An Expert Approach of the Different Ways to Use the New Cabri and the Richness of its Connections with the Online Freeware Cabri Express

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Abstract: *The acceleration of the provision of more and more sophisticated technological tools and more sharp on-line applications, instead of encouraging the teachers of mathematics to integrate more technology into their teaching might lead them rather to take refuge in their fundamental approaches. What I mean is that the profusion of tools like tablets and smartphones, the accessibility to a multitude of online courses (for the teachers as well as for their students), the poverty of their in-service training and a final examination which does not practically take into account the use of the technologies recommended by the curriculum together might bring the teachers to return to a more classic teaching leading to the assessment of technical skills. So we must have a pertinent approach of the role of the use of technology during the math lessons compatible with the actual needs of the teachers. We (experts) must rethink the role of the calculators, the role of software (DGS or CAS) and especially simplify them in order to provide to teachers and students a microworld enhancing a normal but rich pratice of mathematics. The new Cabri is an environment especially dedicated to authors who want to create multipage activities for students. These activities can be modified very easily by teachers (there are three available modes : the author, the teacher and the student modes). Recently the developers of this Cabri, put online a software free of charge, Cabri Express, containing a part of the tools of the New Cabri : this freeware is a calculator connected to a 2D environment that can be extended to a 3D one. This revolutionary environment can be a response to those who want simple, accessible and powerful tools for primary and middle schools. What is exciting is that the files created online can be downloaded on your computer and also files avalaible on your computer can be opened online. More than that, activities created with the new Cabri can be opened with Cabri Express. We aim in this paper to show with examples all the didactical possibilities of both Cabris. We will also present how the restricted 3D environment provided by these Cabris can be extended to a richer one closer to the Cabri 3D one. We will see that the proposals of the expert allow teachers to show the known techniques of DGS to enhance a link with the real world, creativity, discovery and pleasure of proof when it is possible.*

1. From Cabri 2 Plus to Cabri Express via Cabri 3D and the New Cabri

1.1. The new Cabri and Cabri Express

During one of his talks Cedric Villani (Fields medal 2010) said « *The mathematician needs a lot of imagination, perhaps more than in other sciences because he must overtake the constraint of rigor which is very important and also because he cannot use experiments for proving but only for helping him to discover new ideas* ». He always insists on the role of mistakes and on the role of the long time of a normal research. How can technology and how especially can dynamic mathematics supported by technology can help teachers and students to practice such mathematics where experiments can nevertheless be the nestle of conjectures and by the way creativity, where experiments can be a help for building deductive proofs and so respect the constraint of rigor? We must question what we have learnt during the last 20 years and how it can be used with appropriate tools. We have chosen to present all our examples with the New Cabri which is a software containing DGS in 2D and 3D, algebra and numbers. This software is an "activity creator" more dedicated to experts who want to create interactive multipage math activites at the primary school and middle school level. Such software can follow the desires of teachers but nevertheless use the good ideas developed during the past. The richness of such an environment is that the author can create activities that can be modified very easily by the teacher when this one uses his special mode. The student mode is a mode where the student can use only the tools allowed by the author or the teacher. The French ministry of education has already bought a huge set of activities which are available online and used for free by every primary or middle school teacher and his (her) students. In order to help teachers and students to use such software to investigate on their own, Cabri Express has been created: it is a freeware that can be used online linking calculator, 2D and 3D dynamic geometry environments and where the work done online can be saved and reopened when necessary. More than that, Cabri Express can be used to open online activities created with the new Cabri.

1.2. What did we learnt and therefore what do we know about DGS

Let us summarize what we will do in this paper. We will remind ourselves that DGS is best used to foster experimentation, heuristicity, creativity, modelling and eventually another approach to learning and doing math.
We will give examples showing the simplicity and the power of the chosen environments focused on the previous points

1.3. About the special role of experts when using these environments

We will present how the New Cabri 3D environment can be extended to a richer and wider one with a pertinent use of math knowledge and the power of macro constructions embedding 3D objects. We will show how to create lines and planes in all possible positions and how to recreate them with the macro constructions that record the previous creations. It is an illustration of the power of math constructions: starting with an environment containing very few objects and tools we can create a wider and richer environment containing a lot more objects and a lot more tools. This new environment provided to teachers and student can help them to see and manipulate the 3D objects not provided in the original software. We will show that starting from the original 3D environment of the new Cabri, containing only the possibility of creating the platonic solids, prisms, pyramids and cones on the horizontal plane, we will create tools allowing the possibility of creating dynamic lines, planes and half spheres every where in 3D space. We will also give some examples of the use of such richer environments.

2. Illustration of the power of DGS within the New Cabri environment

2.1. DGS and experimentation (Pythagoras's theorem)

We already showed in our invited paper in 2002 how to use the conditional constructions and the soft loci with Cabri 2 Plus to discover experimentally the Pythagoras' theorem ([6]). Let us show now another approach more focused on the experimental work of the learner even if this work is guided in the New Cabri environment. Recall that, in the present context, experimenting is related to a process of production of data leading to the statement of a conjecture or the corroboration or the refutation of a previous conjecture (see Popper and Lakatos [2]).
The mini activity we have created contains three pages where we give the learner the opportunity, to experiment, which means to generate data, to interpret these data in term of conjecture, to experiment again to corroborate this conjecture at a high level of plausibility (in the meaning of Polya [1]). In reality the design of these three pages is a design chosen for a presentation to pairs. A design for students would be more interactive and would need more coding to the author of the

activity. Let us describe the three pages of such an activity that could give a taste of the sort of work that can be led by the student when using such an environment.

First page displayed in Figure 1 on the left

The figure provided is a given segment [AB] and a set of eight pairs of points laying on eight lines perpendicular to the given segment (and moveable on these lines). These points do not appear when opening this page. A movable red circled point M is also given. Displayed on the right part of the screen. The distances MA, MB and AB and the result of $MA^2 + MB^2 – AB^2$.are displayed on the right part of the screen. Also displayed on the right part of the screen, the task for the students with a button allowing them to show the 16 points when necessary during the execution of the task. Here is the task

1. Move point M and tell what you state when dragging point M. 2. Click on the right button and a set of eight pairs of points will appear on the blue verticals. 3. Move the blue points until you reach a position where the previous expression gets the value 0. 4. What conjecture can you state about their position with respect to [AB]

Task 1 : The student must drag point M and record his (her) observations. His (her) work will be an exploration work which aims to collect informations about the positions of point M, without knowing if this exploration will lead to interesting inferences or suggest some unexpected invariant. In doing that we can see that point M could be surrounded by a little circle or not, depending on its position. More accurate, would be to point the fact that point M is circled when the displayed value of $MA^2 + MB^2 – AB^2$ is positive and not circled when this value is negative (as we can see in Figure 1 in the middle).

