Towards Dynamic Mathematics with Cabri
Where do We Stand? Where do We Go?

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Abstract: *It is a common to say that technology and its impact on the society are moving fast.*

Is that so sure? Beginning of the 80, the raise and success of the concept of Graphical User Interface (GUI) in direct manipulation, made possible the invention of dynamic geometry.

It took some time for Dynamic Geometry Systems (DGS) namely Cabri and just after Geometer's Sketchpad to spread in the various education systems. For long time only the most innovative teachers have understood the real potential of the revolution brought by such environments.

Nowadays we have to admit that dynamic geometry*, and more and more widely and recently* dynamic mathematics*, have not yet fully revolutionized the learning and teaching of mathematics.*

In my presentation I will look at the limitations encountered to make profound, technology-based changes, in math education. I will present evidence of such constraints but also explain what evolution and solutions, the Cabri-team is working on, especially with regard to the availability and maturation of the new Cabri Express: Cabri Express is a dynamic mathematics *software, freely available and running through internet on any device.*

1. A condensed view of how dynamic geometry was born and developed with technology

Math can be viewed as the study of what is changing and what is constant. This applies to the sciences in general: many sciences have for object the recognition of what is constant (invariants) among an ocean of changing perceptions.
Let's take a basic example, one of the first human knowledge acquired on earth. The solar globe passes through the sky every day, reappears the "next day" at about the same place as the day before. Extracting more and more regularities from many observations, humanity has forged what we call scientific knowledge. This can be viewed as an extraction of what many different configurations have in common. And this process can be viewed purely as an abstraction, which is essential in mathematics.

We argue that, also in mathematics, the main spring of development is concerned at looking at how things change or do not change along a process.

Consider here again a basic example. When we observe how a measurable quantity varies by Δy depending of the variation Δx of another quantity, we could notice that in many cases Δy is in simple relation with Δx. One of the simplest cases is probably $\Delta y = \Delta x$ or more generally $\Delta y = k*\Delta x$ where k is a constant. In (Anglo-Saxon) textbooks k is called the rate of change. Looking more closely, scientists have been led to recognize the importance of the concept of linearity (in one and later in n dimensions) and eventually to the concept of linear approximation of the local behavior of a phenomenon by its derivative.

By examining many parts of mathematics, we can easily recognize that in many of them, these variations refer to certain variables and, often, explicitly or not, one the variables can be identified with time: *variation* becomes *movement*.

In the absence of technology (as we know it with computers), we can hardly perceive the importance of movement in the development of mathematics. It is clear that computers, especially in simulations, have made it possible to act on time in accelerating, stopping or even inverting time. If movement is as important in mathematics as I claim, we should be able to clearly recognize the role of movement in the historical development of mathematics. One difficulty here is that, unfortunately, most mathematics (from advanced research papers to most textbooks) are written in a very static way (what Lakatos [1] refers to as deductivist style). In fact, professional mathematicians have the ability of resorting to thought experiments to freely look at mathematical objects in different positions, to move them around mentally, passing at will from one to another in a sort of mental video (as an extension of what people call mental imagery).
Before computer burst into the classroom, this ability to think, execute and run a mental video in the brain, was reserved to a very limited number of people with a strong inclination for mathematics. Some mathematicians, among those who were attentive to the teaching and learning of mathematics (such as Clairault [2], Poincaré, …) even took time to describe explicitly in their writings the essence of their mental experiments.

For a long time, computers were merely machines that could, from a set of numbers, apply an algorithm to them and produce another set of numbers (the result). This did no help to facilitate access to mental video-clips because 2D or 3D representations of data (numbers) was clearly missing.

In the 1970s a first generation of computer terminals appeared in the form of heavy CRTs connected to mainframes. It took 10 or 15 years for a second generation of computing devices to appear with graphical possibilities: the images were composed of line segments and the rendering of an image on the screen was made by the rapid move of the electronic spot, from one segment-extremity to the next, along all segments in the image. It was done again and again in the hope that it could be fast enough to deceive our eyes and our brain so that we could *see* an image and not a point in rapid motion.

The next achievement was to be able to individually *address* small screen entities (called later pixels) and turn them on and off at will. For the first method, we are talking about raster device, for the second bitmap images. This was well after the pioneering work by Bresenham [3] at IBM, and his conception of an efficient algorithm for deciding which pixel to activate in order to produce the (jaggy) image of a line-segment.

So, basically, it took 30 years of technological computer development to reach computers, powerful enough to render images in a reasonable way. Not such a fast development… And so, dynamic geometry was born in 1984 [4], when computers acquired enough power to display calculated images of animated geometric objects (points, lines, circles…). The lines depended on 2 previously defined points, the circles also depended on 2 points, its center and another peripheral point… Changing the location of any point resulted in the re-calculation of the figure to be drawn in its updated state.

2. To what extend have computers changed teaching/learning practices?

The best I could do to introduce the above question is to tell an anecdote.

