Empirical Research on Mathematical Inquiry Based Dynamic Software in the Lesson Study Training

Hongyun Li
lhysdnu@163.com
Department of Mathematics
Beijing Institute of Education
Beijing, China
Chunlan Wu
clw821@126.com
Department of Mathematics
Beijing Institute of Education
Beijing, China

Abstract

This research designed activities using theories of teachers' learning, two teachers from different schools participated in the teaching of the law of triangle altitude using dynamic software. The research showed that students were interested in mathematics inquiry using GeoGebra or geometer's sketchpad, found the exploratory direction using the dynamic of the software, built experiences in mathematical activities, and posed good mathematic problems under the proper guidance. The performance of students had encouraged the teachers who participated in the research, changed the belief in the integration of technology into the teaching, deepened the understanding of the content of mathematics, and influenced the understanding of the students' ability to learn mathematics. The research showed that there are some points in an effective mathematical inquiry based on technology: negotiation with the trainees who participate in the teaching to a consensus(motivation); discussion of teaching strategies according the content of mathematics, analysis of the students and the characteristics of technology(understanding); teaching practice using technology (practice); reflection process based evidence (reflection).

Key words: *Dynamic Software; Mathematics Inquiry; Lesson Study*

1. Background

National Mathematics Curriculum Standard for Compulsory Education in Mainland China (Version 2011) [1] suggests "using information technology as a powerful tool for students in the process of learning mathematics and solving problems, letting students be engaged in practical, exploratory mathematical activities". However, the reality in junior middle school mathematics teaching is far from this goal. As the "Action Plan for Education Informatization 2.0" [2] issued by the Ministry of Education of People's Republic of China on April, 2018 has pointed out "After years of exploration and practice, the revolutionary impact of information technology in education has begun to appear, the level of the construction and application of informationized learning environment is relatively low. Teachers' information technology application capabilities are available basically, but innovation ability for informationized teaching is still insufficient. Information technology and subject teaching are not deeply enough integrated".

From March, 2017 to May, 2018, we conducted a survey on the application of information technology in teaching activities of junior middle school teachers; the problems above-mentioned have also been partially confirmed. Statistical results (Table 1.1) show that the cases of application of information technology in teaching demonstrations made by junior middle school teachers account for about 30% in the survey. The cases of letting students use information technology to take part in the process of mathematics learning and inquiry are rare, account for only 6.2% of the total cases investigated.

Table 1.1: Distribution of Junior Middle School Mathematics Teachers according to their Use of Technology

	Percent
1.Frequent using of Interactive Whiteboards	27.7%
2.Frequent using of Multimedia course resources	33.9%
3.proficent in geometer's sketchpad、Z+Z or GeoGebra	6.2%
4.Frequent teaching of mathematics inquiry or experiment based technology	13.9%
5.Frequent learning of mathematics inquiry or experiment based technology by the students	6.2%

"Action Plan for Education Informatization 2.0" [2] suggests that the full coverage of information teaching and learning for all teachers and students of all levels of education should be realized by the year of 2022, promoting the deep integration of information technology and education; improving the information ability of teachers and students fully; promoting the expansion from technical application to information competence; equipping them with information thinking; using information technology to solve problems in their teaching and learning process.

In this view, in the training program of lesson study of junior middle school mathematics teachers in 2017, we designed lessons of students' using dynamic software in their inquiries. The process of the research has four steps: motivation-understanding-practice-reflection. We hope that such a design would affect the belief of using information technology into their teaching styles of the trainees in the program, promote the deep integration of information technology and mathematics teaching further.

2. Design of the research

2.1 Design of the lesson

The theme is inquiry of triangle's altitudes' law using dynamic software as the tool. The dynamic software is GeoGebra or geometer's sketchpad in our research. GeoGebra is open-source software about mathematics teaching and learning suitable for all levels of education, and the geometer's sketchpad can present the location relation and changing of geometric figure dynamically, they're dynamic software frequently used in the mathematic lessons.