Tasks 2 and 3: Clicking on the blue button provides 16 points to help the student to conduct an investigation (experimentation conducted with a known technique). The student must drag these points until positions where the displayed number is 0. You can notice what is done in figure 1 on the right: one right position can be found with point M when its aspect changes from circled to non circled or directly in moving a blue point on its line until the sign of the displayed number changes: you can see in Figure 3 on the right that when the first blue point lies on a right place a yellow cup appears to reward the successful trial of the experimenter (this feedback is programmed in the software very easily in a page behind our working space).

Task 4: The previous work (using the technique of trial and error) must lead the experimenter to a figure looking like that displayed in Figure 2 on the left. At this stage the possible interpretation of the data produced by this technique is: the positions of M where the displayed number is 0 seems to be on the circle having [AB] as a diameter. It is a perceptive conjecture on the screen of the computer; this work is a geometric work led under the praxeology or paradigm G1 Informatique (according to my classification [7])

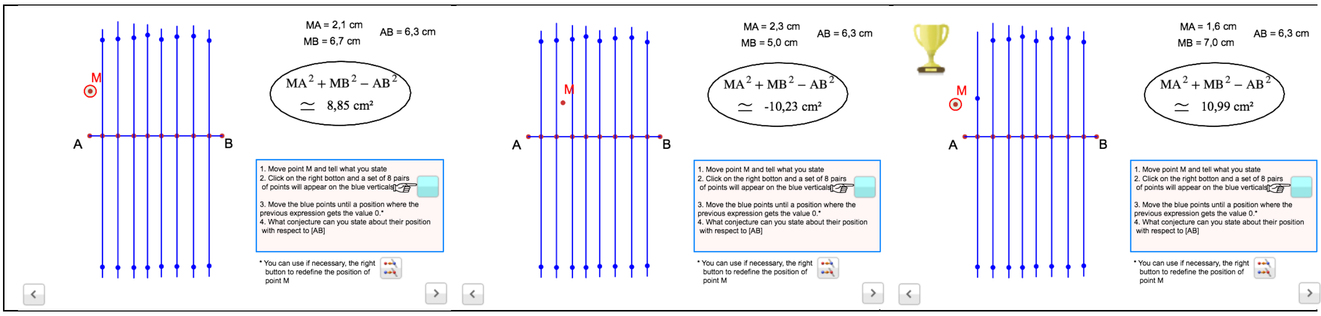

Figure 1: Pythagoras's activity (first step)

Second page displayed in Figure 2 on the left

This second page must be the model of what could be the stage of validating experiments after having stated the previous conjecture. The tools "midpoint" and "circle" as well as the tool "redefinition" are here to suggest to the experimenter the good initiatives for this stage of validation (which means corroboration or refutation). The student must think of the technique of superimposition in trying to check if the circle having [AB] as a diameter is superimposed to the points he has located in respecting the constraint $MA^2 + MB^2 - AB^2 = 0$. Checking that is the verification of a consequence of the previous conjecture (if it is true that the set of points verifying the constraint is this circle, this one must be superimposed to the points). So the students construct the midpoint of [AB] and the circle centred on this midpoint and passing through point A. They must observe the expected superimposition on the screen, which is another G1 Informatique level of validation because checking a superimposition on the screen of a computer is at a higher level than in a paper an pencil environment which correspond to a G1 praxeology (so called by Houdement_Kuzniak and Parzsiz [5] [8]). This experiment is shown in Figure 2 in the middle. A higher level of validation is suggested by the use of the tool redefinition which can be done in page 2 but we will show it in page 3.

Third page displayed in Figure 3 on the right

In this page we have kept only the circle having [AB] as a diameter. We can check in Figure 2 on the middle (page 2) that if we drag point M until a position on this circle the displayed number will approach 0 as expected. On page 3, we will redefine the position of M to move from a free location to a position on the circle and miracle! the displayed number becomes 0 and does not change when we drag point M along the circle (Figure 2 on the right). It is another verification of our conjecture when point M lies on the circle: the truth of the conjecture implies that this number must be equal to 0. The level of this validation is G2 Informatique ([7]), which is the level of deduction got by the software. The level of the usual deduction (we call it the formal proof) is G2 (according to Houdement-Kuzniak and Parzsiz). G2 Informatique is higher than G1 because the verifications are not perceptive, they are based on the respect of the computational results predicted by math properties. We can say that G1 Informatique is the highest level of validation before a formal proof.

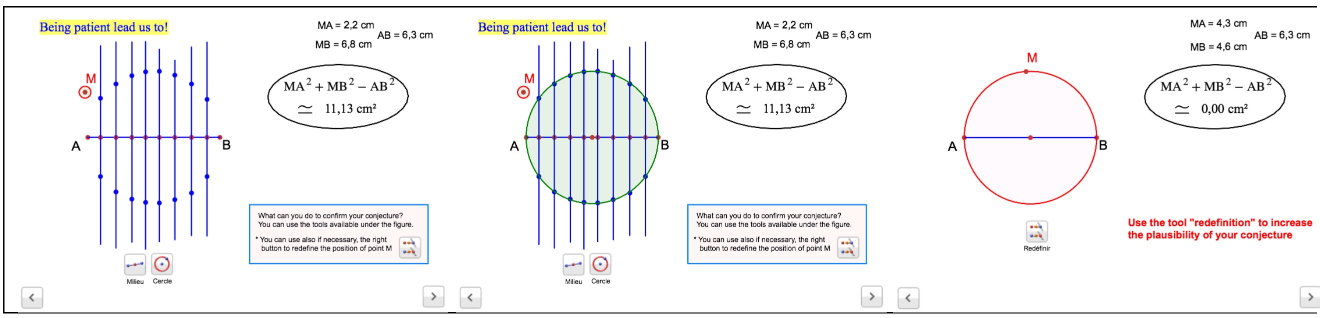

Figure 2: Pythagoras' activity (second step)

Conclusion: with such a task which is guided but nevertheless open, thanks to the design of the software, we are sure to motivate the student to be active, to enjoy observing data, to enjoy discovering some connections between data, to conjecture, to understand the necessity of validations (because they will have at least two different levels G1 and G2 Informatiques), to understand the role of deduction in what we call a verification. Experimenting is not only being in action but a process of generation of data that can be interpreted especially in the process of investigation (where the role of the techniques is crucial).

