Making the Mathematics Enjoyable in Modern Science

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Abstract: This is *an introduction to current Japanese mathematics and science education for enhancing children's motivation to concern about the nature of phenomena around us, for example, the size of the Universe and Atom. How teachers can apply mathematics to science using materials which I developed, details, contents which science teacher can use in mathematics and science education class more enjoyable. This can be explained easily using the exponential function. For example, if you try 1% more keeping an effort every day than the previous day, you will grow 37 times more than you are now in 365 days, a year later. On the other hand, if you continue to take a rest of 1% than today, it will only be 3% of your current one year later. However, it is hardly understanding for young children, even adults. In human history, people have recognized nature through their own senses, especially through vision. Currently, in human science research, the human sense is exponential. This can lead to false recognition with an illusion. I try to explore the ideas and creativity of children under the theme of how they were using exponential function in mathematics and science class. It is important for children to make the mathematics enjoyable in modern science. The importance of mathematics and science is explaining the real world. They can be always explaining the phenomena of nature under the limited condition of approximate value.*

1. Introduction

Starting from next fiscal 2020, the time for 'research activities' in school curriculum will be set up specifically for mathematics and science in the Japanese Course of Study. Until now, the time for such inquiry activities has been set, but a national curriculum that focuses on "exploration of science" is required. In addition to acquiring math and science knowledge and skills in school education goals, students think about themselves, explore math and science problems, and children themselves formulate a method for solving the problems and try to solve those problems. It is important their learning activity that is more deepen scientific learning. Therefore, we developed concrete teaching materials for "exploration of science" for primary to high school students.
Specifically, on the theme of "Jules-Henri Poincaré conjecture" (to think the size and shape of the Universe) in mathematics, ask students what happens to a sheet of paper as you can fold it and continue folding forever where each fold doubles the thickness. In addition, we mention the following sample questions that we ask students how to measure:

- the size of a body,
- the size of a building,
- the height of a mountain,
- the size of the earth,
- the distance to the moon, the distance to the sun,
- the size of the solar system,
- the size of the Milky Way Galaxy,

- the size of the galaxy,
- the size of the universe,
- the size of the end of the universe, and so on.

While introducing the brief history of human civilization, the unit of length, the unit of size, and the unit of weight, etc., we usually start from the part of human body as a starting point of discussion,. Then we move on to the discussions regarding the size of the earth, the weight of the earth, the density of the earth. Based on the distance between the sun and the earth, and the speed of light, with the help of modern science and technology, it will help to measure the size and shape of the universe. How humans recognize the world around them and capture the size and shape. I aimed to explore the ideas and creativity of children under the theme of where things came from.

At present, I organize open lectures for schoolchildren, I am expanding the experimental classes throughout Japan to verify the effects and how they can be used for "exploration of science".

2. Can you imagine?

Please consider what happens to a sheet of paper as you fold it - and continue folding again and again. Each fold doubles the thickness. Since the paper is about 0.05 mm thick, the thickness after each fold is as follows;
0.05, 0.10, 0.20, 0.40, 0.80, 1.60, 3.20, 6.40, ・・・ (See Table 1). Ordinary though doubling is, when we apply the procedure repeatedly, the result soon becomes very large. Imagine the results after folding the paper 100 times. How thick would the paper become?

Number of folds	Thickness / mm
0	0.05
1	0.10
2	0.20
3	0.40
4	0.80
5	1.60
6	3.20
7	6.40

Table 1: Overall thickness of folded paper after each fold

Here are some possible answers choices to the question:

1. Human height (1.8m),
2. School building (10m),
3. Tokyo Sky Tree tower (634m),
4. Mt. Fuji (3776m),
5. Space Station height (400km),
6. Distance to the Moon (384,400km),

7. Distance to the Sun (150,000,000km),
8. More than that!

The correct answer is number 8, "More than that!" which is about 6.34×10^{22} km. This is much larger than the distance to the Sun, which is only 1.5×10^{8} km.