There're many aspects inquired in triangle's altitudes, such as the location relation (relative to the triangle or other altitudes) and quantitative relations, the location and quantitative relations of figures generated by the altitudes, the altitudes about special triangles, etc. The textbook in grade 8[3] arranges the learning of triangle and important segments, and one of the inquiry activities is the law of median using geometer's sketchpad, the method is observing when the triangle is changing after drawing the three medians using geometer's sketchpad. The lesson in our research is an extension based the textbook in two ways: the content and the method.

Students are in grade 7 in the research, they have learned triangle altitude segments, but they have limited geometry knowledge and exploratory ability. As a result, teachers should help students to find the direction of the inquiry using the technology, for example, the location of altitudes (relative to the triangle or other altitudes), to discover and pose problems and to build experiences of inquiry activity.

2.2 Framework of the research

According to the six-element model of the teachers' learning and development [4] raised by the American scholar Shulman, combined with the characteristics of the in-service teachers' training, we select four elements among the six: motivation, understanding, practice, reflection to design a research framework consisting of the four loop steps (Figure2.1).

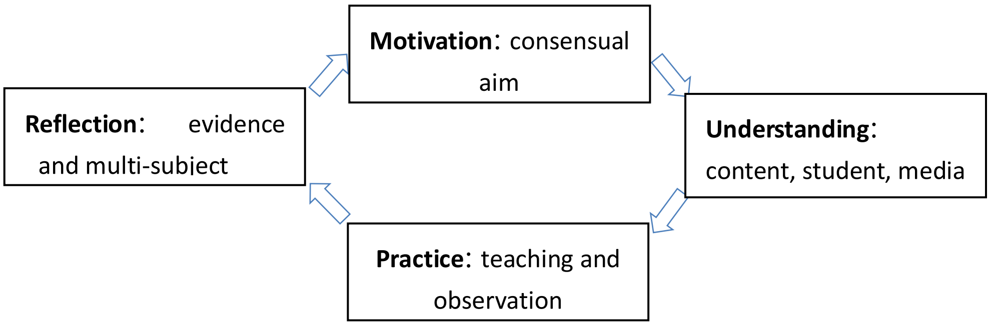

Figure 2.1: Framework of the research

Motivation: Stimulating the impact of information technology on mathematical inquiry activities, promoting the renewal of educational concepts of trainees, especially the teachers who would carry on the lesson, making them realize the importance of students discovering and posing problems and building experiences in mathematic.

Understanding: Negotiating about the contents of the lesson, teaching object (students) and teaching media (information technology); enhancing the level of understanding of the information technology based mathematical inquiry activities of the trainees, especially the teachers; further influencing the teaching behavior of the trainees.

Practice: The teachers design and implement classroom teaching, researchers and trainees observe classroom teaching together.

Reflection: Reflection is based on evidence: the objectives, contents and re-recognition of students; the reflection subject is multi-subject, including teachers, trainees, and researchers.

2.3 Participants

There are 2 teachers and their students, 2 mathematics teacher trainers (the authors) and trainees in two teacher training programs.

The lesson study is comprised by 2 sections. In the first section, the participants include teacher A, the authors, and 87 trainees (non-Beijing teachers, short-term centralism training program). In the second section, the participants include teacher B, the authors, and 34 trainees (Teachers in Beijing, long-term routine training program).

Teacher A is a male teacher who has taught for 9 years, he is skillful in GeoGebra, and his students have learned GeoGebra for some time and begun to explore some mathematic problem.

Teacher B is a female teacher who has taught for 28 years, she has rich experiences, and uses the geometer's sketchpad in her regular lessons, but she never had her students use it in exploring. We caught on the students' behavior of drawing altitude and posing problems across pre-test. Teacher B was not very optimistic about the students' behavior and worried about the lesson implementation.

3. Lesson Episodes

3.1 Two Episodes in Teacher A's Lesson

Episode 1: Given 3 altitudes segments figures about right triangle acute triangle and obtuse triangle (Figure3.1-3.3) drawing by students.

