Creating a Symbiotic Relationship between Epistemology of Combinatorics and STEM Teaching Process

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Abstract: *Combinatorics is a domain in mathematics that concerns with procedures of arranging of objects into patterns of specified rules. Meanwhile, STEM (science, technology, engineering, and mathematics) is a term used to group these related subjects. Recently, educators are looking for practical ways to teach STEM. In combinatorics domain there are steps that need to be followed to design any specific problems. We are optimistic that these steps in combinatorics will be the basis to establish a framework for teaching STEM. Thus, in this paper, we would like to present the historical roots of combinatorics towards proposing a framework for teaching STEM disciplines. Based on combinatorics approach, we attempt to establish a practical method when resolving problems namely explore, discover and develop (ExDiD). This method was demonstrated to a group of students from several schools in Kedah, Malaysia who attended the "I C D' BEAUTY IN STEM" workshop. We illustrated the three steps involved to establish the procedure/method needed in resolving any complex scenario. The method was well received by the participants of the workshop.*

1. Introduction

Combinatorics, a fancy word for counting, is a branch of pure mathematics concerning the study of discrete (and usually finite objects). Historically, combinatorics has its roots in mathematical recreations and games. This area of mathematics is about problem solving that leads to theory building.

Meanwhile, STEM (science, technology, engineering, mathematics) term started as early as in the 1990's in the United States through the US government policies [3]. The goal is to ensure that every citizen of the United States is willing to pursue STEM and finally be able to work in careers related to STEM fields such as scientists, engineers, mathematicians, and technologists. STEM can be defined in the following manner (see [10]):

Science: a study of the law of nature associated with physics, chemistry, and biology and the treatment or applications of facts, principles, concepts, or conventions related within these disciplines.

Technology: the entire system of people and organization, knowledge, processes, and devices that go into creating and operating technological artifacts, as well as the artifacts themselves.

Engineering: a body of knowledge about the design and creation of products and a process for solving problems.

Mathematics: a study of patterns and relationships among quantities, numbers, and shapes.

Based on these definitions of STEM, we conclude that the disciplines in STEM should be the integration of all these four subjects: science, technology, engineering, and mathematics as presented in Figure 1. Specifically, STEM can be defined as a root in the real-world problems that are complex and interdisciplinary in nature. (see [10]).

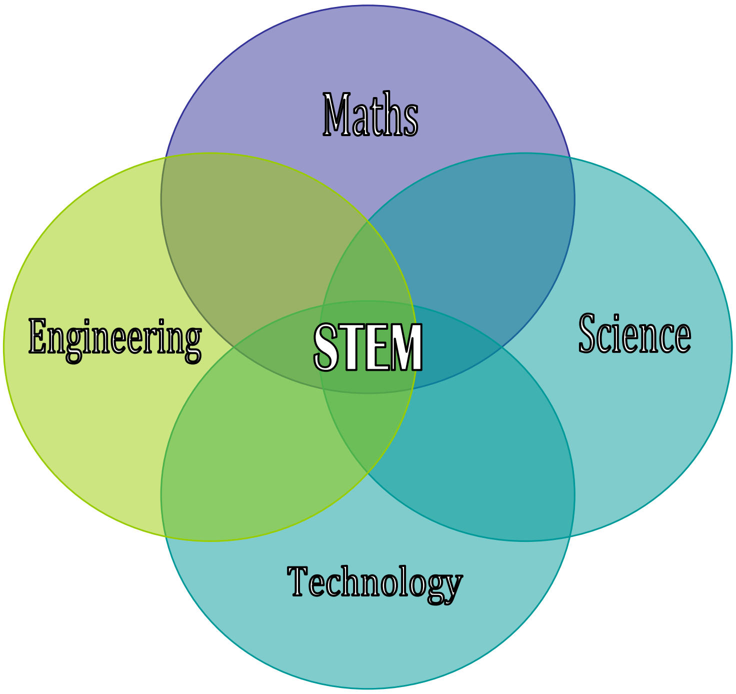


Figure 1.1 STEM relationship

Thus, a practical and systematic teaching process must tally with the definition of related disciplines of STEM to ensure that students have a fundamental basis to embark on in solving complex phenomenon. Therefore, this paper serves as a catalyst to design a framework for teaching STEM disciplines using epistemology of combinatorics. We would like to bring readers in a voyage of exploring combinatorics and then embed the steps in combinatorics into the designing methodology of teaching STEM subjects.