2.2. DGS and heuristicity (discovering Brocard's Theorem)

We already know that a dynamic approach to a geometric problem is more heuristic than a static one ([3]), which is the case in a paper and pencil environment. We will illustrate that in showing the investigations I have conducted when this problem was proposed to me (at this time I had never heard of it before). I will show how to use the New Cabri (like an interactive slider) to facilitate the description of this work. The Brocard problem is the following one : Given a triangle ABC, we want to know if it is possible to construct inside it a point verifying the equality of angles shown in Figure 3 on the left. In reality there are two problems in one, depending of the orientation of the angles.

Part 1 of this research work (dynamic experimentation leading to a conjecture) : we will use a technique of investigation already used during the « Journées APMEP Nice » about a bisectrix problem where reasoning by necessary conditions is crucial ([4]). After the construction shown in Figure 3 in the middle, we can investigate all the possible values of angle α and state that this problem has a unique solution. We have created three rays Ax, By and Cz controlled by the slider located below on the page. These rays can rotate around their origin as the value of angle α increases (in using the slider) from 0° to the minimum value of the three angles of triangle ABC. They define a red triangle uvw which become smaller and smaller as α increases until a value where these points seems to be superimposed (Figure 3 on the right) and then becomes bigger and bigger. This intermediate position obtained experimentally can be interpreted as a conjecture at level G1 Informatique : a point given by the constraint of the equality of three angles exists and is unique. Another conjecture can be found similarly in changing the orientation of the three angles in the triangle.

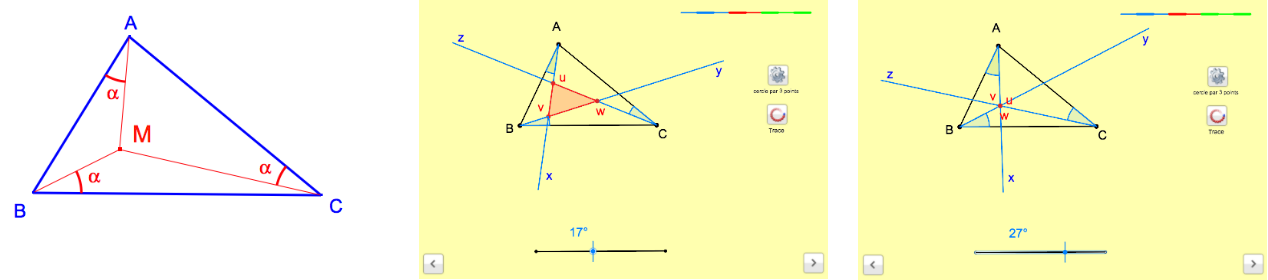

Figure 3: Brocard's points (investigation 1)

Part 2 of this research work (experimentation leading to the possible position of the solution point) : you can notice that two tools are available for the students to experiment. Let us use the tool « Trace » to activate the trajectories of points u, v and w. Then, let us move point P of our slider to increase the value of α until we obtain the perceptive superimposition of the three points u, v and w. We can see on Figure 4 on the left that the trajectories of these three points seem to be arcs passing respectively through points A, B and C and having a common point which would be the solution of our problem. As the length of the trajectories is not as long as we would want, we can chose another screenshot during this experiment to obtain another part of them in Figure 4 in the middle. If we have a good sens of observation we can suppose that these arcs seem also to pass through a second vertex of ABC where they are tangent to the next side. If we have not been able to conjecture this last result, we could provide to the students the tool « Locus » (not provided here) and obtain what we can see on Figure 4 on the right. The quality of the displayed loci can lead us to confirm this second conjecture.

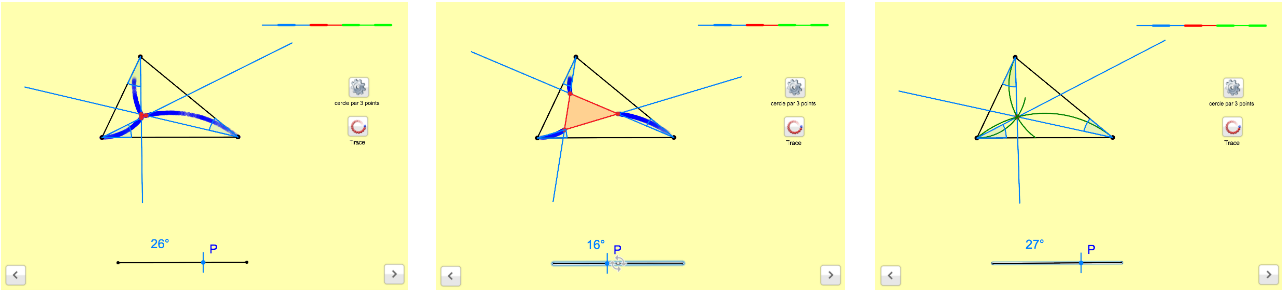

Figure 4: Brocard's points (investigation 2)

Part 3 of this research work (experimentation leading to the corroboration of the previous conjecture) : now we start a stage where we will use a reasoning by necessary conditions. If our conjecture is correct (trajectories are arcs verifying the previous constraints) and if we construct for each locus a circle passing through three of its points we might obtain a circle superimposed to the locus. It is exactly what we obtain in Figure 5 on the left (the tool « Circle by three points » can be provided for the students). This experiment is located at level G1 Informatique (perceptive validation).