In August 2016, in Hamburg, I attended sessions of TSF 43, devoted to the *Use of technology in upper secondary mathematics education* [5]. One of the presenters were rather enthusiastic about how *Dynamic Geometry Systems* (DGS) were an opportunity to motivate students and deepen their learning by exploring mathematical constructs. During the question-and-answer session, one of the attendees asked how students could interact with the geometric constructions, what kind of ease or difficulty they encountered with the SW. In fact, a very interesting question.

The presenter then explained that she could not answer the question because, as she explained, it was out of the question for her to let the students access the mouse or another pointing device; she said her teaching was based on the use of a whiteboard, thus preventing the student from **touching** the mathematics (my emphasis).

Below is a German cartoon showing that all math teachers have not yet understood how to use the new technological tools: "*But not so bad this new device ...*"

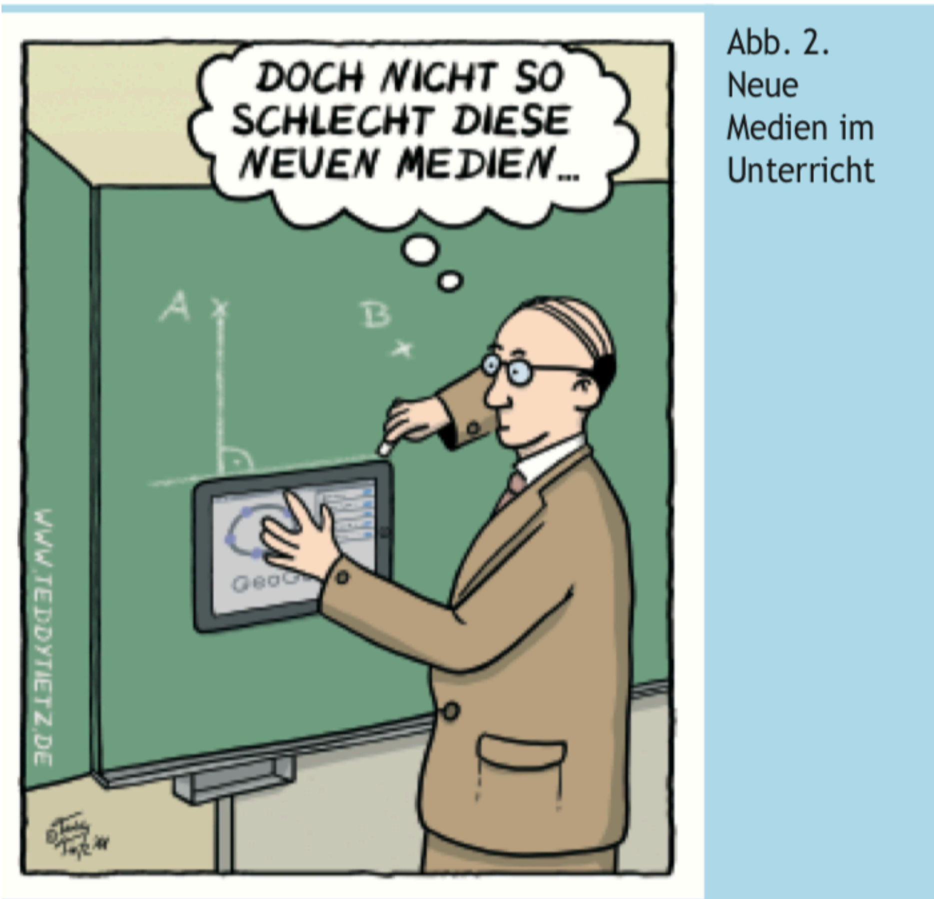

Figure 2.1 Teacher Issues about using new technologies in math classroom

This means that still today many teachers, even some of the most technologically advanced, are far from understanding what technology and dynamic mathematics are. This is not new and we have been confronted from the beginning with similar situations, with shared responsibilities also with senior officials at MOEs.

For that, I will have two more anecdotes, this time about Cabri.

In 1996, many years after Cabri and Geometer's Sketchpad were already widely known, it turned out that my University (recognizing Cabri's innovative power) had presented Cabri for the *European Academic Software Award*. We were 33 years after the invention (by Douglas Engelbart) of the mouse… and Cabri were rejected because Cabri needed a mouse to function. (BTW Geogebra won that award 6 years later with its first version, at that time definitely a direct Cabri clone).

Second anecdote is in 2008, in Morocco where the Ministry of Education launched an ambitious program to introduce ICTs in all schools in the country. The National License proposed by the MOE contained a provision for the training of a maximum number of teachers. At the time of implementing the training the government realized that it would be too expensive and that it would be cheaper to train only Math Inspectors (around 200 people). Our team went to Rabat and spend 4 days in training all the inspectors divided in 4 groups. In each group, the inspectors were asked to work by two or three people. On the second day, one of the trainers noticed that the inspectors from one of the subgroups were not really active and were just looking at their computer screen. The trainer therefore asked them why they were not trying to work themselves on the proposed geometry activities, why they did not use the mouse connected to the computer. The answer was: *"But you know, we are Math inspectors, we are here to understand the basic concepts beyond software, to be able to pass this knowledge to teachers in the countryside. We do not have any need to learn how to use a mouse in practice."*

Many researchers have questioned the effectiveness of the ICT use in education and particularly in math education. The results do not seem particularly optimistic.