Next, I show the model of a terrestrial globe of 13cm diameter, in front of my desk. I ask them if the Earth were to be this size, how big the moon and the sun would be in comparison.
"Please discuss, use your imagination and share your ideas." Immediately, some of my students challenge me and state: the Earth is not a circle because mountains are more than 8,000 meters high and seas are more than 10,000 meters deep around the world. Another student also say: the Earth is not exactly circular but is elliptical because the diameter of the Earth at the equator and at the poles are different by 22 km. There are usually many ideas about the shape of the Earth in my mathematics class.
Then I show the following Table 2:

Size	Diameter	1,000km →1cm	Looks like	Distance from the Earth
The Earth	12,756 km	=13.0 cm	Baseball size	0 m
The Moon	3,476 km	= 3.5 cm	Ping Pong ball	380,000 km = 3.8m
The Sun	1,391,400 km	1,391.4cm =14 m	8 Floor Building	14,960,000 km =1.5km

Table 2. (The Size and distance of the Earth, the Moon, and the Sun)

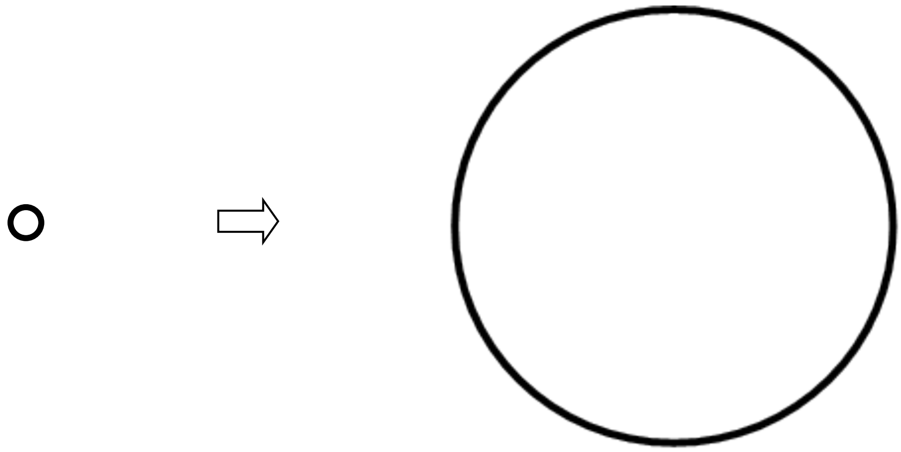

Figure 1. When the diameter of this circle becomes 10 times, how is the size of the circle?

The Earth is not exactly circular however if a circle of 13cm diameter is drawn using compass, to represent the diameter 12,756 km, the pencil line (assumed to be 0.4 mm thick) would represent 40 km on the same scale. If the difference between 8000m (8km) mountains and 10000 m(10km) deep seas is total 18 km and the difference between the equatorial and polar diameters of the Earth is 22 km, then the maximum possible variation in the surface of the Earth is expected to be no more than 40 km. This means that all variations in the surface must be within the line drawn to represent it.
In addition, the orbit around the sun is an elliptical orbit, just as the earth is an ellipsoid, which was found by Johannes Kepler (1571-1630) who estimated that all the planets take an elliptical path

from the study of Mars' orbit, and that in all cases the sun is one of the focal points of the planet's orbit.

The law of universal gravity was derived based on the law of motion which Newton (1642-1727) discovered and the law of Kepler. On the other hand, Kepler's law is the universal gravitation law, and the mass of the planet compared to the mass of the sun under the condition that the sum of the potential energy of the planet and the kinetic energy is negative (that is, the planet can't fly to infinity). It can be derived by performing an approximation and solving that is sufficiently small (that is, the sun can be regarded as static and interactions between planets can be neglected). What Kepler describes about the motion of planets in the solar system holds similarly between any two mass points, which are sufficiently small compared with that around it.

A planet orbiting an elliptical orbit passes through the perihelion point closest to the sun and the farthest apocalypse yearly. On July 4, the earth will reach the apogee and move away from the sun to 152,212,196 km. The perihelion passage was January 3rd, approaching 149,590,787 km. On average, the difference between the aphelion and the perihelion differs by about 4.8 million km, or about 3%. As a result, the apparent size of the sun also decreases by about 3%, but the difference may only be seen with a telescope.

That is, as with the shape of the earth and the orbit around the sun, even if it draws a 10 cm circle with a 3 mm core pencil, it will be included in the error range.