2. Combinatorics Design Process

This section will present classical problem in combinatorics. Since the nature of combinatorics problems usually involves solving real world problems, it often leads to theory building. Before that, we shall introduce in this section the procedures needed in solving combinatorics problems. Basically, combinatorics deals with the following procedures:

i. Existence of the arrangement – Does such an arrangement exist?

ii. Enumeration of the arrangement – How many valid/possible arrangements are there?

iii. Classification of the arrangement – Can they be classified in the same way?

iv. Algorithm – if such an arrangement exists, is there a definite method for constructing one or all of them?

v. Generalization –Does the problem under consideration suggest other related problems?

Since combinatorics problems occur in the real world problems, we start our presentation by introducing the idea of graph theory. Graph theory is an interesting branch of mathematics that can model several networks in our daily life. The origin of the graph began as early as 1783 in Prussia (now Kaliningrad, Russia) when a mathematician named Euler solved the problem of the Königsberg bridge, whereby the people in the region experienced difficulties in crossing seven bridges that connected two islands [11]. Figure 2.1 shows a map of Königsberg with the seven bridges and the river Preger.

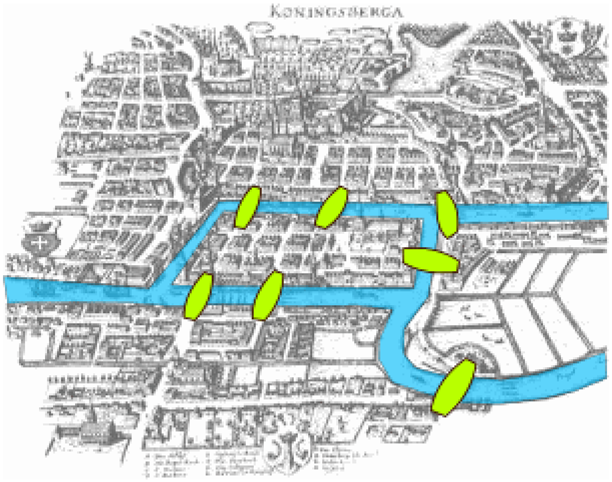


Figure 2.1 The Königsberg Problem

At the time, Euler attempted to solve the problem such that the people in the city could cross each of the bridges for only one time. He illustrated this solution using dots to represent the land masses and lines for the bridges. The outcome of the scenario appeared to be similar to Figure 2.2 shown below.

