The Delivery Role and Assessment Role of Computer-based Technology in a Flipped University Mathematics Course

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Abstract: *In recent years, computer-based technology (CBT) has enabled university lecturers to teach their courses using non-traditional pedagogies. One such pedagogy is the flipped learning model. Under this model, students learn the basic content on their own using pre-class tasks and then come to class to engage in more challenging work such as solving difficult problems. CBT can play two important roles in flipped learning, namely to deliver learning materials efficiently and to assess student achievement effectively. This paper describes how these two roles were applied to a flipped Linear Algebra II course in the National Institute of Education (Singapore), taken by a group of student teachers (n = 15) over a 12-week period from January to April 2018. Their perceptions of flipped activities were gathered using weekly surveys, mid-semester survey, end-of-course survey, and end-of-course interviews. They generally agreed that flipped learning using CBT was helpful and enjoyable. As flipped learning becomes more common among university lecturers in Asian countries, it is beneficial to share experiences of utilising CBT to promote active learning of mathematics among university students.*

1. Computer-based Technology (CBT) and Flipped Learning

CBT comprises a collection of tools using the computer as the essential interface, such as word processing, multi-media presentations, social media on the Internet, learning management system (LMS), and discussion forum. The easy availability of powerful CBT in recent years allows teachers in schools and lecturers in universities to use several different pedagogies, ranging from traditional lecturing to open-ended investigations, to teach academic contents and to help students inculcate higher order thinking and self-directed learning.

One significant utilisation of CBT in university mathematics education is the flipped learning model. Most definitions and implementations of “flipped” learning (also known as “flipped” classrooms, “inverted” classrooms, “inverted” instruction) require students to learn basic content on their own (using pre-class tasks) before coming to classes, so that class time can be used effectively to resolve students’ confusions arising from pre-class learning and for them to solve more difficult problems, under the lecturer’s guidance and peer discussion. Thus, the traditional approach of delivering lectures during class sessions has been “flipped” as a pre-class task undertaken by the students themselves. In addition to learning academic content, students can also develop self-directed learning, through completing pre-class tasks, and collaborative skills, through in-class discussion and problem solving. Self-directed learning is a critical skill for adult learners [5], and the students in this study, who were just in their second year of tertiary education, would need much practice to develop this skill.

The principles underpinning flipped learning include succinct aspects of individual and social constructivism, mastery learning, formative assessment, metacognition, and domain-specific teaching-learning theories. For mathematics education, domain-specific theories are especially influenced by Bruner’s multiple representations and Dienes’ theories of perceptual and mathematical variability. Research into flipped learning of university mathematics such as pre-Calculus, Calculus, and Linear Algebra typically reports weak effects on student achievement with the effects depending on the types of courses and flipped activities [6,7,8]. Student perceptions of flipped learning are more positive than negative, especially when the courses are well organised, allowing the students to re-watch video-lectures at their own pace [2,6].

Two important roles served by CBT in flipped learning are to deliver learning materials efficiently and to assess student achievement effectively. These two roles must be supported by the appropriate theories noted above. Indeed, using technology as a teaching-learning tool just because it is readily available or currently popular may not bring about the intended learning outcomes. The rest of the paper describes how these two roles were actualised for a flipped Linear Algebra II (LA) course over a 12-week semester from January to April 2018, conducted at the National Institute of Education, Singapore.

2. The Participating Students

Most of the students taking the LA course in the January 2018 semester had strong mathematical background, and all 19 of them were awarded teaching scholarships by the Singapore Ministry of Education to enrol in the Bachelor of Science (Education) programme that integrates the study of Science and Education over a four-year period. They are trained to be mathematics teachers in Singapore schools. At the time of this study, they were in the second year of this programme.