Figure 5: Brocard's points (investigation 3)

Part 4 of this research work (experimentation leading to the corroboration of the previous conjecture at level G2 Informatique) : at this moment of our work, our conjecture state that there is a unique point respecting our intial constraint located at the common point of three circles, passing through A and B and tangent to (BC) in B, passing through B and C and tangent to (CA) in C and passing through C and A and tangent to (AB) in A. Therefore, if this conjecture is true we can deduce that such circles have a common point and this point satisfies the constraints of equality of angles. Let us construct these three circles (Figure 5 in the right). We construct these three circles and the sofware knows that according to the properties of this figure there is only one common point and if we measure the three angles with the accuracy allowed by the software (7 digits), we can state a perfect equality. We always get this equality when we change the shape of the triangle. So we have obtained a corroboration at the G2 Informatique level which increases at the maximum the level the plausibility our conjecture.
The way we have conducted all these investigations is very rich because the proof will be only the stage of verification in our reasoning.

Part 5 of this research work : this final part must be the formal proof . We will not give any of the possible proofs. One of them is a consequence of the trigonometric version of the Ceva's theorem.

2.3. DGS and creativity

We know after all the experiments conducted during several years with a math teacher (during Cabri 2 Plus and Cabri 3D workshops mostly with middle school students) that teaching students the techniques of animation involving the transformation tools of the DGS used by them enhances a new power of creativity ([3'] and ([4'] for example). Knowing that led me to use the New Cabri and its special tool « Net ». The idea is to present some techniques of creation of 3D objects with this tool. Here are some ideas that can be used in future experimentations with Cabri Express which need to be presented to students in videos avalable on the net.

2.3.1. Techniques of modelling letters

As we modelled letters with cubes within Cabri 3D ([2']), we can model letters with the tool « Net ». In Figure 6 on the left, we have displayed the technique of modelling letter S. Starting with a blue square, we have created eight other squares equal to the first one in using reflections with respect with the sides of them. A click on the first square with the tool « Net » displays the foldable red object composed with 9 squares. The square 2 of this foldable object always stays on the ground floor but can be dragged everywhere on this plane. If folded correctly, this net can give letter S modelled in yellow on the back of the screen. Similar work can be done to model letter A (see Figure 6 in the middle). Such a technique can be used to model letter Delta (Figure 6 on the right) : here the tool « Net » gives a real net because it is the net of a prism.

We are confident that, teaching such examples will allow students to enhance their motivation and their creativity.

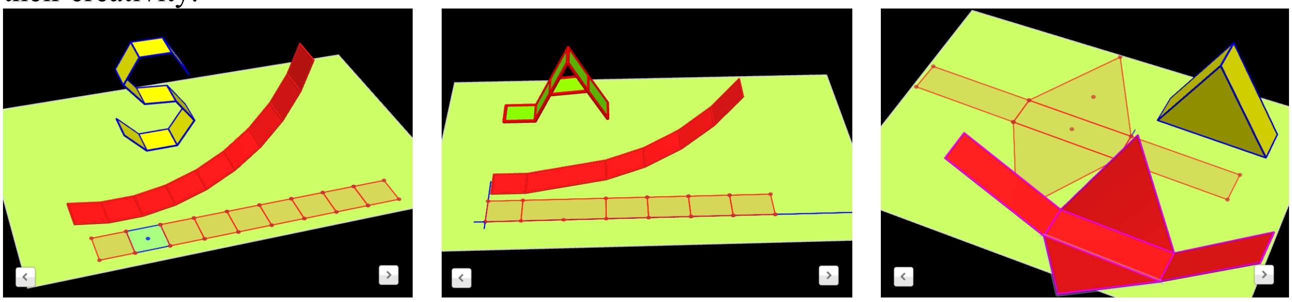

Figure 6 : Modelling letters

2.3.2. Techniques of modelling objects of the real world

Artistic modelling : It is possible to model an artistic vase like the one shown in Figure 7 on the left. The starting point is a cross created with a square (that can rotate around its centre) and four rectangles. The tool « Net » applied to this cross in clicking on the initial square gives a folding object like the red one. With the same cross we can create several nets : two nets when the cross is in the position shown and another one obtained after rotating the cross of 45°. The last manipulation after different folding of the three « nets » is the superimposition of these three nets.

Realistic modelling : It is also possible to model a handball field like the one shown in Figure 7 in the middle. It could be a motivating screen to show to students during the stage of initiation of the techniques such as those presented in 2.3.1. In this example we have two nets which are obtained from two sets of rectangles symmetric with respect to the middle line of the field. We can see these rectangles created on a grid of the horizontal plane in Figure 7 on the right.

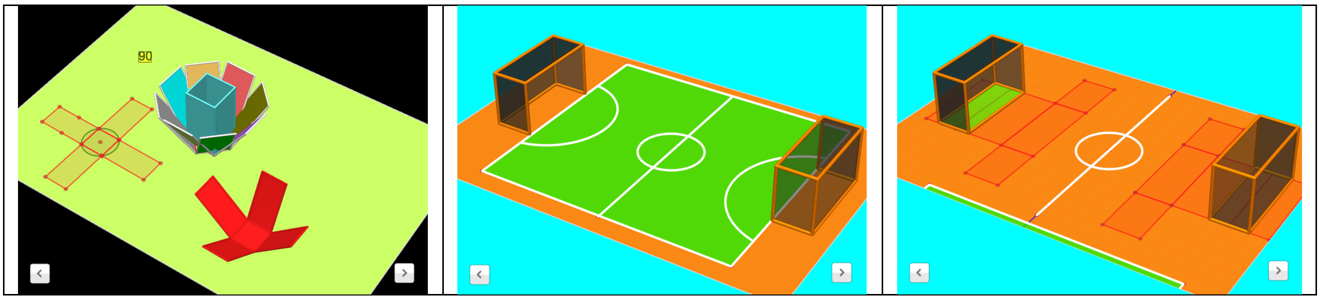

Figure 7 : Modelling objects of the real world

2.4. DGS and modelling

We will see now that it is possible to obtain a 3D representation of the earth with the new Cabri and also a dynamic visualization of its meridians and parallels (Figure 8 on the left) even if the new Cabri is not featured like Cabri 3D (in which this work was performed : [1']). We need to represent a 3D object representing the earth (with a radius that can be modified), a horizontal plane whose height can be modified with a slider and whose intersection with the earth models a parallel, a vertical plane that can rotate along the axis and whose height can also be modified with a second slider of the earth and finally the vertical axis (which we know is not perperdicular to the ecliptic plane). We will use some techniques that will be shown in the last paragraph where we will see how to increase the possibilities of the new Cabri in the domain of the 3D representation.