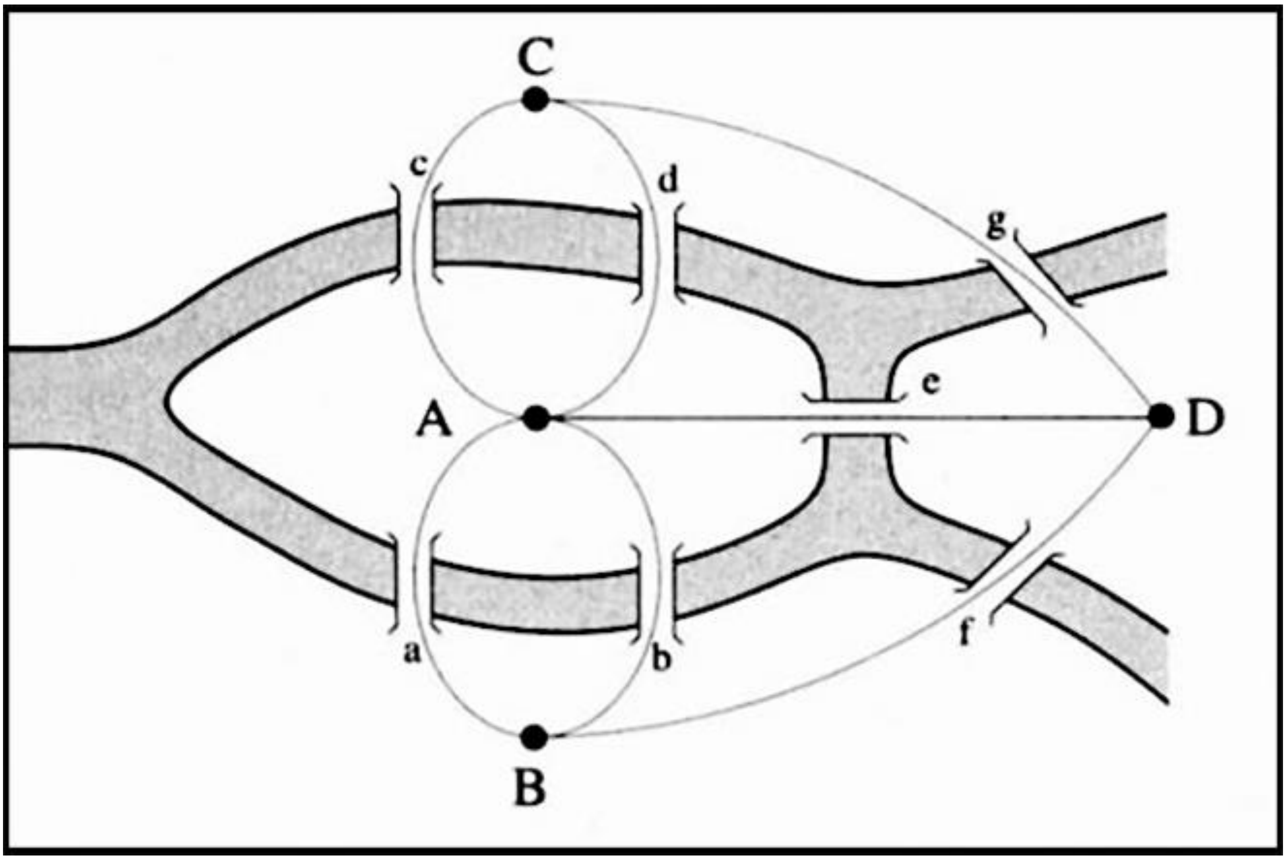


Figure 2.2 Representation of Königsberg in dots and lines

Based on this historical story, numerous studies have used the idea of graph theory to model related applications. If we see the origin of combinatorics, the majority of the work actually relates to real world story upon which the researchers would apply that idea and eventually, they would come up with various theories to solve these problems.

Now, let us illustrate the classical root of combinatorics by using one of the famous problem in combinatorial design. Consider this problem in combinatorial design theory, which is Lady's and Gentleman's Diary [4].

“A schoolmistress has 15 students and she wishes to take them on daily walks for a week. On each day, the students are to walk in five tunnels of three students each. It is required that no two students should walk in the same tunnel more than once over the week”.

This problem was posed by Reverend T.P. Kirkman in 1850 and is generally known as Kirkman’s schoolgirl problem. Now, we would like to discuss how we can achieve the solution by employing the procedures used in solving combinatorics problem.

Existence

This is the first step to understand the requirements from the scenario that we want to solve.

15 students
5 rows of students at one time since 15/3= 5 we want all the students to walk in a single day
Time constraint – A week for 7 days
No two students should walk in the same tunnel more than once over the week – e.g. 123, then the combinations 124, 135, 236 will not be allowed since pair 12, 13, 23 have already occurred in the beginning.

Enumeration and Classification.

We start to play with simple combinations for the first day without any rule but only through trial and error.

1,2,3	4,5,6	7,8,9	10,11,12	13,14,15

Now, perhaps we try to fill in the blanks the second day as below.

1,4,7	2,5,8	3,6,9	A	B

But, if we start by taking a combination of the students in column 1, 2 and 3, then we will have problems for column A and B. Thus, this combination is not suitable to be used. For this part, we have to play with different combinations until we can see some pattern forming to establish the design.

1,10,7	2,14,8	13,6,9	3,11,4	5,12,15

This is another possible combination for the second day, but to continue with this pattern for the third day means we will still have the same problem. For this step, we have to continue doing more possible combinations until we can see some similar pattern. This process takes up too much time and sometimes it is easy to have a pattern and sometimes, it is not.

Algorithm and Generalization

This is the crux of solving any problems. Based on the above step, after several enumeration and classification we have to find the most suitable pattern and thus, we need to establish the algorithm for the general case. In this step, we have to carry out our plan and check each step. We have to ensure the step is clear and correct, and eventually prove the algorithm.