Prior to taking LA, these students had passed Calculus I, Finite Mathematics, Number Theory, Linear Algebra I, Calculus II, and Computational Mathematics. The second author (Xxx) taught Calculus II to these students in the August 2017 semester, also using the flipped model. Their overall performance in Calculus II was very good, with mean score of 79 and median score of 78 (out of 100). Concurrent to LA, they also enrolled in Complex Analysis, Statistics I, and Differential Equations. Differential Equations was taught under the flipped model by another lecturer. Through these three flipped courses which spread over two semesters, these students would have intensive but slightly different experiences in flipped learning of university mathematics. These courses had provided the students with first-hand experiences of this learning model, thereby preparing them to consider its use in their future teaching of school mathematics to align with the recent vision of the Singapore Ministry of Education to include flipped learning in its schools [3]. Hence, this study is relevant to research in using ICT to prepare pre-service teachers.

Nineteen students were enrolled in this LA course, but only 15 of them (11 females and 4 males) gave written consent for their course data to be analysed for this study. Only these 15 students were asked to respond to the written surveys, to be interviewed about their perceptions, and to be observed during class sessions. All the students, however, had to complete the learning and assessment tasks as part of the course requirements.

3. The Linear Algebra II (LA) course

The LA course is a second-year course for the Bachelor of Science (Education) programme. It comprised a total of 12 weeks of in-class sessions and 11 weeks of pre-class flipped learning. It was conducted from January to April 2018, with a one-week mid-semester break after Week 7. The final written examination was held two weeks after the end of the course.

Three types of learning activities were implemented: pre-class tasks, in-class interactions, and post-class consolidation. Both CBT and printed materials were used to provide rich learning experiences for the students. Pre-class tasks comprised six types of weekly activities: synopsis, videos, summary sheet for every video, pop quizzes embedded within the videos, worksheet problems and activities, and online quizzes. These materials were delivered through Blackboard, the online LMS used by the Institute for its courses.

In-class interactions were conducted in a two-hour "lecture" and a one-hour "tutorial" per week for 12 weeks. At these class sessions, active learning was encouraged with students working on more complex problems and proofs through peer discussions and presentations of their mathematics solutions to the whole class. These in-class sessions were video-taped and transcribed. On the other hand, post-class consolidation required the students to solve problems and post their solutions of assigned "procedural" problems in the problem sets on Blackboard (see below).

4. Data Collection

The main research question to be addressed in this paper is student perceptions of the delivery role and assessment role of CBT for this flipped course. The following instruments were used to collect the research data.

1. Weekly surveys of pre-class tasks (printed). The students rated each of the six pre-class tasks on a 6-point scale (1 to 6), on three dimensions: "Easy to understand", "Help me learn", "Useful" (Weeks 1 to 6), "Enjoyable" (Weeks 7 to 12). They also reported the amount of time they spent watching each video and to complete the other tasks. Each survey took only a few minutes to complete. They submitted the completed survey to the lecturer at the first in-class session of the following week.
2. Mid-semester survey (printed) at the end of Week 7. The students indicated the workload for each activity (1 = not enough; 2 = just right; 3 = too much) and rated their perceptions on two dimensions ("helps me learn" and "enjoyable"). They also responded to three open-ended questions: re-watch the videos; use of additional learning resources; one change they wished to see in the second half of the semester.
3. End-of-course survey (printed) at the last session of Week 12, covering similar questions as for the mid-semester survey.
4. Interviews of 14 of the 15 students, conducted just before the final examination.
5. Student achievement in the course. Their course grade was computed using four components: in-class and online participation (10%), three 10-minute written quizzes conducted in Weeks 4, 7 and 12 (10%), one 1-hour mid-semester test in Week 9 (20%), and one 2½-hour end-of-course final examination (60%).

5. Delivery Role of CBT

The delivery role of CBT has, among other characteristics, the advantages of flexibility, robustness, and access at anytime and anywhere, compared to face-to-face interactions. It allows the lecturer the flexibility to create and design learning materials in different formats, the robust delivery of these materials through CBT platforms, and 24/7/365 access of materials by the students anywhere and anytime they wish. Conversely, the students can use CBT platform to seamlessly deliver their mathematics solutions, comments, and questions to the lecturer and peers.