3. How does human being recognize the length, area, and space around the world?

Historically, the original name of the unit to measure the length is based on human body, for example, 1 cubit, 1 foot, 1 inch, 1 尺（1 Shaku), 1 寸（1 Sunn), 1 分（1 Bu), and so on East and West civilization. In general, the diameter of human head is about 15 cm. If you multiply that by 100 million, it will be 14,000 km, larger than the diameter of the earth (≒13,000 km）. If you multiply the oxygen and nitrogen molecules in the air by 100 million, it will be about the size of a ping-pong ball (3.5cm). Then, if we assume that our human head is as large as the earth, we can compare it to a ping-pong ball.

Let us image the volume of a circle and a sphere that is 10 times the diameter in terms of the recognition of the object's length, width, and space (Figure 1). There is a theory that human cognitive ability regarding the length, size, and space of objects is influenced by the experience of early childhood. This teaching material is effective for enriching space recognition ability. When traditional educators hear that we use space models in our classes, they regularly claim that elementary school children cannot possibly understand or accept such a difficult concept. However, the children have completely welcomed the contents of this kind of experimental class using NASA Video without any concern for these educators' misgivings. Why do children like lessons that use space models? Modern science education should keep this in mind and be careful not to betray children's trust when teaching them what they want to know. We provide interesting and challenging problems with experiments and let every student guess and discuss what the results might be before performing the experiment. The children are deeply motivated by the fact that they can discover the problem by themselves.

Children will lose the sense of pleasure and challenge in investigating the problem by themselves if they already know the result. They acquire knowledge of space science through group studies in their class. The learning environment is very relaxed and enjoyable. Moreover, nowadays, children can see computer simulations of space motion like Power of Ten (1998). It is very important for us to be familiar with an easy and understandable the size of the object's length, width, and space from our childhood. We hope that this plan will be a good way of sharing the magnificent experience of thinking scientifically with space models.

In Japan, how can children use origami to draw such ellipses, hyperbolas and parabolas? Let us unfold using traditional Japanese square origami. There are educational materials that can be used to open paper and experience geometry in a fun way for not only children but also adults. Now, origami engineering is applied to the latest technology like STEM education in Japan.

4. Conclusion

In this case study, cause and effect relationships are not searched yet, thus farther research is required. The skills of mathematics and science knowledges are obtained through notebooks from classroom lessons and the students repeating the learning processes under their scientific discussion during mathematics and science classroom. I should consider identifying and instructing the students recognition about mathematics explaining for space science. I deeply emphasize how the universe is clearly using mathematics and very interesting.

The importance of mathematics and science is explaining the real world using the geometry of mathematics. Mathematics and Science can be always explaining the phenomena of nature under the limited condition of approximate value. Mathematics and Science education should be useful for making students understand our nature and phenomena around the world and the universe using mathematics.

Acknowledgements

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Appendix

① The brief history of the original name of the unit to measure the length.

The original name of the unit to measure the length			
1 cubit	From arm elbow to fingertip	Human body is a unit of length	BC5000; Ancient Egypt and Mesopotamia
1 Foot	From heel to toe		BC 2500; Sumerian culture
1 inch	Width of tip of thumb	1/12 of a foot ; The meaning of 1/12 in Latin	BC 2000; Ancient Rome
1 尺（1 Shaku）	The word “Shaku” is a character with the thumb and index finger spread		BC4000;　Ancient Chinese Zen Period
1 寸（1 Sunn）	One Sunn is one tenth of 1 Shaku ≒ 30mm		In Japan AD 700 from Korea (高麗尺こまじゃく)
1 分（1 BU）	One BU is one tenth of 1 Sunn		Kaneshaku（曲尺かねじゃく）
1 束（1 Tuka）	A unit of length representing the length of the arrow		Kujirashaku（鯨尺くじらじゃく：主に和裁）
1咫（1 Ata）	Extended length of thumb and forefinger		
1 尋（1 Hiro）	The length that an adult fully extended his arms		

② The following are the units and notation commonly used in physics and mathematics education in general.