Construction of Kirkman Problem Solution (see [4] and [13])

Let $v = 6n+3$ and let $(Q,\circ)$ be an idempotent commutative quasigroup of order $2n+1$, where $Q = \{1,2,3,\ldots,2n+1\}$. Let $S = Q \times \{1,2,3\}$, and define T to contain the following two types of triples.

Type 1: For $1 \le i \le 2n+1, \{(i,1),(i,2),(i,3)\} \in T$.

Type 2: For $1 \le i < j \le 2n+1$, $\{(i,1),(j,1),(i \circ j,2)\},\{(i,2),(j,2),(i \circ j,3)\},\{(i,3),(j,3),(i \circ j,4)\} \in T$

Table 2.3 The Kirkman Problem solution

Monday	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Tuesday	1	4	11	2	5	10	3	8	13	6	7	14	9	12	15
Wednesday	1	5	9	2	6	8	3	11	15	4	12	14	7	10	13
Thursday	1	6	13	2	4	15	3	9	10	5	7	12	8	11	14
Friday	1	7	15	2	9	14	3	6	12	4	8	10	5	11	13
Saturday	1	8	12	2	7	11	3	5	14	4	9	13	6	10	15
Sunday	1	10	14	2	12	13	3	4	7	5	8	15	6	9	11

Table 2.3 presents the final solution for constructing the Kirkman Problem based on the general algorithm.

Based on these steps in combinatorics we are optimistic that these steps are practical to be adopted in teaching any related STEM disciplines to instill the imaginative thinking skills that are needed in skilled workers of future demand. We illustrate the whole procedures in the workshop "I C D' BEAUTY IN STEM". Before we discuss on the workshop, we will guide the reader to see the STEM scenario in Malaysia

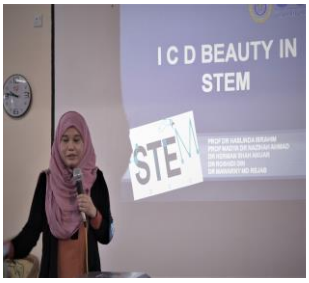
3. STEM Education in Malaysia

The advancement of STEM is the key driver for the future economic growth in the country. Thus, Malaysia has to take actions in order to be competitive and relevant to future global demand. For instance, "as a nation that is progressing toward a developed nation status, Malaysia needs to create a society that is scientifically oriented, progressive, knowledgeable, has a high capacity for change, forward-looking, innovative, and a contributor to scientific and technological developments in the future. In line with this, there is a need to produce citizens who are creative, critical, inquisitive, open-mined, and competent in science and technology." (pp. 1, Ministry of Education of Malaysia, 2005).

Based on this, the focus of STEM initiative in Malaysia blueprint 2013-2025 is: (1). To prepare students with the skills to meet the STEM challenges, (2). To ensure Malaysia has a sufficient number of qualified STEM graduates. Measures undertaken under this initiative: (1) raising student's interest through new learning approaches and an enhanced curriculum, (2) sharpening skills and abilities of teachers, (3) building public and students' awareness. The Malaysian government instituted the 60:40 Science/Technical: Arts (60:40), however the target is not achieved in the currently 45% enrollment in science stream [6]. Contributing factors for the declining enrolment in STEM: (1) Limited awareness about STEM, (2) Perceived difficulty of STEM, (3) Content-heavy curriculum, (4) Inconsistent quality of teaching and learning, (5) Limited and outdated infrastructure.

Concentrating on the aforementioned discussion about the scenario STEM in Malaysia, a practical teaching approach must be designed to cultivate an interest in learning the subjects. So, what should the criteria for an effective STEM teaching look like in a classroom? Morrison suggested that in a STEM integration classroom, students should be able to perform as (1) problem solvers, (2) innovators, (3) inventors, (4) logical thinkers, and also be able to understand and develop the skills needed for (5) self-reliance and (6) technological literacy [see in (7)]. Thus, the rest of the section will illuminate the reader on the teaching instructions using "Explore, Discover, Develop" (ExDiD) method that can cover the seven criteria suggested by Morrison.