For this study, the key delivery role of CBT was in the weekly videos. These videos were created using Camtasia to capture mini-lectures narrated by the lecturer in his office. These videos covered standard definitions, theorems, simple examples, and routine procedures of the weekly content. Complex proofs and difficult problems were dealt with during in-class interactions. Camtasia is a screen recording and video editing software. It records the presentation slides shown on the laptop screen, including things written on the screen using stylus pen as the slide is being recorded, and the voice-over of the lecturer. Once the recording is done, we can edit the recorded video. This allows us to add effects and annotations to highlight some statements on the recorded slides so as to focus viewers' attention on them. We can also do simple editing of the audio to reduce the ambient noise and to modulate the voice-over. During the editing process, we can add quizzes to make the video more interactive. However, the quiz questions can only be typed using simple text and no mathematical symbols can be input. There is also no option for multiple-answer questions. This limits the type of mathematical questions in a quiz.

There were three to five videos per week, totalling 46 videos for the whole course. Each video was 3 to 13 minutes long, and the total duration of all these videos was around 330 minutes. Each video was quite short so that the students could watch and re-watch them and take notes under flexible setting, controlled by the students [1]. Every video was associated with a printed summary sheet which comprised a few questions, and by answering these questions, the students could understand the main points of the video. This learning strategy was meant to engage the students actively with the content rather than just watch the videos passively. An example of a question in the summary sheet for Week 3 is given in Figure 1 below.

2. Video clip 3.2:

(a) Give an example of a vector space V and a subset W of V such that W is a subspace of V.

(b) Give an example of a vector space V and a nonempty subset W of V such that W is **not** a subspace of V.

Figure 1 A question in the summary sheet for Week 3

There are 4 videos in Week 3 on the topic of Subspace. Video 3.1 gives a definition of a subspace of a vector space, and states necessary and sufficient conditions for a nonempty subset of a vector space to be a subspace. After the video, students were asked to fill in the definition of Subspace and its necessary and sufficient conditions in Summary Question 1. Video 3.2 explains with two examples how to apply the theorem stated in Video 3.1 to determine whether a subset of a vector space is a subspace. In Summary Question 2, students are then asked to write down an example and a non-example of a subspace of a vector space (see Figure 1). Video 3.3 defines the term "Linear Combination", and illustrates the definition with two examples. In Summary Question 3, students are asked to fill in the blanks in a given definition of linear combination. Video 3.4 gives a definition of the term "Span", and illustrates it with some examples. In Summary Question 4, students are asked to fill in the blanks in a given definition of span and to give an example of a spanning set of a vector space.

The students accessed these videos through Blackboard, but they could not download them onto their own devices. This is because quizzes were embedded in the videos, and these videos were uploaded to Blackboard as content packages instead of in mp4 file format. When students accessed the videos online, the scores of the embedded quizzes they took would be reported to the lecturer via email. Moreover, the lecturer could also check whether individual students watched the videos. The disadvantage of not being able to download videos is that students must have access to the Internet in order to watch the videos.

From the weekly surveys, the amount of time the students reported spending on watching these videos over the 12 weeks ranged from 169 minutes to 492 minutes, with the mean being 349 minutes. We recall that the total duration of all the 46 videos is around 330 minutes. On average, they reported spending about half an hour watching the weekly videos. The weekly surveys show that by and large the students were conscientious in watching these videos over the weeks, spending longer times in later weeks of the course as the videos were slightly longer (see Figure 2). Note that there is no data point for Week 9 because in that week the students were required to take a mid-semester test and thus they did not have to complete any pre-class tasks.

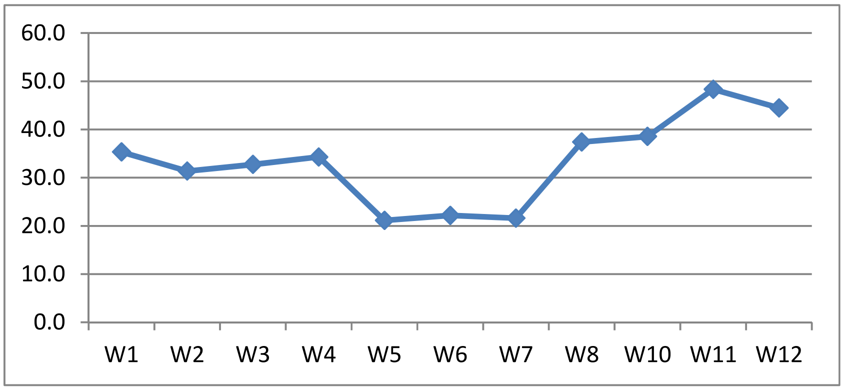


Figure 2 Mean times (in minutes) spent on watching videos over the weeks

The chart in Figure 3 below shows, for each week, the number of students who did not watch the videos. For example, "W1V1" stands for Week 1 Video 1 and "W2V3" stands for Week 2 Video 3, and in Week 2, one student did not watch Video 1.