We start our work in creating two sliders : one for the height of the horizontal plane and another one for the common height of the axis of the earth and the height of the vertical plane. We continue in displaying two numbers that can be changed at any time : one for the radius of the axis and a second one for half of the thickness of the vertical plane, for example 0.5 (r) and 1 (t) as shown in Figure 8 in the middle.

– **Initial constructions** (Figure 8 in the middle) : a point O centre of a little circle which radius is r (here 0.5), midpoint of segment [AB] and centre of a big circle where point C lies and controls the rotation of segment [CD] around O. We have also created a square with side is equal to CD parallel to (CD) and tangent to the big circle.

– **Representing the axis of the earth** : it is a cylinder whose heigth is controlled by the second slider and admitting as a base the little circle.

– **Representing the vertical and rotating plane** : we create a quadrilateral which width is 2.t and the vertical plane is modelled with a prism based on this quadrilateral and which heigth is also commanded by the second slider. The rotation of this plane is commanded with point C.

– **Representing the horizontal plane** : we create a prism based on the square created before and whose height is controlled with the first slider. We then hide all its faces apart from the upper one modelling the horizontal plane and whose level is controlled with the first slider

– **Representing the spheric earth** : We use a model of the media : a 3D model of the earth provided by the software ; we drag it until segment [AB] to attach the earth on it (the diameter of the earth will be AB).

To obtain figure 8 on the left, we change the value of r from 0.5 to 0.1 and the value of t from 1 to 0.0001

– **Representing a parallel** : we see it as the intersection between the sphere-earth and the horizontal plane.

– **Representing a meridian** : we see it as the intersection between the sphere-earth and the vertical plane.

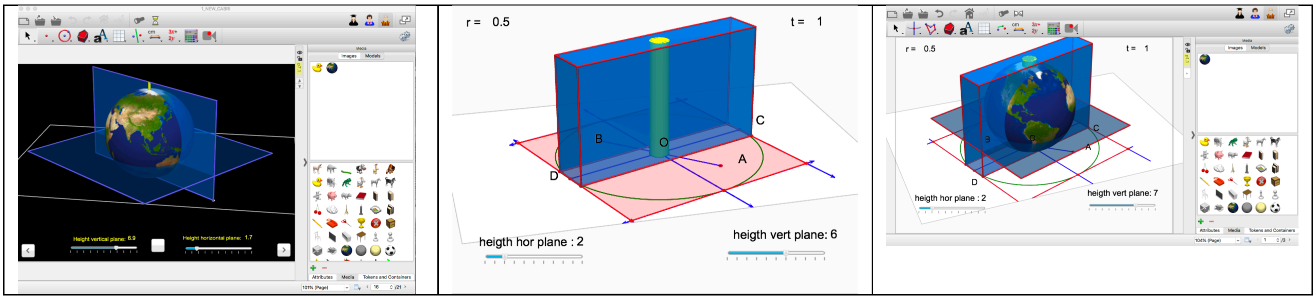

Figure 8: Representing the earth, its meridians and parallels

2.5. DGS and another approach to mathematics (link with Cabri Express)

A possible research work for the future will be to see how files created under the new Cabri could be used online with Cabri Express This paper is a first attempt in this direction.

3. The role of the expert to enrich a learning environment

3.1. The 3D features of the New Cabri and more

The two screenshots of figure 9 summarize what we can create in 3D with the New Cabri. As shown in Figure 9 (on the left) and detailed in Figure 9 (on the right), we can create, lying only on the 2D plane, cylinders, cones, prisms, pyramids and spheres; we need only to click on the appropriate tool and also on a number if necessary. We can also extract from the provided list of 3D models of the media, real 3D objects like an aircraft, a ball or a cup that can be attached or not to a segment. In this case the length of the segment controls the size of the object. Not represented in Figure 9, but nonetheless available are the Platonic solids that can be created with the same technique with one or two clicks.

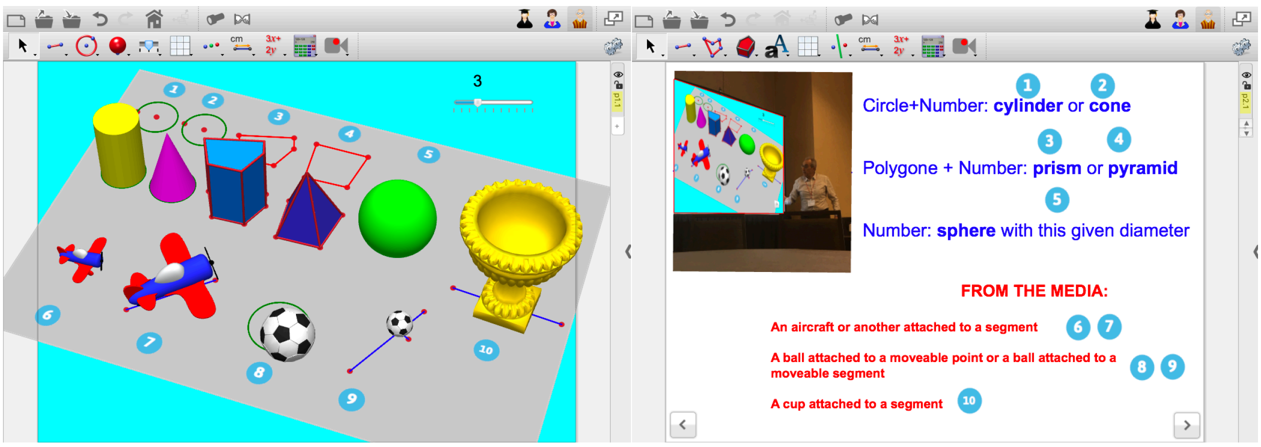

Figure 9: The 3D environment of Cabri 3D

Remarks:

- It is possible to attach pictures to segments or polygons to obtain the page represented in Figure 9 on the right.
- It is not possible with available tools to create lines or planes everywhere in the 3D world provided by this new Cabri, even spheres centred on the 2D plane.

It is the goal of the following part to show how to create these tools as macros. It is the role of the expert to go further because in doing that, he can be a model for the learners: he can convince them that the use of simple tools with a lot of imagination leads generally to a surprising creativity.

