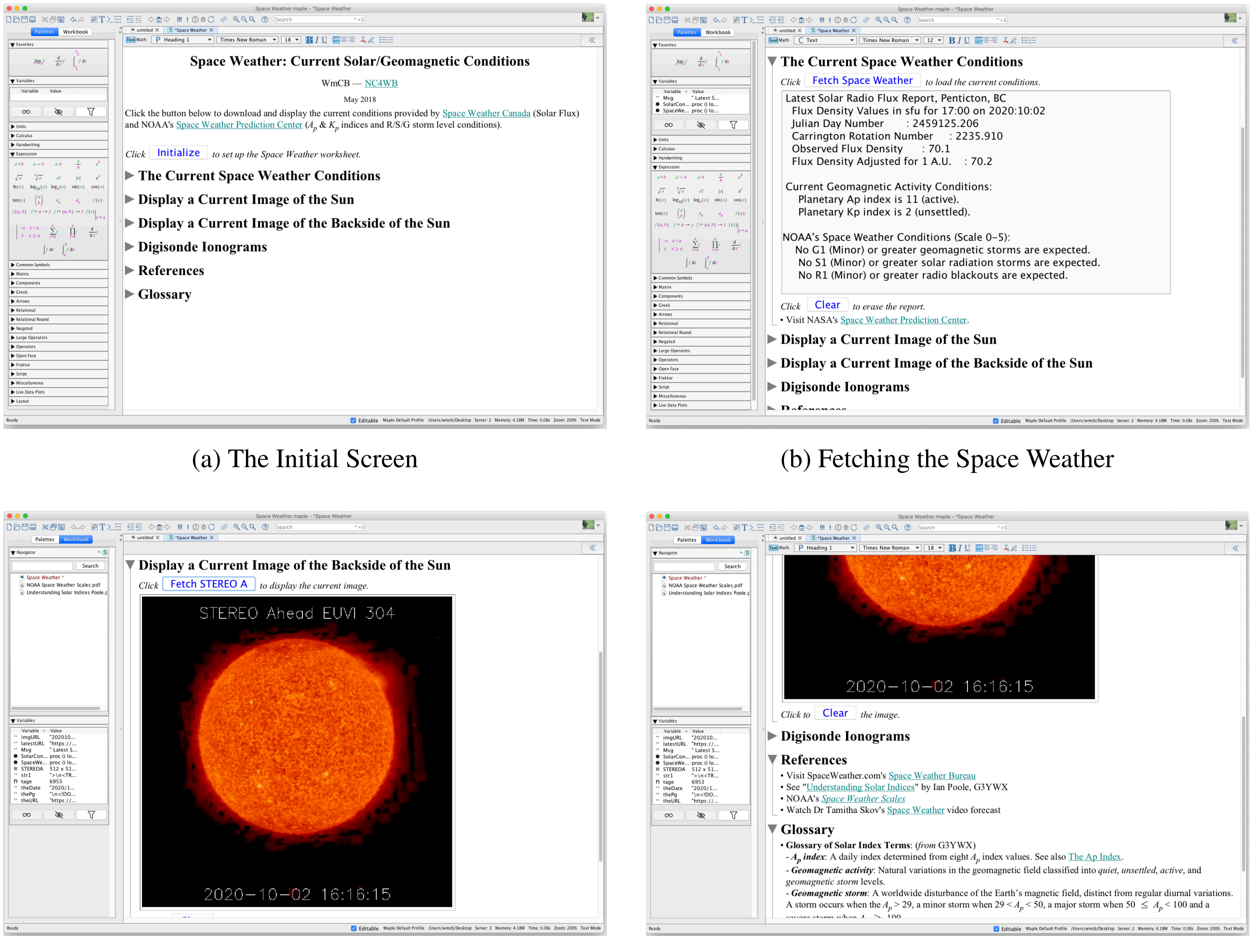



Figure 18: Maple's Insert Hyperlink Dialog

Figure 18 shows the various types of hyperlinks that can be embedded in a Maple document.