Below is a quote from an official report of the Western Australian Government [6] (p.147)

Empirical research summaries have consistently suggested that the use of graphics calculators and CAS calculators by secondary school teachers and students can result in improvements in conceptual understanding in mathematics, although the improvements are typically modest and depend on the extent to which teachers and students make effective classroom use of them. Definitive large-scale studies on the effectiveness of sound use of CAS in secondary schools are not yet available.

Currently, there is neither a national nor an international consensus on which technologies are most appropriate for use in senior secondary schools, although it is clear in recent years that technology is regarded as an increasingly important part of the school learning environment for mathematics.

In the same manner, reading from an ICME report [7] (p. 5):

The overall image is that the use of technology in mathematics education can have a significant positive effect, but with ***a small effect size****. Given that any innovative educational intervention usually has a positive effect anyway (Higgins, Xiao, & Katsipataki, 2012), these studies do not provide overwhelming evidence for the effectiveness of the use of digital tools in mathematics education.*

The results reported above are mixed, and interpretations reported by authors seem ambiguous. The results of the OECD study show negative correlations between mathematics performance and computer use in mathematics lessons and lead to the conclusion that there is little evidence for a positive effect on student achievement: Despite considerable investments in computers, internet connections and software for educational use, there is little solid evidence that greater computer use among students leads to better scores in mathematics and reading. (OECD, 2015, p. 145)

Today, however, it is recognized that the use of ICTs in education has more advantages than disadvantages. While it is true that we still lack a great amount of serious large-scale studies about the effectiveness of technology for math education, some do exist. Surprisingly they have never been seriously taken into account by governments and/or education policy makers. I recall here in a note[1] about two of them showing significant improvements in student achievement when technology is part of the curriculum.

One other point where research has not yet been sufficiently developed, is the dimension of software quality. When examining the various small-and larger-scale studies of technological efficiency in math education the various characteristics of SW used for the study, are never analyzed or even considered. It is as if, in order to examine student performance in a music school, the actual quality of the instruments on which every student is playing, would be considered irrelevant, that is, the result obtained by a given student would have no connection with the quality of his instrument. For example, in the case of a piano, if it is perfectly tuned or unbalanced and detuned, would be irrelevant. It seems to me that in the same way that not all the pianos are not identical, all dynamic geometry SW, are not the same. I recommend here studies by Kate Mackrell [8] (2011, as referenced in Sinclair & al. [9]:

Mackrell [2011] discusses the matter of design decisions for interactive geometry software comparing *Cabri*, *Sketch-pad*, *Cinderella* (Richter-Gebert & Kortenkamp 2012) and *GeoGebra*, and shows that a certain variety of approaches is desirable. She also highlights the fact that consistent educational design of a DGE is difficult to achieve.

It is clear that not all SW are equal, even if they are sometimes considered superficially equivalent. This may be due to different global visions, different design decisions, different qualities in their underlying mathematical engine. In reality, students are surely not performing independently of certain characteristics or the SW.

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[1] One study is from Spain [10] with 15,000 students, followed during their 6 years of secondary schooling, and the other from the SRI [11], on the use of innovative SW SimCalc in two states in the US, Texas and Florida

3. So how could we achieve better in enhancing teaching/learning with technology?

At the turn of the millennium Dynamic Geometry Systems (DGS) had conquered a large audience worldwide, essentially with Geometer's Sketchpad or Cabri. Nevertheless, they have never gained 100% of acceptance. In fact, to follow a well know pattern of acceptance evolution[2], DGS went only from an *Early Adopter* stage to some *Early Majority* stage. Then their adoption has been then stagnant, more than half of the potential users do not routinely take advantage of that technology.

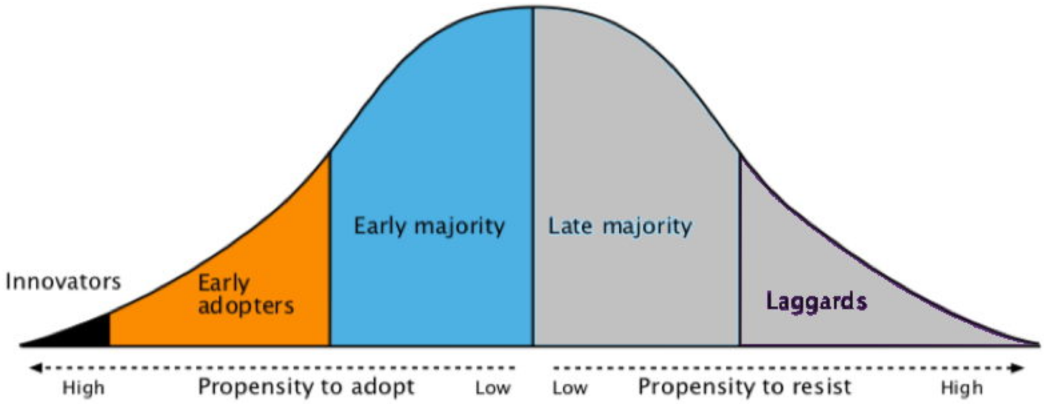

Figure 2.1 The right part of the potential users of DGS (in grey), have actually no use, or at most a marginal use, of DGS.