Paper Fold	the units and notation
a sheet of paper	0.1mm(=1×10^{-4}m)
1st; 0.2 mm	0.2mm
14th; 1638.4 mm	1.6384 m(≒1.6m)
24th; 1677721.6 mm	1.6777km(≒1.7×10^{3} m=1.7km)
37th; 1.37×10^{4}km	beyond the diameter of the Earth(12,756.274 ≒1.3×10^{4} km)
41st; 2.20×10^{5}km	beyond Jupiter's diameter (the largest planet in the Solar system≒1.4×10^{5} km)
44th; 1.76×10^{6}km	beyond the diameter of the Sun≒1,392,000 =1.4×10^{6}km)
51st; 2.3×10^{8}km	beyond 1AU(Astronomical Unit: average distance from the Sun to the Earth)
67th; 1.5×10^{13}km	beyond 1 light year(≒9.5×10^{12}km ; The distance light travels in a year)
84rd; 1.9×10^{18} km	beyond Disk diameter of the Milky Way Galaxy ≒1×10^{18}km
88th; 3.1×10^{19}km	beyond Distance to the Andromeda Galaxy≒2.3 $\times10^{6}$ light year=2.3×10^{18}km
100th; 1.27×10^{23}km	≒ 14 billion light year≒ 1.4 $\times10^{10}$ light year

Paper Fold; If the paper is 0.1mm (=1×10^{-4}m), the thickness after each fold is as follows;
1st; 0.2 mm
2nd; 0.4 mm
3rd; 0.8 mm
4th; 1.6 mm
5th; 3.2 mm
6th; 6.4 mm
7th; 12.8 mm【beyond 1cm】≒1.3×10^{-2} m＝1.3 cm (centimeter) = 0.013m
8th; 25.6 mm
9th; 51.2 mm
10th; 102.4 mm

11th; 204.8 mm
12th; 409.6 mm
13th; 819.2 mm
14th; 1638.4 mm ≒10^0 m【beyond 1 m】= 1.6384 m
15th; 3276.8 mm
16th; 6553.6 mm
17th; 13107.2 mm ≒ 10^1 m =13.1072 m
18th; 26214.4 mm
19th: 52428.8 mm
20th; 104857.6 mm ≒ 10^2 m =104.8576 m
21st; 209715.2 mm【beyond Giza's Pyramid(138.74 m)】
22nd; 419430.4 mm
23rd; 838860.8 mm
24th; 1677721.6 mm ≒ 10^3 m = 1×10^0 ㎞【beyond 1 km】= 1,000 m (*10^{-3}m = 1 millimetre)
25th; 3355443.2 mm
26th; 6710886.4 mm【beyond Mt. Fuji (3,776.24 m)】
27th; 13421772.8 mm ≒ 10^4m = 1×10^1 ㎞【Beyond Mount Everest (8,848 m)】= 10,000 m
28th; 26843545.6 mm
29th; 53687091.2 mm
30th; 107374182.4 mm ≒ 10^5m = 10^2 ㎞ ≒107,374 m
31st; 214748364.8 mm
32nd; 429496729.6 mm
33rd; 858993459.2 mm
34th; 1717986918.4 mm ≒ 10^6 m = 10^3 ㎞ ≒ 1,717,987 m (*10^{-6} m = 1μm = 1 micrometer)
35th; 3435973836.8 mm
36th; 6871947673.6 mm【beyond the diameter of the Moon (3,474.3 km) 】
37th; 13743895347.2 mm ≒10^7 m = 10^4 ㎞【beyond the diameter of the Earth (12,756.274 km)】
38th; 27487790694.4 mm
39th; 54975581388.8 mm
40th; 109951162777.6 mm ≒ 10^8 m =10^5 ㎞ ≒ 109,951.163 km ≒ 109,951,163 m
41st; 219902325555.2 mm【beyond Jupiter's diameter (the largest planet in the Solar system) 142,984 km) 】
42nd; 439804651110.4 mm【beyond 1 light second; 299,792.458 km】
43rd; 879609302220.8 mm
44th; 1759218604441.6 mm ≒10^9m=10^6 ㎞【beyond the diameter of the Sun (1,392,000 km)】
45th; 3518437208883.2 mm (*10^{-9} m = 1nm = 1 nanometer)
46th; 7036874417766.4 mm
47th; 14073748835532.8 mm ≒ 10^{10}m = 10^7 ㎞
48th; 28147497671065.6 mm
49th; 56294995342131.2 mm
50th; 112589990684262.4 mm ≒10^{11} m = 10^8 ㎞
51st; 225179981368524.8 mm【beyond 1 Astronomical Unit (average distance from the Sun to the Earth) 】