3.2. How to create lines in any 3D direction?

3.2.1. Lines parallel to the 2D plane (Figure 10)

Similar to the technique presented in 2.4., we start with a segment [AB] giving the direction of the line we want to create, a number called half thickness (t) representing the width of the line and another number controlled by a slider representing the height (h) of the line. Then we create a quadrilateral whose base is a rectangle with two sides parallel to (AB) and the others perpendicular to (AB); its length is equal to AB and its width is equal to 2t (see Figure 10 on the left). Finally, we create the prism with the given height and based on the rectangle. The line is modelled by the top of the prism (the other faces are hidden). We can see the result in Figure 10 (in the middle) with t = 0.5, the line is modelled by a rectangle at height h.

At this stage we create a macro with a segment and two numbers (t and h, here 0.5 and 3.1 respectively) as initial objects and the upper face of the prism as the final object. This macro can be used by clicking on the button located in the screen. You can see that we obtain a nice representation on the left when t = 0.02 and h = 2.2 that can be modified with a slider. Such lines can be extended by extending segment [AB], its direction can be changed by rotating this segment around its midpoint and its height can be modified with the slider. Figure 10 (on the right) shows the use of this macro three times to represent three lines parallel to the horizontal plane the heights of which are controlled by three different sliders and the width of which is given by 0.01. It is interesting to observe that the range of the slider can be defined with negative numbers allowing the representations of lines located below the horizontal plane!

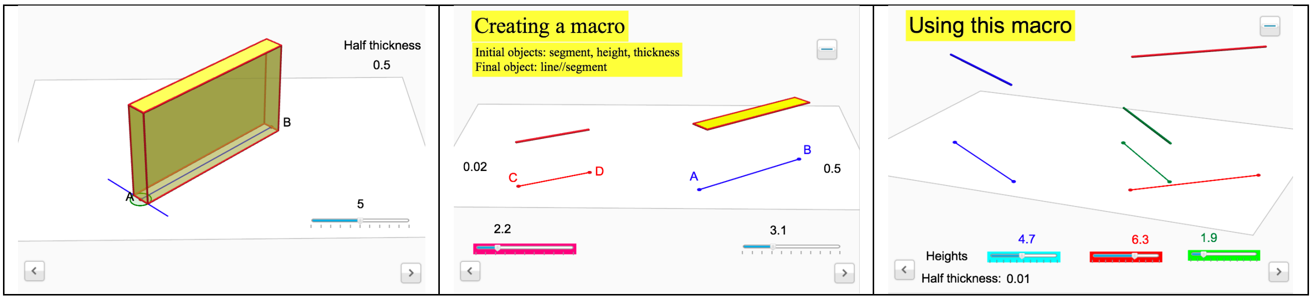

Figure 10: Lines parallel to the 2D plane

3.2.2. Lines perpendicular to the 2D plane

Also similar to the technique presented in 2.4., we start with a point on the horizontal plane. We construct a circle centred at this point with radius a given number. We use a second number generated by a slider (but not necessarily) for the height of the cylinder based on the previous circle. This cylinder models a vertical line passing through the initial point with a given height when the radius is a small number (Figure 11 on the left). In Figure 11 (in the middle) we have created the macro having a point, the height and the radius as initial objects and the cylinder constructed as a final object which models a vertical line passing through the given point with the given height. You can see that this macro can be used with its button. In this figure the cylinder obtained with this macro is an acceptable modelling of a perpendicular line because the given radius is 0.05. In Figure 11 (on the right), we have used this macro several times with the different heights commanded by different sliders. One of these perpendicular lines is located below the horizontal plane because of the range of its slider.

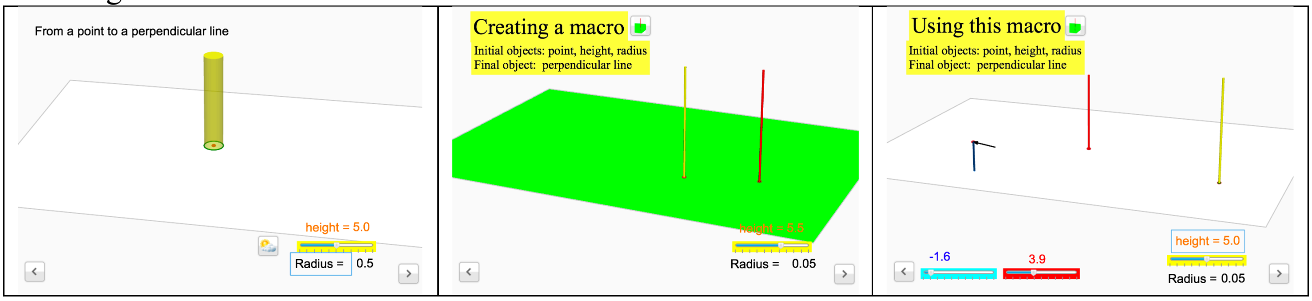

Figure 11: Lines perpendicular to the 2D plane

3.2.3. Sloped lines

If [AB] is a given segment, d a given number and a another given number (between 0° and 180°), we want to construct a segment in the plane containing [AB], perpendicular to the horizontal plane, passing through A which length is d and inclined at a°. The trick here is to use one face of a pyramid to model such a line

The technique is shown in Figure 12 (leftside): starting with [AB], we construct ray [AB) and on it, point C such as AC = d.cos(a). Then the rectangle which will be the base of our pyramid centred in C (length = 2.AC and width = 2.e = 2.AE) and finally the pyramid based on this rectangle the height of which is equal to d.sin(a).

We can visualize this pyramid in Figure 12 (in the middle) where e is a given displayed number and where d and a a are given with sliders (d belongs to a range of positive numbers and a can be modified from 0° to 180° but it is possible to chose a > 180° to obtain lines below the horizontal plane). The sloped line is modelled by the inclined face containing A when e is chosen as a very small number.

Eventually we create a macro with a segment ([AB]) and three numbers as initial objects (length d, angle a and half width of the base e) and the inclined face as final object. This macro is used in Figure 12 (right).

It is possible to create another macro giving the previous inclined line and its symmetric with respect to its origin to get a more realistic modelling (we have to use before that the previous macro in changing only the value of a from a to a+180°).