A similar phenomenon can also be seen about Seymour Papert's famous Logo. Logo is an incredibly powerful concept and tool to enhance education; nevertheless it has never gained 100% adoption. This may be because it does not really match the real needs of teachers. The same applies to GSP and Cabri.

I would dare to mention here the case of Geogebra: At least in Europe, and also to a lesser extend in the US, the policy makers, decision makers, influential teachers have largely explained that Geogebra was THE solution because it was free. I think it has been a serious mistake by too many policy makers to think that one of the main obstacles for DGS adoption, was the price. From this it was deduced that *national organizations* had to recommend a SW like Geogebra because it was free (in addition to be *perceived* as Open Source).

Today we have enough distance to analyze what has happened since the appearance Geogebra in 2004, that is 15 years ago. One of the published concluding outcomes of the ICME-13 *Survey Team Report* on Recent Research on Geometry Education (Sinclair & al, 2016) [9], reads as follows:

The availability of digital tools, in particular DGEs, has rapidly changed over the past 8 years. The arrival of the freely- available *GeoGebra* made DGE affordable and available for

[2] The categories are from Rogers [12], the picture (CC BY-NC-SA 4.0) is from http://www.enablingchange.com.au/Summary_Diffusion_Theory.pdf

anyone, but it also undermined the educational software market in Geometry that originally had room for several different DGEs—not only the original pack-ages *The Geometer's Sketchpad* (Jackiw 1989) and *Cabri-Géomètre* (Baulac, Bellemain & Laborde 1988) but also more than 50 other products with various licensing models. Some of them, including *Sketchpad* and *Cabri*, are still available, but it has become difficult for independent developers or companies to maintain or launch alternative DGEs.

Today in most places, it is true that software implementing *Dynamic Mathematics* (DGS, CAS… etc.) is actually available in and out of the classroom. Nevertheless, there is a big distance still to run from such a state to the actual use of technology, routinely, by teachers and students. In France, the situation since 2000 have not fundamentally changed, there are many teachers having embraced the technology and implemented IBL (Inquiry Based Learning) but they are a minority next to a majority not using any technology routinely [3].
Focusing on DGS and France, the only change is that, when Cabri was almost in every school at a cost of 0.20 € per student, it is today replaced by Geogebra which is free, in practice not more used than Cabri in the past, but, according to many researchers, with less student usability [4].

Why do Logo or more recent Dynamic Mathematic software package not meet the real needs of teachers? It seems that in their majority (the *Late Majority* and the *Laggards* of Rogers's categorization [12]), in addition to not having received adequate training on the changes inherent in the adoption of digital technology in their classroom, they also have little time to implement themselves the technology. Teachers lack digital resources fulfilling their needs. Textbook manufacturing is a millennial activity in all cultures, while the creation of digital resources is just 10- or 20-year-old. Therefore, there is no mystery that there are not yet sufficient resources to satisfy the teachers and convince them to radically change their teaching style. Again, it is hard to see how stakeholders in the field (the resources providers) can hope a sustainable eco-system, when implicitly it is widely expected that digital resources are free.
Such a wait can probably be explained at least by two factors: one factor it that, if one is inclined to attach a price (which can be different from its *value*) to pay for a material item, there is difficulty to admit that immaterial items have a price[5]. In the case of purely material items, the price of production (creation) of one item can be similar (or of the same order) to the price of its reproduction. For intangibles the price of the initial creation is still recognized but the price of its reproduction is perceived as close to zero. One other factor is again the result of the mistake made by policy makers in advocating, sometimes exclusively, free-resources (see above the observation about *undermining* [9]).

4. What can be done to change the situation?

What are the ingredients needed to boost the production of good resources?

[3] It seems that as long policy maker will be reluctant to the use of technology at exams, teachers will reflect (or even advocate) this ideological position, producing a strong barrier against any wider use of technology, other than scientific pocket-calculators.
[4] See Makrell [8]
[5] This is reflected in the long time it took for the record industry to stabilize a model where authors and creators receive a decent revenue from their works (and talents) and where the companies organizing the distribution of the works have enough revenue to be sustainable. By the way similar fight had to take place some hundreds of years ago, if not longer, for musicians and writers to be recognized in their rights and to receive payment for their work.