52nd; 450359962737049.6 mm
53rd; 900719925474099.2 mm
54th; 1801439850948198.4 mm ≒10^{12} m = 10^{9} ㎞ (*10^{-12} m = 1 pm = 1 picometre)
55th; 3602879701896396.8 mm
56th; 7205759403792793.6 mm
57th; 14411518807585587.2 mm ≒10^{13}m = 10^{10} ㎞
58th; 28823037615171174.4 mm
59th; 57646075230342348.8 mm
60th; 115292150460684697.6 mm ≒10^{14}m = 10^{11} ㎞
61st; 230584300921369395.2 mm
62nd; 461168601842738790.4 mm
63rd; 922337203685477580.8 mm
64th; 1844674407370955161.6 mm ≒10^{15}m =10^{12} ㎞ (*10^{-15} = 1 fm = 1 femtometre)
65th; 3689348814741910323.2 mm
66th; 7378697629483820646.4 mm
67th; 14757395258967641292.8 mm≒10^{16}m=10^{13} ㎞【beyond 1 light year; the distance light travels in a year】
68th; 29514790517935282585.6 mm
69th; 59029581035870565171.2 mm
70th; 118059162071741130342.4 mm ≒10^{17}m =10^{14} ㎞
71st; 236118324143482260684.8 mm
72nd; 472236648286964521369.6 mm
73rd; 944473296573929042739.2 mm
74th; 1888946593147858085478.4 mm ≒10^{18}m =10^{15} ㎞【beyond 1000 trillion ㎞】
75th; 3777893186295716170956.8 mm (*10^{-18} = 1 am = 1 attometre)
76th; 7555786372591432341913.6 mm
77th; 5111572745182864683827.2 mm ≒10^{19}m =10^{16} ㎞
78th; 30223145490365729367654.4 mm
79th; 60446290980731458735308.8 mm
80th; 120892581961462917470617.6 mm ≒10^{20}m =10^{17} ㎞
81st; 241785163922925834941235.2 mm
82nd; 483570327845851669882470.4 mm
83rd; 967140655691703339764940.8 mm【beyond Disk diameter of the Milky Way Galaxy; ≒ 10^{18} light year 】
84th; 1934281311383406679529881.6 mm ≒ 10^{21}m =10^{18} ㎞ (*10^{-21} m = 1 zm = 1 zeptometre)
85th; 3868562622766813359059763.2 mm
86th; 7737125245533626718119526.4 mm
87th; 15474250491067253436239052.8 mm ≒10^{22}m =10^{19} ㎞
88th; 30948500982134506872478105.6 mm【Distance to the Andromeda Galaxy; 2.3 million light year】
89th; 61897001964269013744956211.2 mm
90th; 123794003928538027489912422.4 mm ≒10^{23}m =10^{20} ㎞
91st; 247588007857076054979824844.8 mm

92nd; 4951760157141521099596496896.6 mm
93rd; 9903520314283042199192993792 mm
94th; 19807040628566084398385987584 mm ≒10^{24}m =10^{21} ㎞(*10^{-24} m =1 ym = 1 yoctometre)
(*Neutrino (ν): they pass through matter undisturbed almost at the speed of light, not in a quark.)
95th; 39614081257132168796771975168 mm
96th; 79228162514264337593543950336 mm【Great Wall Length (the largest structure in the Universe)】
97th; 158456325028528675187087900672 mm ≒10^{25}m =10^{22} ㎞
98th; 316912650057057350374175801344 mm
99th; 633825300114114700748351602688 mm (≒ 6.34×10^{22} ㎞)
100th; 1267650600228229401496703205376 mm ≒10^{26}m =10^{23} ㎞
101st; 2535301200456458802993406410752 mm
【Distance of the age of space multiplied by the speed of light ≒ 14 billion light year】
102nd; 5070602400912917605986812821504 mm ≒ 5×10^{27}m = 5×10^{24} ㎞【Co-motion distance from the Earth to the edge of the "visible" Universe (the ground plane of the cosmic light; ≒ 46.5 billion light year)】(*Planck Length: 10^{-35} m ; any length shorter makes “no physical sense”)