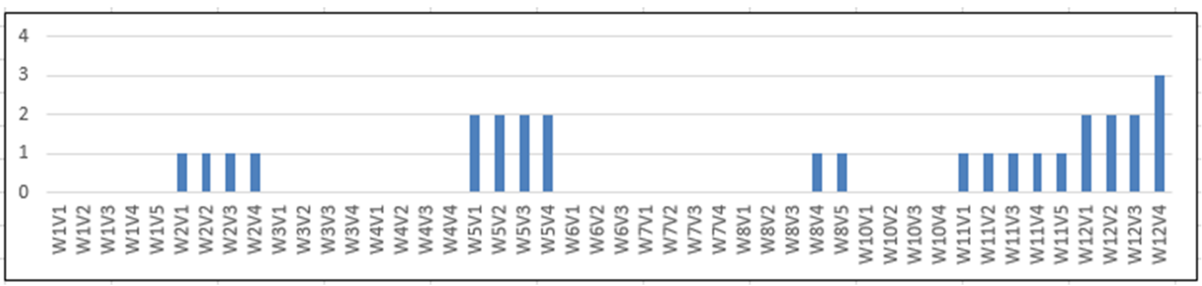


Figure 3 Number of students who did not watch each video from Week 1 to 12

Some students mentioned that they did not watch all the videos due to tight deadlines from all the courses they were taking in the semester; as Student A explained, "I didn't get to watch all the videos. … Like the summary questions and see if I can look at the textbook instead of watching the videos". On the other hand, most of them did not report re-watching the videos. The chart in Figure 4 shows the number of students who re-watched each video. For example in Week 4, there were three students who re-watched Video 1.

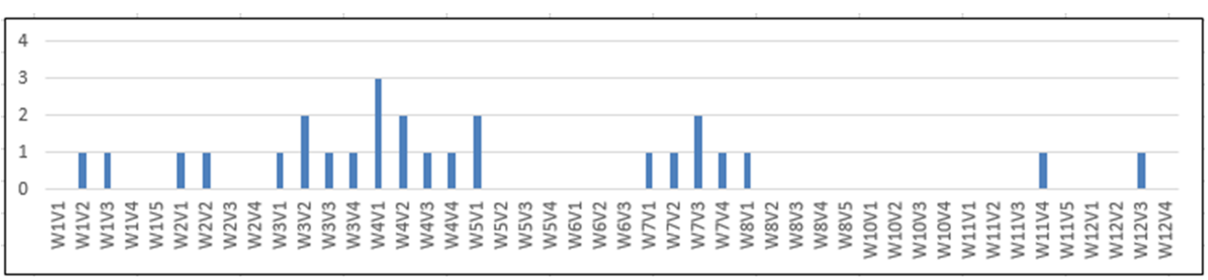


Figure 4 Number of students who re-watched an entire video

As shown in Figure 5, the amount of time spent on watching these videos had non-significant negative correlations with the final course marks on a previous course (Calculus II) (r = -.167) and the grade for that course (r = -.381). The "stronger" students seemed to spend less time on watching the videos than the "weaker" ones.