Since the late 2000s we have begun to realize that a microworld (say of type LOGO, a recent high-end handheld device —TI-Nspire, Casio ClassPad or others—, a CAS, Mathematica, Maple, a DGS) would not *per se* fulfill the needs of teachers. We came to the conclusion that it was necessary, in addition, to enrich the system with carefully designed digital activities. In a sense, we wanted to facilitate the creation of constrained direct manipulation microworlds.

A constrained microworld with direct manipulation appears as a coherent environment (like LOGO) and then offers to the student the opportunity to explore mathematically the task at hand. In addition, its user interface (GUI) is governed by direct manipulation and direct engagement [14]. Unlike early microworlds, it is not completely open-ended and is strongly linked to a task that the student must tackle. It is limited in the sense that everything cannot be done on the surface of the screen. For example, it should not be possible to completely mess with the objects represented on the screen.

Older Cabris have been designed as versatile dynamic math environments. Their design is user-centered without actually making any strong assumptions about the user, who could be a student, an advanced student, a math or physic teacher, even a professional mathematician. In fact, even if it was possible to conceive enriched math activities designed for students to explore and learn science, nothing was really provided to support the design of interactive learning activities, able to provide teacher-level feedback to the user. In contrast the *new* Cabri appeared around 2010 offers some specific tools for a teacher to design interactive activities. It provides support for automated adequate feedbacks to the user in case the user gets lost or makes mistakes.

Below I am talking about what has changed between 1985-2008-Cabris and the new generation Cabris (2008-2019). To understand what is changed it is important to keep in mind what has NOT changed: actually, essentially the foundations of the overall design based on four main principles, commented and published elsewhere[6] [13]. I just recall them:

- Direct manipulation,
- Direct engagement,
- User centeredness,
- Domain fidelity.

I remember introducing this new generation of Cabri, (called at that time the *New Cabri*) during ATCM 13 in Bangkok in 2008. One of the major changes was the possibility of developing a multi-page activities with an easy way for the author, to work freely and put notes in the margin outside of the main window. At that time we also introduced three different usage modes: On one end the *Student Mode*, with access to tools as decided by the teacher or the author; at the other end the *Authoring Mode*, with access to all available functionalities of the SW, and in between, the *Teacher Mode*, in which the teacher can adjust certain parameters of the activities (as they were prepared in the authoring mode) and also replay verbatim, the activity previously done by a student.

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[6] *Cuatro Principios fundamentales para el diseño de recursos digitales profundamente interactivos en matemáticas*, Pleanry address, act of the 8e congreso de modelation, Medellin-Colombias May 2016,
For the two first principles, refer to Kearsley, G., & Schneiderman, B [14]

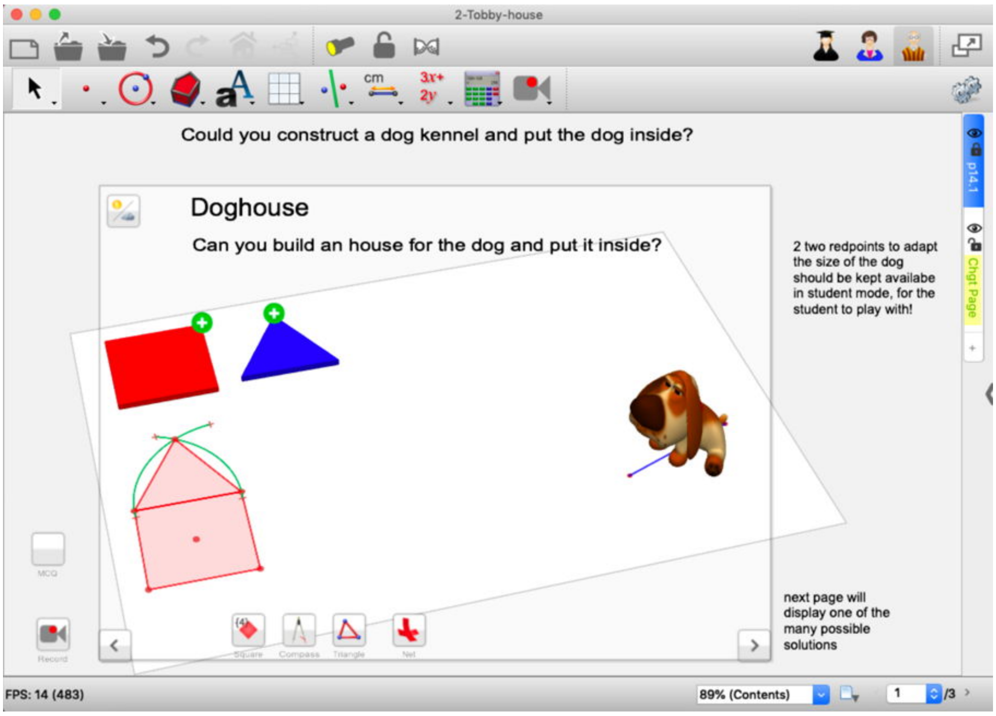

Figure 3.1 Interface of the new Cabri in *authoring mode*
(note the work in the margins and the complete tool bars)