4. Workshop: I C D' BEAUTY IN STEM

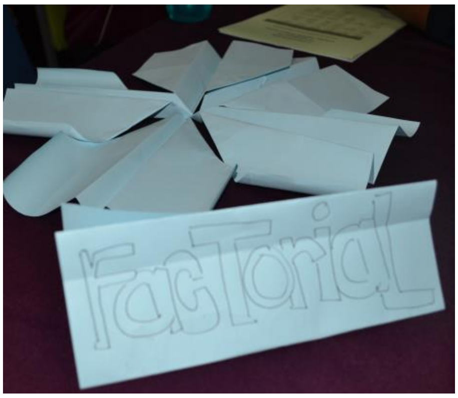


Six schools in Kedah, Malaysia have participated in the "I C D' BEAUTY IN STEM" workshop.

Objective: i) to expose students to the procedure involved
ii) to create awareness about STEM disciplines

Activity 1: Ice breaking:
The "ice-breaking" session will be the starting point of getting to know the participants from different schools. This is how the session will be handled:

1. Divide the group into a mix of six students from different schools.
2. Each group is assigned two facilitators (undergraduate students in the Mathematics Department).
3. Each group has to give a name for their group, some sample as shown in Figure 4.1
4. Then as a warming up activity, the students have to make paper airplanes.
5. To end up the ice breaking session the students have to write up the procedures on how they make the airplanes.



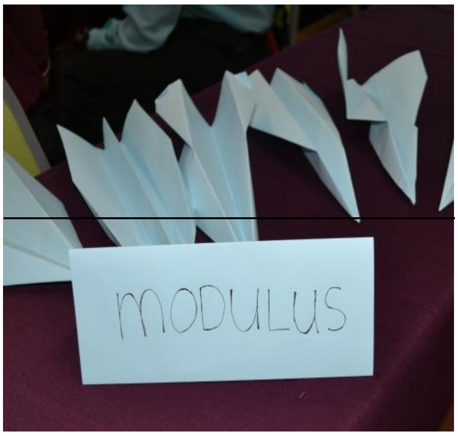


Figure 4.1 Name of the group

Objective for writing the procedure:
This is to see how students articulate the process that is needed in solving any particular problem.

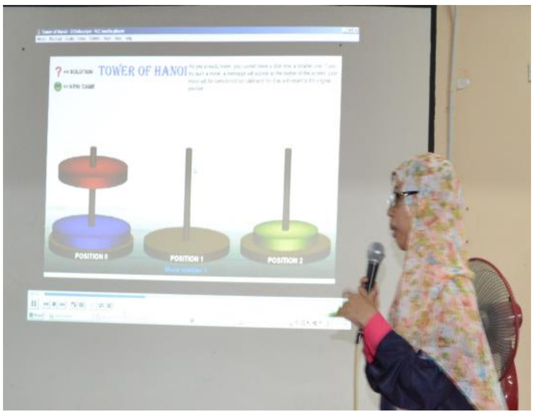
Activity 2: Warming up activity: Energizing with Fibonacci Sequence

We asked the facilitators to create the steps to dance to the Fibonacci sequence.

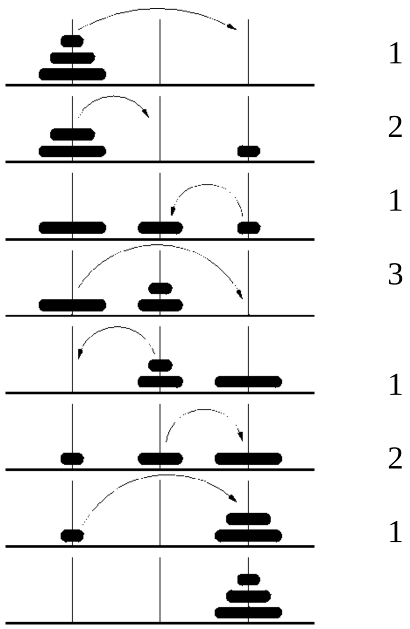
Fibonacci Sequence:
1, 1, 2, 3, 5, 8, 13, 21,….

Then, we ask the students to observe how we get the sequence. The reason for doing this is to expose to the students the way to explore, discover, and develop method. This will hopefully shed some light to the students when they start to do the problem in groups. This brings us to the next activity

Activity 3: Hanoi Tower



Hanoi Tower: Given a stack of *n* disks arranged from largest on the bottom to smallest on top placed on a rod

Objective: Formulate the minimum number of moves required to move the stack from one rod to another **Rules:** 1. Only one disk can be moved at a time. 2. Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack. 3. No disk may be placed on top of a smaller disk
EXAMPLE Sequence of moves for solving the Tower of Hanoi problem with three (3) disks. With 3 disks, the puzzle can be solved in 7 moves. 
EXPLORE & DISCOVER 1. Solve the Tower of Hanoi problem with four (4) disks. How many moves does it take? 2. Next, try to solve the Tower of Hanoi problem with four (5) disks. How many moves does it take? 3. Continue doing this until you can see some pattern.