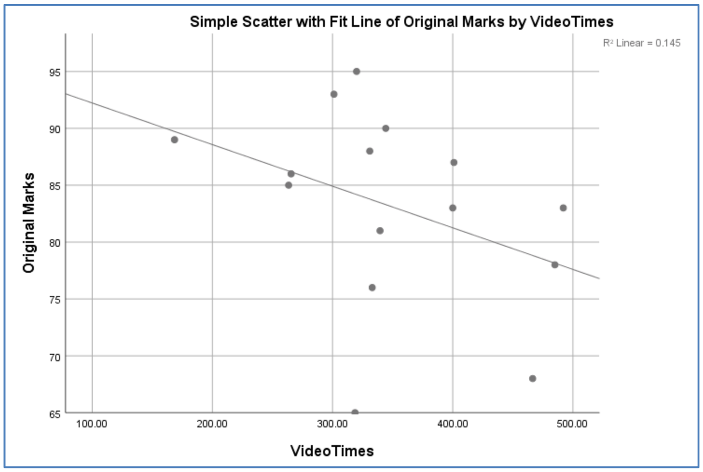


Figure 5 Final marks vs. time spent on watching videos

On the mid-semester survey and the end-of-course survey, the students on the whole rated the videos as helpful (means: 5.5 & 5.8) and enjoyable (means: 5.2 & 5.3). This finding is corroborated by data collected at the end-of-course interviews. For example, Student B commented thus: "I feel that I actually understand a lot more in this course and I actually enjoy the course a lot more compared to other courses … because you just watch the videos ... when you come to class there's a lot more interactions between the lecturer and the students … I seldom flip through the textbook because actually I feel that his videos are … enough to cover everything".

Likewise, Student A commented that "when you do online lectures it gives you the main important concepts that you need to know. And when we go for class, our Prof clarifies the mistakes or misconceptions very easily. So we have a deeper understanding … That's the best thing about this! Because in lectures you can't stop the Prof … For video clips we can pause and write notes. But in class you have to simultaneously write and listen". On the other hand, Student C commented that "it saves time during class because we can learn at home ourselves", and that the downsides were "like we have extra lectures to attend every Sunday and when we are not sure we also can't directly ask questions on the spot when we watch the videos".

6. Assessment Role of CBT

Formative assessment, especially in the form of specific and actionable feedback to students, has been widely acknowledged as having positive effects on student learning in schools and universities (e.g., [9,10]). In this study, CBT was used to provide three types of formative assessment: pop quizzes embedded within every video; weekly online quizzes; posted solutions by the students. These activities were not onerous but they provided many opportunities for students to consolidate their understanding of content through the retrieval or testing effect (e.g., [4]). CBT enables these activities to be delivered and captured seamlessly.

6.1 Embedded pop quizzes

Almost every video had an embedded pop quiz comprising one or two items, in either multiple-choice, fill-in-the-blanks or true/false format, which are placed usually at the end, but sometimes in the middle of, the video. These items assessed basic understanding of the content in the videos. An example of a pop quiz reads: *If W and V are vector spaces such that dim(W) = dim(V), then W = V; true or false?*

The students submitted their answers to these pop quizzes online anonymously, and immediately their answers were automatically checked. This immediate feedback is a key feature of effective formative assessment. Their responses were not graded, and this arrangement was to encourage the students to take responsibility of their own learning.

These pop quizzes were rated as less helpful (means: 4.7, 4.9) and enjoyable (means: 4.7, 5.0), compared to other pre-class tasks. Five students commented on these pop quizzes at the end-of-course interview and noted that they were quite easy and straightforward; Student D: "Pop quiz is very simple so it doesn't really help … Not really a waste of time … don't really need that much brain? … just definitions". Thus, formative assessment needs to be at an appropriately challenging level, certainly beyond definitions, in order that the students will engage actively in answering these quizzes.

6.2 Weekly online quizzes

The weekly online quizzes comprised about eight true/false and multiple-choice items (some items had more than one correct answer) per quiz. They covered the contents in all the videos for that week. The submitted answers were automatically "marked" immediately as right or wrong. Figure 6 below shows what a student sees after he/she has completed the online quiz. It shows that the student has chosen the options A and C, but only C is the correct answer. A is not the correct answer because the vectors 1 and x are not in the subspace W.