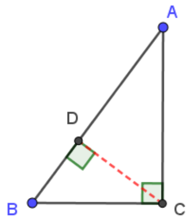

Figure 3.1

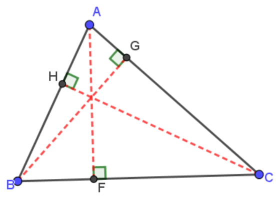

Figure 3.2

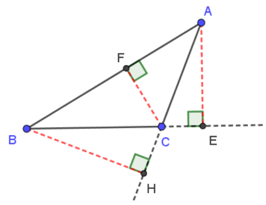

Figure 3.3

Dialogue in the episode (T represents teacher and S represents students)

T: *What relations about altitudes relative to the triangle can you find?*

S: *For the right triangle, one altitude is interior and the other two are on the side; for the acute triangle, all the altitudes are interior; and for the obtuse triangle, one altitude is interior and the other two are exterior.*

T: *What relations about the three altitudes* ?

S1: *For the right triangle, they intersect at one point, and the point is the vertex.*

S2: *For the acute triangle, they intersect at one point, and the point is interior.*

S3: *For the obtuse triangle, they didn't intersect*.

T: *Ok, we find that for right triangle and acute triangle, three altitudes intersect at one point, but is seeing true?*

In episode 1, the teacher didn't give the fact of "three altitudes intersect at one point" directly; he guided the students to observe the location relative to the triangles and their altitudes themselves instead. When student gave the conclusion that the three altitudes of an obtuse triangle don't intersect at one point, the teacher didn't give a negative response immediately. He commented on the student's conclusion: Is seeing true? This is exactly a key transition point in the geometric learning from elementary school to middle school, the transition from "looks like" to rigorous argument.

Episode 2: "Proof" of the proposition: three altitudes of triangle intersect at one point

T: *How to illustrate three altitudes intersect at one point for the right and acute triangle?*

S1: *For the right triangle, three altitudes are AC、BC、CD , they all have C , therefore they intersect at one point.*

S2: *For the right triangle(pointing to the Figure3.1), AC、BC intersect at C, C is the vertex* *and the third altitude must pass the C, so they intersect at one point.*

T: *You have two ways to illustrate, what about the acute triangle?*

S: *For the acute triangle, we can draw it using GeoGebra, and observe whether three altitudes would separate when the triangle changing.* (Here all the students validated using GeoGebra)

T: *"Looks like" doesn't mean true in mathematics, so do you have better ways to illustrate?*

T: *Could the second illustration way of right triangle give you some hints?*

S: *First, two altitudes must intersect at one point, suppose AF and BG intersect at the point O, connect C and O, and extend CO(Figure 3.4), If CO⊥AB, then we can prove it.*

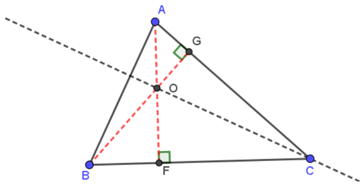

Figure 3.4

All students tried using GeoGebra and the teacher guided, then the teacher raised the question: *what about the obtuse triangle? Please explore using GeoGebra.*

In the process of "proof" that "three altitudes of triangle intersect at one point", the teacher followed students' cognition, instead of using information technology to prove it as the textbook does, he let the students argue firstly. Based on previous classification, the student discussed from right triangle to acute triangle and combined the ideas with the manipulation of GeoGebra in the "proof" process of acute triangle. Finally, the teacher responded to the problem of "the three altitudes of an obtuse triangle don't intersect at one point" raised in episode 1 and let the students explore it by themselves using GeoGebra.