DEVELOP

1. Find the pattern for 1 disk, 2 disks, 3 disks, 4 disks, 5 disks and 6 disks.

Number of discs	Sequence for discs movement	Minimum discs movement
1	1	1
2	1 2 1	3
3	1 2 1 3 1 2 1	7
4		
5		
6		

2. Based on our observation, come up with a formula for the minimum number of moves
   required to move the disc?
3. Finally, find the general formula for the minimum number of disc movements?

We gave the students about one hour and a half to play with the tools and finally, try to solve this scenario as shown in Figure 4.2

Figure 4.2 An activity during Hanoi Tower problem solving

For this activity, we provide a rubric based on partition of our work in three ExDiD categories.

Rubric for the assessment of PROBLEM SOLVING SKILL

Characteristic	Very Weak 0	Weak 1	Fair 2	Good 3	Very Good 4	MARKS
EXPLORE *Problem identification*	No attempt to explain any part of the problem/task	Able to give unclear explanation for only a small part of the problem/task	Able to give clear explanation for only a small part of the problem/task.	Able to give clear explanation for a large part of the problem/task.	Able to give clear explanation for the whole problem/task.	
DISCOVER *Identify Strategies*	No attempt at using a strategy or below grade level work shown.	There is an attempt to solve the problem. No strategy is applied that could lead to an answer.	Used an appropriate strategy. Reasonable strategy selected, minimally developed.	Used an appropriate strategy. Reasonable strategy selected, moderately developed.	Used an appropriate strategy. Reasonable strategy selected and developed.	
DEVELOP *Generate solution* *Looking back - reflection stage*	No attempt to identify correct solutions	Incorrect solution Students identify unworkable solutions with little reasoning. They rarely check their solution.	Arrived at a correct solution that comes from conceptual errors. Students identify partially correct solutions with some reasoning and limited ability to check their answer and if they do so are unable to make adjustments in their planning or execution stages.	Arrived at correct solution that comes from computation errors. Students include reasoning behind the evaluation of most options, and identify one correct/workable solution. Incorrect solutions lead to reflection and adjustments in planning.	Arrived at a correct solution. Students include reasoning behind the evaluation of each option. They can reflect upon solutions to make adjustments in and provide insights about their plan.	
TOTAL MARKS						

Activity 4: Closing

This was the end of the workshop. Each group had to display their findings as shown in Figure 4.3. Marks ware given according to the rubrics.

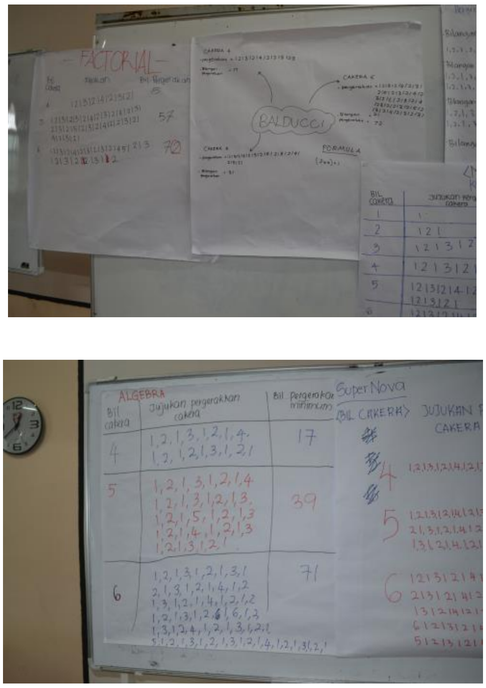


Figure 4.3 Solution for Hanoi Tower

Figure 4.3 displays the process on how they arrived at their solutions. Some of the groups managed to arrive at the final step, “Develop”. The majority of the group only managed to do the “Discover” steps to see the pattern before moving towards establishing the generalization of the algorithm.