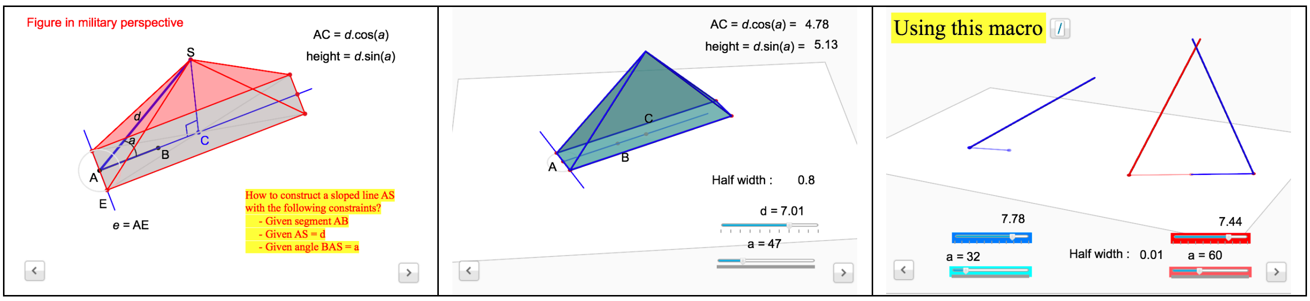

Figure 12: Sloped lines

3.3. How to create planes in all positions?

3.3.1. Planes parallel to the horizontal plane

The technique is summarized in Figure 13 :

– On the left how to create a rectangle which is the base of a box when starting from segment [1,2] and whose width is d. Then we use a rotation to create segment [1,4] which angle is defined by the formula d – d + 90 (trick for the following macro : 90 will not be an initial object because it depends on d) and therefore we create the rectangle [1,2,6,5].
– In the middle we create the prism defined by this rectangle and the given height. Finally our macro uses as initial objects the initial segment and the number defining the height of the box and gives as final objects the top of the box which models our parallel plane at a given level (previous height) and the rectangle of the 2D plane which is its perpendicular projection on this 2D plane.
– On the right, we can visualize the result of the use of this macro.

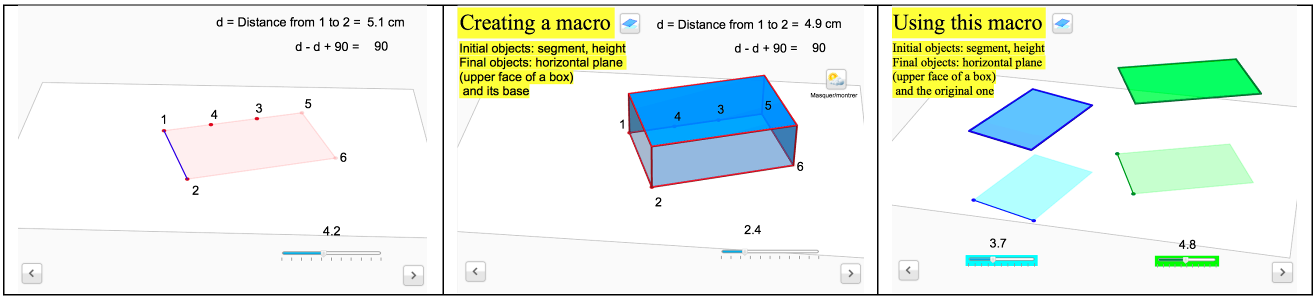

Figure 13: Planes parallel to the 2D plane

3.3.2. Plane perpendicular to the horizontal plane
See Figure 14 to discover the technique
– On the left how to create a vertical box from a given segment, a given height and a given number t (the width of the base of the box is 2.t).
– In the middle, creation of the macro having a segment, the height and half of the thickness of the box as initial objects and givig the front face of this box modelling a vertical plane when t is a small number.
– On the right, we can visualize the result of the use of this macro.

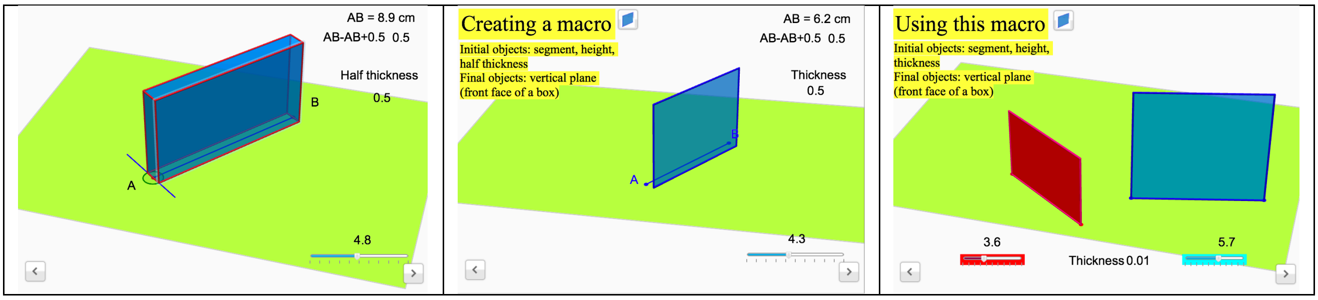

Figure 14 : Planes perpendicular to the horizontal plane

3.3.3. Sloped planes (Figure 15)
As a sloped line was modelled with a face of a pyramid, a sloped plane will be modelled similarly with another trick related to the length of the face of this pyramid. Figure 15 on the left shows that we start from a segment [AB] of the horizontal plane and we construct a pyramid where distance IS is a given number and angle a also a given number (commanded with a slider). The technique of creation of this pyramid is exactly the same as the one constructed in 3.2.3. But here the original idea is to chose a big number for IS (for example 200) because with such a constraint, the face ABS

of the pyramid will look like a plane (we have the impression to watch a parallelogram of the screen and not a triangle). We create then the macro having this face as a final object when the initial objects are the segment [AB], a first number defining the angle between the horizontal plane and our plane and a second number as big as possible. You can state in Figure 15 (in the middle) that the illusion is nearly perfect when the value for the big number is only 50 for an angle of 55°. Figure 15 on the right shows examples of use of such a macro. Let us note that, if we use this macro for an angle of a+180°, we can visualize the extension of our plane under the horizontal plane and eventually create a more powerful macro giving the two representations with one click.