Our next example, the Space Weather workbook, illustrates hyperlinks. This document includes hyperlinks to web pages, to web-based data, to PDF documents attached to the workbook, and to help topics. The workbook can be downloaded from the Maple Cloud via the web or through Maple. Figure 19a shows the initial screen of the Space Weather workbook.




(a) The Initial Screen

(b) Fetching the Space Weather

(c) The Current Sun Image

(d) The References and Glossary

Figure 19: The Space Weather Document

The turquoise colored links in the text area of Figure 19a lead to websites of Space Weather Canada or NOAA. The blue initialize button defines all the functions and variables we'll need.

Figure 19b shows the result of opening the "Current Space Weather Conditions" section and clicking the "Fetch Space Weather" button which downloads current data from the web, then formats and places it in a *Math Container* DocumentTools component. A hyperlink to NASA's "Space Weather Prediction Center" is below the math container.

The next section displays a current image of the sun dowloaded from NASA's website. See Fiure 19c.

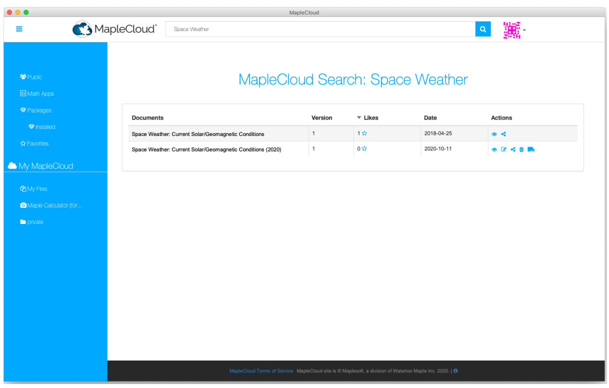
The last image, Figure 19d, of the Reference and Glossary sections shows hyperlinks that go to websites or display PDFs that are embedded in the workbook. The document "NOAA Space Weather Scales.PDF" is seen as the second item in the navigator pallet (upper left). Clicking on this link will display the PDF in a reader program, such as *Preview* on MacOS.

Since website addresses change rapidly, at times I have used an indirection technique from programming, "handles," to avoid having to recode the Maple document. A "handle" points to a pointer to a variable; this is a version of "double indirection." Thinking of a URL as a pointer, we form a URL pointing to the desired URL. The Maple hyperlink goes to a local website that has an immediate redirect to the desired website via an "onload" parameter in the "body" html tag. Then I need only change the local website to point to the new web address. Students using the Maple document won't notice the change.

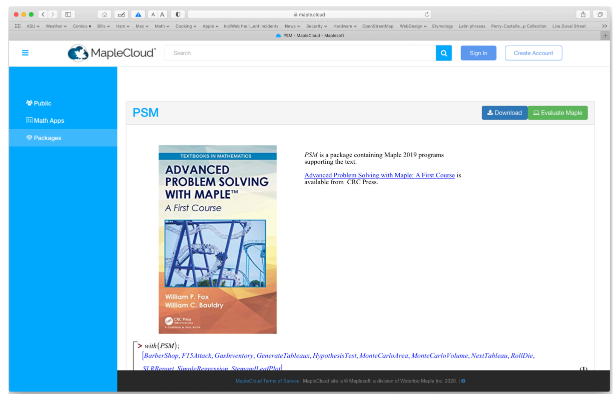
## 3.2 Documents in the Maple Cloud

The Maple Cloud (https://maple.cloud) is an online repository for Maple documents that gives public access. An instructor can easily create a private area in the Maple cloud to distribute Maple documents to students. There is a large number of Maple applications available to all on the cloud. Click the "Math Apps" tab to see the list. There is also a collection of Maple packages; click the "Packages" tab to see the list. Downloading any item is quite simple.

The Maple Cloud is also accessible from inside Maple. Click the icon at the right edge of the toolbar in a Maple window. Figure 20a shows the result of searching the Maple Cloud for the term "Space Weather." Two results are shown, the document we looked at earlier has "2020" in the name. Clicking on a document brings up a preview and a download button. When accessing the cloud from within Maple, the document is downloaded into Maple as an active worksheet.