5. Discussion on Explore, Discover and Develop Steps

This section is oriented on developing the framework for teaching STEM. A study on diverse STEM program and curriculum designs revealed that many researchers and educators agreed on the two major foci of STEM integration: (1) problem solving through developing solutions and (2) inquiry (see [1], [2], [8] and [14]). Thus, we can summarize that teaching STEM integration should not only focused on content knowledge alone but must also include problem-solving skills and inquiry-based instruction.

A compendium of literature suggested that STEM could (1) develop 21st century skills in students via engineering thinking, (2) increase science and mathematics achievement among students using engineering design approach, (3) increase students’ interest in learning science and related STEM disciplines [see in (12)]. However, in this paper, we would like to propose a STEM instruction via the combinatorics approach that could (1) cultivate interest in STEM, (2) instill artistic and innovative thinking.

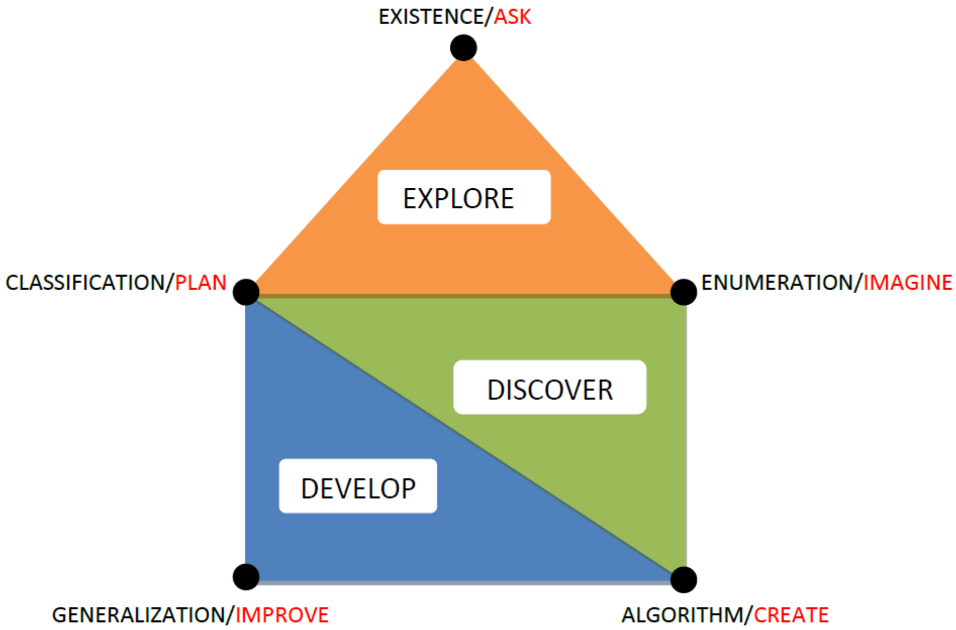
We will provide two designs for teaching STEM, Engineering Design Cycle and Combinatorics Design as depicted in figure 5.1 and Figure 5.2 respectively. Then, we will propose the framework for STEM instruction as presented in Figure 5.1

Table 5.1 Engineering Design Cycle (Museum of Science-Boston, 2009)

Design Process	Description
Ask	What is the problem? How have others approached it? What are your constraints?
Imagine	What are some solutions? Brainstorm ideas. Choose the best one.
Plan	Draw a diagram. Make lists of materials you will need.
Create	Follow your plan and create something. Test it out!
Improve	What works? What doesn't? What could work better? Modify your designs to make it better. Test it out!

Table 5.2 Combinatorics Design Approach

Design Process	Description
Existence	Does the problem exist? What is the problem?
Enumeration	Brainstorm ideas. Try some feasible solutions.
Classification	Try to identify/observe similar pattern.
Algorithm	Attempt several designs. Create the feasible coding to solve the problem. Test it out!
Generalization	Provide the optimal solution for this work.



Remark: Red → Engineering Cycle Design
Black → Combinatorics Design

Figure 5.3 ExDiD Method

We can represent STEM instruction as the following function

f (problem) = Solution(method/algorithm/steps/process/etc)

Problem → Explore, Discover, Develop (ExDiD) → Solutions

The teaching process of embedding ExDiD criteria cultivates artistic and innovative thinking in reaching the solutions.

"*Drilling activities sleep the artistic inquisitive mind*"

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