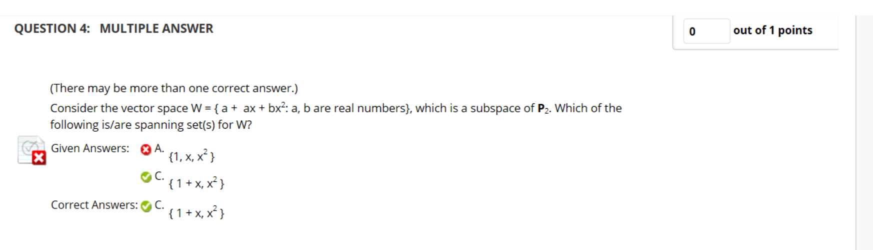


Figure 6 Screenshot of a completed online quiz

The scores on the online quizzes contributed to an "effort" component (less than 10%) as part of the final course assessment. This provided a minor motivator for students to take such learning activities seriously. Figure 7 shows, for each week, the total online quiz mean scores (= sum of the mean scores of the 15 students). The mean score of each student for each week is obtained by dividing his/her total score for the quiz by the total number of questions in the quiz. For each question, the student is awarded 1 mark for a correct answer. The maximum total mean score for each week is thus 15. By and large the students did well in the online quizzes as the total mean scores varied within a reasonable range of 7.8 and 12.2 over the duration of the course. All the students had submitted virtually all the online quizzes.

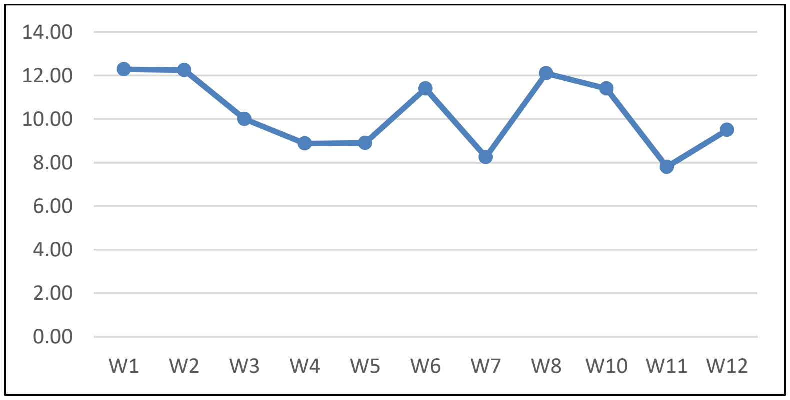


Figure 7 Total online quiz mean score for each week

Unlike the pop quizzes, these online quizzes were rated highly in terms of helpfulness (means: 5.5, 5.5) and enjoyable (means: 4.8, 4.9). These quizzes were found to be more "tricky" than the pop quizzes and took time to complete, but they helped the students to notice what content they had missed, to prompt thinking, and to build confidence. Student E explained the learning progression: "It's like you can do the basics, now you can move on to the next level. So it feels more of systematic progression so like building up our confidence on the topic rather than just throwing us off the deep end and just letting us to solve stuff".

In the subsequent in-class session, the lecturer would explain corrections to the mistakes students made in the online quizzes, so that the students' answers can be used in a formative way to improve learning. He would give mini-lectures to extend the content covered in the videos, such as alternative solutions and complex proofs. After the tutorials, the students could re-do the weekly quizzes again, but their subsequent attempts would not be graded. This was another opportunity for the students to benefit from the testing effect, if they so desired.

6.3 Posted solutions

There were nine problem sets for the course, and each problem set consisted of three to five "procedural" problems and four to seven "challenging" problems. Each problem set covered the contents for one to two weeks. These problem sets were delivered through Blackboard about two weeks before the respective tutorial sessions. An example of these two types of problems are shown below; Figure 8 shows an example of a procedural problem while Figure 9 shows an example of a challenging problem in the problem set for Week 3.

2. Determine whether each of the following vectors is a linear combinations of $(1, 0, 1)$, $(0, 3, -2)$, and $(1, 3, -1)$.