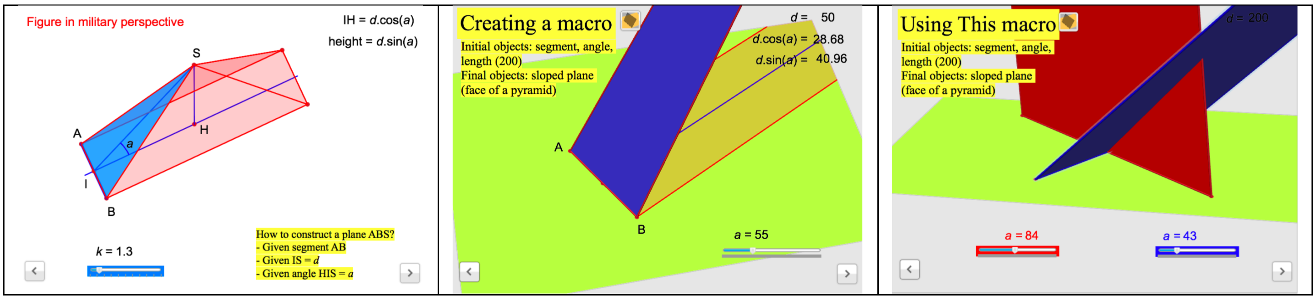

Figure 15 : Sloped planes

3.4. How to create half spheres centred on the horizontal plane

The screenshots displayed in Figures 16 and 17 show the algorithm of construction of such spheres with the New Cabri. These screenshots has been taken from Cabri 3D files.

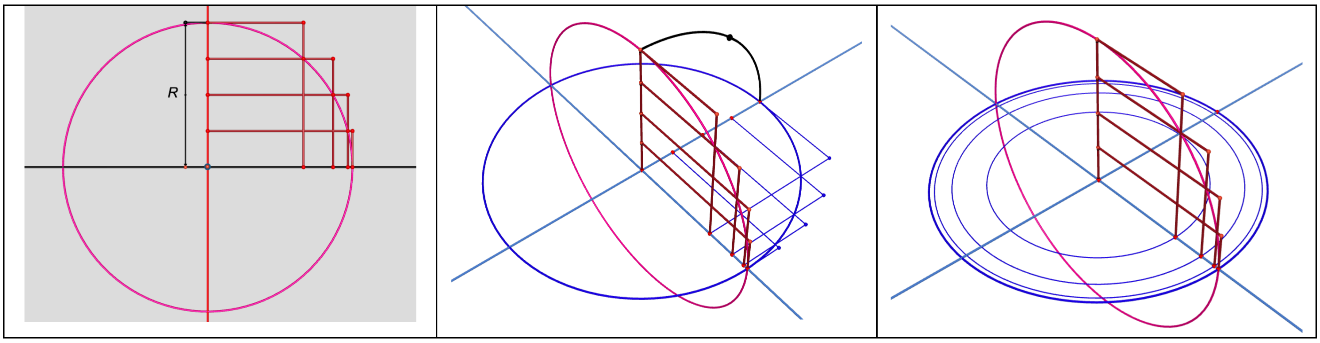

Figure 16 : Modelling half a sphere with 4 cylinders (1)

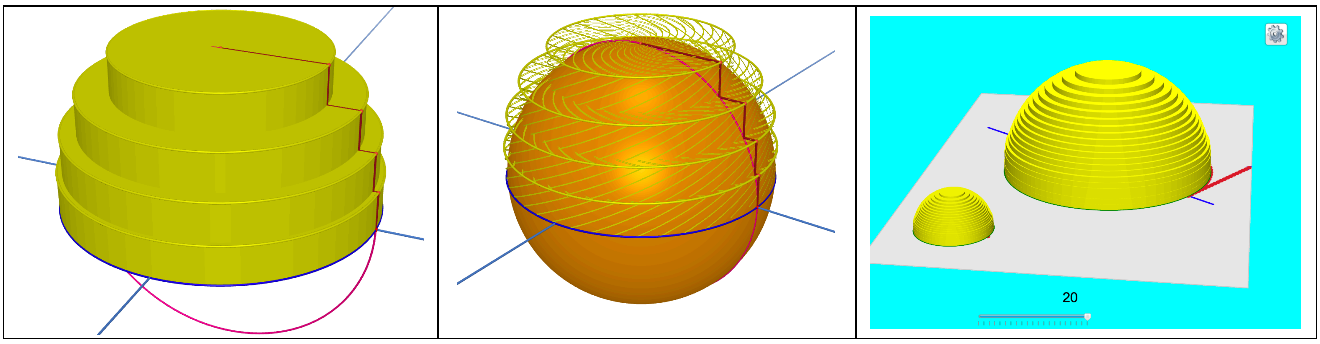

Figure 17 : Modelling half a sphere with 4 cylinders (2)

In the fifth first screenshots, we discribe the algorithm when modelling half a sphere with 4 cylinders. Figure 17 (on the rightside) shows what we can obtain when using this technique in the New Cabri : here the modelling is obtained with the construction of 20 cylinders.
You can notice on this last figure that we have recorded this construction as a macro construction having the base circle of the half sphere and the the number n of cylinders (here 20) as initial objects and the n cylinders as final objects. On the leftside of this figure we have applied this macro to a little circle.

3.5. Possible new 3D micro-environments from the expert to the users

It is possible to provide to the users all the files I have created for this paper. They can open these files online with Cabri Express and work with them. They cannot create them and especially the macros but they can modify them with the tools available in Cabri Express. It would be wonderful that the macros created with the New Cabri worked under Cabri Express : it would provide to the users a richer environment to experiment 3D geometry dynamically online.

4. Conclusion

This paper shows how an expert can use its expertise to enrich the tools of the New Cabri (a dynamic software focused on the creation of multipage activities) via its knowledge, its creativity and the tool macro construction . It shows also that the activities created can be opened and used online with Cabri Express which is an online freeware compatible with the New Cabri. Finally it shows how to enrich the microword provided to student to deal with 3D geometry online.

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YouTube videos (links)

[1'] Longitude et latitude https://www.youtube.com/watch?v=aB9p_Vl6O0o&t=22s

[2’] 08LETTRER https://youtu.be/RNB4nZwxD0I
[3’] Les animations d’Alexandre https://youtu.be/k-Ldyn26SsY
[4’] APM TOULOUSE Un atome et ses électrons par ALEXANDRE
https://www.youtube.com/watch?v=pXbkW5yK0tw

Software

Cabri 2 Plus and *Cabri 3D* by Cabrilog at http://www.cabri.com
Cabri Express at https://cabricloud.com/cabriexpress/