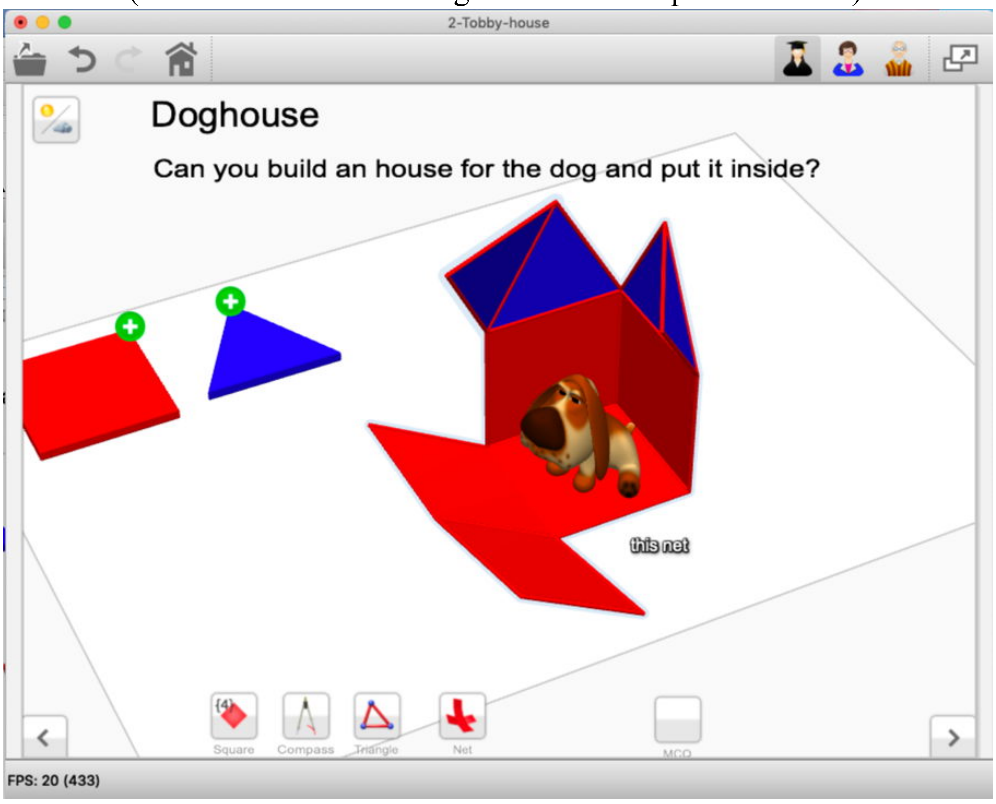

Figure 3.2 Interface of the new Cabri in *student mode*

Increasingly, everyone feels the need to access to digital information not only from a dedicated device, such as a main computer at home or at school, but from very different devices, phones, tablets, laptops, computers at school or elsewhere. In most cases access is via a network, a LAN or even the *cloud*.

The way to make this possible is to have the same SW regardless of the specificities of any device to use. One of the possible options is to make the environment work via a web browser, directly or indirectly. Advances in SW design and programming techniques allow complex applications to run in a browser with essentially the same performance as if the app were running natively. We no longer need, as in the past (10 years ago), to develop different apps, even similar ones, to work separately in Windows, MacOS, Android, iOS or Linux.

Thus, three years ago Cabri underwent a deep rewrite: from pure C,C++, OpenGL technology it was ported to a mix of C, C++ translated into WebAssembly, Javascript, CSS and webGL. This new version exists under different forms, the most common one is called Cabri Express:

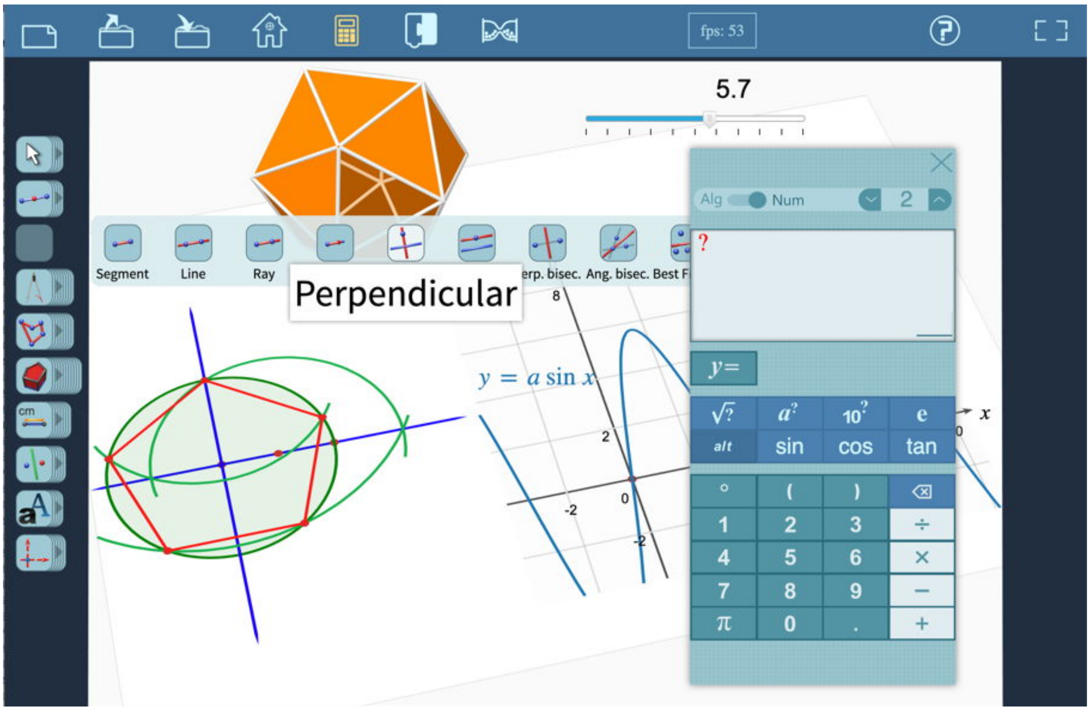

Figure 3.3 Interface of the new Cabri Express

In practice students nowadays access digital resources, either on their own behalf or as part of an institution. At the same time, unless people access some specific services clearly associated with a financial transaction, it is implicitly expected that accessing resources from the Internet has to be for free[7].

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[7] It is interesting to relativize above assertion. Indeed, one phrase has recently gained popularity on Internet: "When something online is free, you're not the customer, you're the product".
It is often attributed to Douglas Ruskoff [16].
Readers who are more interested in the subject, may wish to consult Brideron & Hussler's recent Ph.D. at Umeå University: IF IT IS FREE, YOU ARE THE PRODUCT [17]. By the way, according to a blog form Harvard university [2012] the

So, we decided to free the access to this piece of software for individual users. On line, the easiest access is at following address [14]: https://cabricloud.com/cabriexpress/

Schools, academic institutions or publishing houses on their side may eventually install this *new generation Cabri* on their server through a specific license, in two different flavors.
One flavor is to let the students access Cabri Express through the school server or a specific portal. In this case the student can store her productions online. This allows a given student to retrieve work from another device at home or from a different location while staying connected through the school portal. In this case the student's teacher can also access the student's work to review, annotate it and perform a "manual" assessment.

The second flavor is the Cabri Author flavor. With the help of Cabri Author, teachers and activity designers (say authors) have access to an extensive set of features. One of these features is the ability to insert an *external component* into a page.

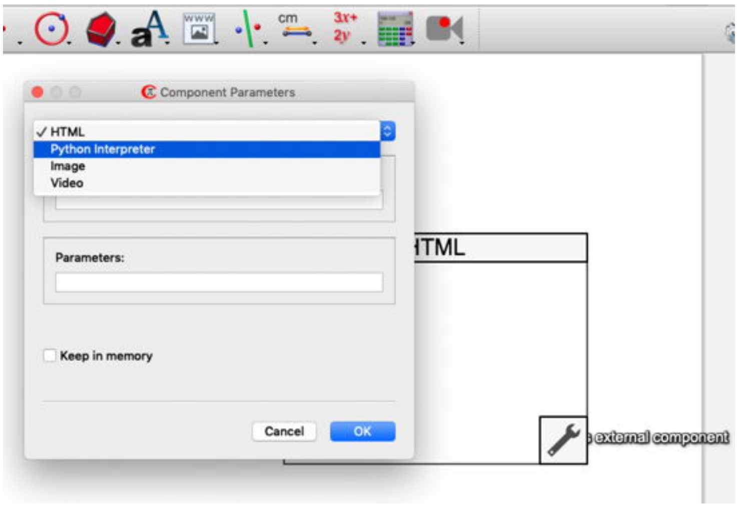

Figure 3.4 Cabri in Author mode to insert external HTML components

The *External Component* tool lets you insert various elements into a user-defined rectangle in the page. In addition to standard images and/or videos it is possible to import a complete HTML

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phrase (or its content) can be traced back much earlier, e.g. Schneider in 2010, Ken (2004) quoted with "Remember, in commercial radio, you are not the customer. You are the product being sold", or even Noam Chomsky in the 90s.

Remember, to accept advertising in exchange for free access to services, or to "benefit" from free access in exchange of letting the system collect information, at least about the way you interact with your computer, or possibly about you, sometimes without your consent. If the system works, because it cannot operate just "in the air", it is because collecting information, extracted through you, creates value that can be monetized.
It is probably too late to reverse the situation. Nevertheless, it seems to me that to overcome this serious difficulty, there is a way to allow individual students to access free educational resources on Internet without being the "product" of some organization, which might not necessarily be motivated by the interest of the students only. This way would be to have such resources financed by national organizations reporting directly to National Ministries of Education. This important point, from my perspective, deserves to be developed more widely.

page. The advantage is to allow then the student to access specific Web pages without compromising the security or running the risk of downloading inappropriate content.