3.2 Two Episodes in Teacher B's Lesson

Episode 1: Conjectures about the location of altitudes

T: *In the works before the lesson, someone stated that all the altitudes are interior for acute triangle, what about right and obtuse triangle?*

S: *For the right triangle, one altitude is interior and the other two are on the side; and for the obtuse triangle, one altitude is interior and the other two are exterior.*

T: *what are the location between lines ?*

S: *Parallel or intersecting.*

T: *what are the location between three altitudes ?*

S: *For the right triangle, they intersect at one point.*

S: *For the acute triangle, they intersect at one point, too.*

S: *For the acute triangle, they don't intersect at one point (showing the Figure3.5).*

T: *We get different conclusions. Do the three altitudes intersect at one point for the acute triangle?*

In the pre-test, we didn't make any hint for classification (acute, right, obtuse triangles), we only give two progressive and relatively open questions: *(1) Please write a conjecture about the location of the triangle's altitudes and draw the corresponding figure; (2) In addition to the location of the triangle's altitudes, what other conjectures do you have about the triangle's altitudes? You can express your conjecture with figures.* Through pre-test, we gave the students more space to think about, on the other hand, we avoided the low cognition activity of letting students draw altitudes on acute, right and obtuse triangles separately because of the limited time (as teacher A does). At the same time, the teacher could show characteristic conjectures, and deepen the cognition with students' conjectures: discussing the classification (triangles); the conclusions of location between altitudes and the triangle; the conjecture about the location of the three altitudes and complete the conjectures about the location of a triangle's altitudes (Table 3.1).

Table 3.1: Findings or conjectures about the altitudes' location

	Relative to triangle	**Relative to other altitudes**
Acute triangle		
Right triangle		
Obtuse triangle		

Episode 2: "proof" of the proposition: three altitudes of triangle intersect at one point

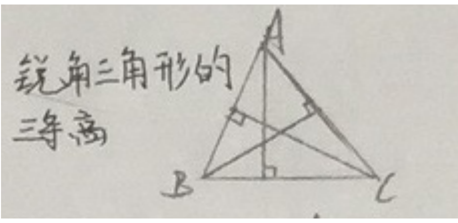

Figure 3.5

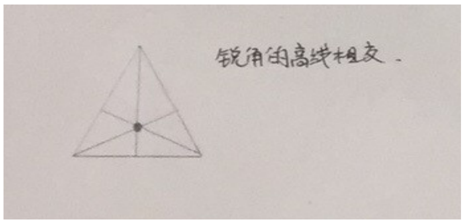

Figure 3.6

T: *Let's observe the two figures（Figure3.5 and3.6）, do the three altitudes intersect at one point for the acute triangle?*

S1: *I'm not sure.*

S2: *I think they intersect at one point, my Figure is the same as Figure3.6, and Figure3.5 may be not precise.*

T: *Let's testify using geometer's sketchpad.*

The students testified, and the teacher guided.

S: *I testified that three altitudes intersect at one point.*

T: *Is it really? How to illustrate more rigorously? Let's discuss in your groups, and you can use geometer's sketchpad.*

Some groups begun to discuss, and some groups begun to testify using geometer's sketchpad.

Group1: *We can illustrate each two of three altitudes must be intersected on one point,* *thus we have three points, if we could verify the three points are coincide, then we can prove it.*

Group 2: *Our discussion direction is: First, two altitudes must intersect at one point, if only we could illustrate the intersection point is on third altitude, we can prove it. And we have used the geometer's sketchpad testified our idea.*

Using the resources of the "incorrect" figure made by the student in the pre-test, the teacher tried to incur a contradiction and let the student testify it using geometer's sketchpad, reflecting the accuracy and dynamic characteristics of geometric drawing for testifying many acute triangles. Through further rigorous argument procedures, let the students get the feeling of the transition from "looks like" to rigorous proof.

4. Analysis

We analyzed the result from three aspects: students, two teachers and trainees.

4.1 Students

In our research about the case of teacher B, we conducted tests on students before and after the lesson, we focused on the students' making classifications and posing conjectures, 38 students participated in the tests. The results are as follows:

Table4.1: Percent of Students in Classification and Posing Conjectures before and after the Lesson

	classification	**location conjecture**	**quantitative conjecture**	**No conjecture**
Pre-	24（63.2%）	8（21.1%）	12（31.6%）	23（60.5%）
Post-	34（89.5%）	28（73.7%）	6（15.8%）	4（10.5%）

According to the students' performances before and after teaching, there is a significant increase in the percent of students who could make classifications and pose conjectures about the locations, and the proportion of students who failed to make conjectures decreases significantly. In teacher B's lesson, the teacher mentioned quantitative relations in the end only, that's why in the posttest, only 6 students raised conjectures concerning the quantitative relations.