(a) Searching the Maple Cloud




(b) Packages in the Cloud

Figure 20: The Maple Cloud

To save the current Maple worksheet to the cloud, simply choose "Save to Cloud..." from the File menu. Then choose the type of access: private, group, or public.

### Maple Packages in the Cloud

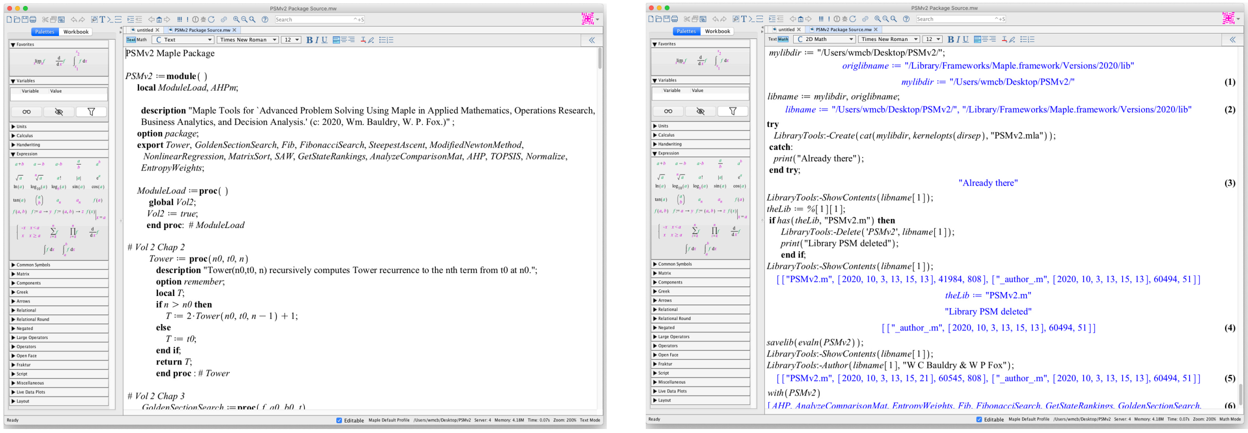
A Maple *package* is a collection of programs/functions, data, and code to execute on loading or unloading collected into a *module* that has the option *package*. Figure 20b shows the preview of the Maple package *PSM* that supports the text *Advanced Problem Solving with Maple: A First Course* [14]. Visit the web page MapleCloud packages for specific details on uploading a package.

## 3.3 Creating a Maple Package

A package is an excellent way to collect a set of routines and/or data for a student investigation into a single, easily downloadable, and maintainable repository. Creating a Maple package is actually rather simple. The routines and data are written as subroutines of the package module that are made available to through an *export* statement.

The definition of the *PSMv2* package module for our new textbook *Advanced Problem Solving with Maple: Applied Mathematics, Operations Research, Business Analytics, and Decision Analysis* [15] is shown in Figure 21a. We see the code begins with the description, then option package is declared. The exports give the routines and data available to the user. Individual functions follow.

The statements used to create the package file sre shown in Figure 21b. We begin by setting the library's pathname. Next, we create the package file, unless it already exists. If the package file already exists, we delete it, and then recreate it for a new version. (Cell (4) in the worksheet.) The command *savelib*(*evaln*(*PSMv2*) saves the



(a) A Maple Module for the PSMv2 Package (b) Creating and Saving the Package

Figure 21: Creating a Maple Package

module *PSMv2* in the package format to the file "PSMv2.m" just created. The rest of the commands are to verify the package creation and add author names to the file.

# 4 Conclusion

We have looked at several aspects of using CAS for didactic activities. Classroom demonstrations are the natural entry for instructors to begin using CAS with students. In these difficult times, prepared CAS documents distributed on the internet help students discover mathematical concepts and become independent learners. The *ACR* paradigm underlying the design of these CAS activities greatly improves their utility and effectiveness. Hyperlinking in worksheets allows us to build cohesive collections of materials for students where they may follow a developing path or access relevant references assisting their studies. The MapleCloud provides an easy to use repository of CAS documents for students to access.