   (a) $(0, -3, 4)$ (b) $(3, 3, 1)$

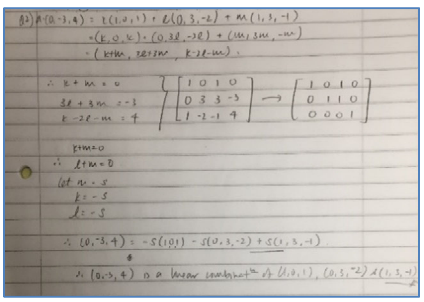
Figure 8 A procedural problem in Problem Set 3

10. Let W_1 and W_2 be subspaces of a vector space V. Prove that $W_1 \cup W_2$ is a subspace of V if and only if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$. [*Hint*: $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$ implies that $W_1 \cup W_2$ is a subspace of V is straightforward. We prove the other implication by contradiction. Assume that $W_1 \cup W_2$ is a subspace of V and $W_1 \not\subseteq W_2$ and $W_2 \not\subseteq W_1$. Take $\mathbf{u} \in W_1 - W_2$ and $\mathbf{v} \in W_2 - W_1$.]

Figure 9 A challenging problem in Problem Set 3

The students were divided into two groups. Students within each group were assigned to post their solutions to specific "procedural" problems on Blackboard. Most students were expected to be able to solve these "procedural" problems, so that tutorial sessions could focus on the "challenging" problems.

The group members could read and comment on the posted solutions of their own group only. The lecturer would also comment online on the posted solutions and might suggest alternative methods. The goal of this approach was to stimulate active group discussion and to obtain immediate feedback given by the lecturer and peers. CBT allows this to be conducted more flexibly than in face-to-face meetings. Figure 10 shows a student's solution to Question 2(a) and the lecturer's comments which were posted in Blackboard.



Your answer for part (a) is not correct. Note that after you reduced the augmented matrix to reduced row-echelon form and wrote down the corresponding linear system, you missed one equation (highlighted below). The linear system corresponding to the reduced row-echelon form is

k + m = 0, l + m = 0, 0k + 0l +0m = 1.

The last equation has no solution in k, l, m. Therefore the linear system has no solution. This implies that (0, -3, 4) is not a linear combination of the three given vectors.

For part (b), your solution is correct. However, instead of giving all possible solutions for the scalars, you can just give one solution, for example, k = 3, l = 1, m = 0 (i.e., put s = 0).

Figure 10 A solution posted by a student and the lecturer's comments

During the course, each student had posted five to six solutions to the "procedural" problems online. The students found this activity, as well as solving worksheet problems in the pre-class tasks, to be most helpful (means: 5.7, 5.8) and quite enjoyable (means: 5.1, 5.2) among all the flipped learning tasks. Many positive comments about this activity were made during the end-of-course interviews: learned from helpful comments given by peers and the lecturer; learned from the mistakes made by others; boosted confidence and a sense of achievement; these problems and the solutions could be used to prepare for tests; Student F noted that "my classmates are very detailed in what they are writing" while Student G explained how she had changed her feelings through this activity: "Especially embarrass when I like erm presented the wrong solutions … as of now I think it feels okay, … in future becoming a teacher we have to prepare solutions also. So I think he's giving us a training we need".

A few students commented on the time taken to post the solutions, and some read the solutions only when they were stuck. Most students uploaded scanned copies of their handwritten solutions, while some typed their solutions online, using an equation editor. A more streamlined procedure may help to cut down on the time taken to deal with the CBT interfaces with Blackboard.

7. Conclusions

This paper has reported on how CBT was used to deliver learning materials efficiently and to assess student achievement effectively in a flipped Linear Algebra II course taken by a group of 15 student teachers. Their perceptions of flipped activities were gathered using weekly surveys, mid-semester survey, end-of-course survey, and end-of-course interviews.

Although students often express favourable perceptions of a new learning model because of novel effect, these perceptions may change under extended exposure. It is encouraging that in this study, the students consistently rated very highly their flipped experiences in terms of enjoyment and helpfulness throughout the semester-long course.

As flipped learning becomes more common among university lecturers in Asian countries, it is beneficial to share experiences of utilising CBT to promote active learning of mathematics among university students. Further research can study the long-term perceptions and impacts of flipped learning, including the inculcation of fruitful learning strategies, such as be prepared for upcoming classes, writing own notes, engaging in active in-class discussions, and using resources to prepare for the final examination.

Acknowledgement

This study was funded by Singapore Ministry of Education (MOE) under the Education Research Funding Programme (OER 12/17 NWL) and administered by National Institute of Education (NIE), Nanyang Technological University, Singapore. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the Singapore MOE and NIE.

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