One other component allows to create a page where students can edit or design Python programs. On the page, left side shows a syntax-color-oriented Python editor and, on the right side, the results obtained when running the program.

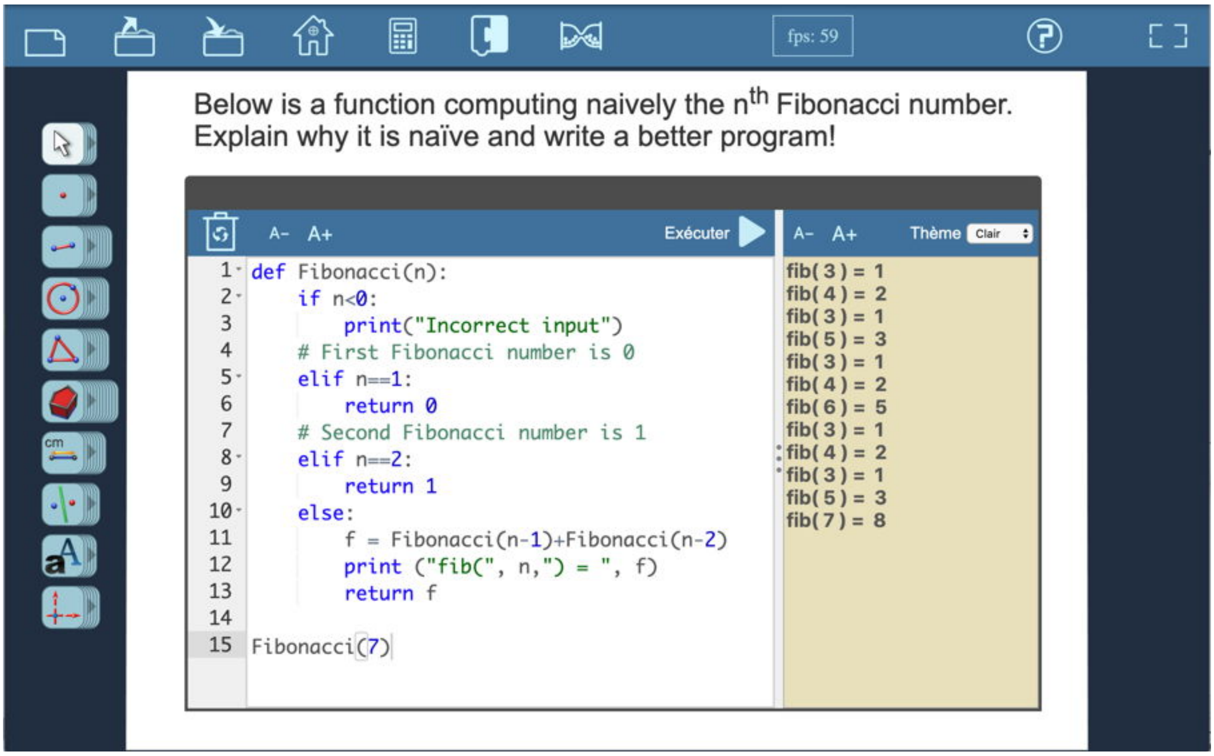

Figure 3.3 Example of a student page containing a basic exercise in an introductory activity to Python coding

The other flavor concerns teachers capable of preparing more or less complex digital activities. This flavor comes also with a special option we call Cabri Premium allowing for automated assessment.

Evaluating their work is a crucial issue for students working with technology. In a paper-and-pencil environment, this can be a tedious job for the teacher or the examiners, but technically it's easy because the student's work can be summarized on one or more pieces of paper. In a technological environment, it is technically possible to automate the evaluation, including even the evaluation of geometric constructions. Cabri Express offers a special support for automated geometry assessment.

Concluding remarks

For a long period of time, financial constraints, objective difficulty of providing "computing power" for every child in a classroom, teacher's issues have constituted real barriers towards the adoption of digital technology on a large scale in education.

For instance, graphing calculators have been massively adopted but other technologies have not. Today, this is about to change with the spread of tablets that are easy to carry and can be set up in no time.

A tablet offers a more direct interaction between the user and the digital environment. Quite recently, the mouse represented immense progress in the way people interacted with their devices, as they could manipulate things directly on the screen. But a mouse is an indirect pointing device, and a tablet offers a more direct means of interaction: the finger.

For a long time, running apps on different devices has been a nightmare because of hardware disparity, but running apps through the Internet in a browser is nowadays definitely a realty. HTML5 and associated technologies (Javascript-CSS-SVG-WebGL), painless connectivity (i.e. online and offline access), are very attractive, and from now highly and deeply interactive apps can run smoothly, thanks to those technologies.

Due to progress in computer technologies and the cognitive sciences, including in math education, we now gaze at the birth of new, sophisticated educational resources, fulfilling the teacher's and student's needs.

In short, we have renewed reasons to hope for a brighter and even more exciting education for everyone.

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