The Maple CAS provides a very rich pedagogical environment for creating material and for student explorations. Maple's extensive capabilities will serve students well after their studies as a computational engine or as a research assistant. These tools have become the necessary foundation for an instructional platform in these challenging times. As Holmes said, "Watson! The game's afoot!"


# Acknowledgements

I am very grateful to the organizers, with special thanks to Prof. Wei-Chi Yang, for the opportunity to present at ATCM 2020.

This material is based on several projects with Wade Ellis, and on joint work with Michael Bossé, Eric Marland, Katrina Palmer, Jaehee Post, Greg Rhoads, and Joel Sanqui.


# Links

- Presentation slides are available at http://mathsci2.appstate.edu/~wmcb/ATCM2020
- The Maple documents are available at http://mathsci2.appstate.edu/~wmcb/ATCM2020/Maple

## References and Further Readings


[1] Apple, D. K., and Ellis, W. Learning How to Learn: Improving the Performance of Learning. *International Journal of Process Education 7*, 1 (2015), 21–28.

[2] Bauldry, W., and Ellis, W. Dynamic applications for learning and exploring mathematics using computer algebra. ACA 2019.

[3] Bauldry, W. C. Using maple in advanced calculus and modern algebra. In *Proc. of the 20th ICTCM* (2009), pp. 13–17.

[4] Bauldry, W. C. *Introduction to Real Analysis: An Educational Approach*. John Wiley & Sons, 2011.

[5] Bauldry, W. C. CAS in the Classroom: Yesterday, Today, and Tomorrow. In *Proc of TIME 2012* (Tartu, Estonia, 2012).

[6] Bauldry, W. C., Bossé, M. J., and Otey, S. H. Circle constructions of polynomial complex roots. *Electronic Journal of Mathematics & Technology 11*, 2 (2017).

[7] Bauldry, W. C., Evans, B., and Johnson, J. *Linear Algebra with Maple*. John Wiley & Sons, Inc., 1994.

[8] Bauldry, W. C., and Fielder, J. R. *Calculus Laboratories with Maple*. Brooks/Cole, 1990.

[9] Bauldry, W. C., and Fielder, J. R. *Calculus Projects with Maple*. Brooks/Cole Publishing Co., 1996.

[10] Bossé, M. J., Bauldry, W., and Otey, S. H. Locating complex roots in the graphs of rational functions. *Electronic Journal of Mathematics & Technology 12*, 2 (2018).

[11] Dick, T., Burrill, G., and Brady Gill, L. New technologies offer new ways to engage students. *NCSM Newsletter 38*, 1 (2007), 19–21.

[12] Ellis, W. Technology and calculus. In *Calculus Renewal*. Springer, 2000, pp. 53–68.

[13] Ellis, W., Bauldry, W., Bossé, M. J., and Otey, S. H. Employing technology to visualize complex roots of real polynomials. *Proceedings in TIME-2016 (Technology in Mathematics Education). June 29* (2016).

[14] Fox, W. P., and Bauldry, W. C. *Advanced Problem Solving with Maple: A First Course*. CRC Press, 2019.

[15] Fox, W. P., and Bauldry, W. C. *Advanced Problem Solving Using Maple: Applied Mathematics, Operations Research, Buisness Analytics, and Decision Analysis*. Chapman and Hall/CRC, 2020.

[16] Marland, E., Rhoads, G., Bossé, M., Sanqui, J., and Bauldry, W. $Q^2$: A Measure of Linearity. *PRIMUS* (to appear).

[17] Palmer, K., Bauldry, W., Bossé, M. J., and Post, J. A complex number raised to a complex exponent: What does this mean? *PRIMUS* (2020), 1–12.

[18] Saber, H. A. Using maple, maple 3d graphics, and tablet pc to teach calculus ii and iii. In *2016 13th Learning and Technology Conference (L&T)* (2016), IEEE, pp. 1–6.

[19] Snow, J. E., and Weller, K. E. *Exploratory Examples for Real Analysis*, vol. 20. American Mathematical Soc., 2003.