We selected 6 students for interviews, 4 students among them can retell the idea in the proof of the location that the triangle's three altitudes intersect at one point. And 2 students can retell two proof ideas. They all think that the geometric sketchpad is interesting and accurate, it is helpful for understanding mathematics in a DIY way. 1 student mentioned that in addition to the content taught by the teacher, he also uses the geometer's sketchpad to explore other problems.

4.2 Two teachers

Before the lesson, teacher A never thought that the law of the triangle's altitudes could be taught in this way, because this content is provided to students as reading materials in math textbooks with median. And the testified process is drawing three medians and dragging the triangle to observe, they get the conclusion that three medians of a triangle intersect at one point, and then they explore altitudes themselves. In teacher A's lesson, students had their own thinking when students facing three altitudes of an/a acute, right and obtuse triangle, they could make arguments starting from the right triangle, and their rigorous ideas about three altitudes of a triangle, these were all beyond teacher A's expectation.

Before the lesson, teacher B discussed with the researchers (the authors), she rated very low of the results of students' pretest, and she thought the proof of the theory that three altitudes of a triangle intersect at one point is a mission impossible for her students. Neither teacher B thought about how to prove this proposition but admitted it as a fact. In the reflections after the lesson, teacher B mentioned that: *This is the first attempt to let students use geometer's sketchpad to explore mathematical problems. I think it is very beneficial. The students were interested in the inquiry, and they did mathematical thinking while they did operations. The process of drawing can reflect the process of knowledge formation. The two "proof" ideas the students raised in the lesson also surprised me very much, the using of geometer's sketchpad can inspire or verify the directions of students' inquiries.*

4.3 Trainees

We also conducted pre- and post-test for the participants in the second section, and 34 trainees participated in the tests.

According to the result, the fact that the higher proportion in classifications as well as the posing of location conjectures has no significant change between pre- and post-test, the proportion in posing conjectures beyond the textbooks is not high either. The teachers' level of posing true conjectures is not high either.

Trainees participated in the observation of the teaching were all feeling shocked by the using of technology, since 92% of the teachers do not know how to use dynamic software. They were also very impressed with the performance of students. However, some teachers believe that the learning tasks have exceed students' cognition and teaching requirements, they also think it's not necessary for students to use information technology.

In the reflection process of teaching observations, all trainees agreed with the idea of applying dynamic software to students' mathematical inquiries. They all agreed that with the dynamic function of the software, students can personally operate, observe figures changes, make conjectures and testify them. The perceptual knowledge of geometric figures was increase and experiences in mathematics inquiry activities were accumulated. They also mentioned that both the space and time guiding students to make conjectures in this lesson are not sufficient enough. There are two trainees who improved the teaching designs based on the teaching of teacher B, one teacher was inspired to apply dynamic software to the teaching of other dynamic geometric contents.

Table4.2: Percent of trainees in Classification and Posing Conjectures before and after the Training

	Classification	**Location conjecture**	**Conjectures beyond the textbooks**
Pre-	27（79.4%）	28（82.4%）	4（11.8%）
Post-		29（85.3%）	8（23.5%）

5. Conclusion and Reflection

We think that high-quality inquiry activities should focus on the worthy problems or conjectures posing by the students under proper context created by the teacher, and the problems or conjectures should be open for the students. The students have their own mathematical thinking in the process of inquiry rather than the manipulations under the teacher's guidance, and the information technology is one of the powerful tools. Thus in the teacher training program we focus on the trainees' posing problems or conjectures, verifying them using information technology and laying foundation for the deduction of the problems or conjectures.

5.1 The basic understanding of mathematics and teaching affects the construction of teachers' high-quality inquiry activities

Taking the Pythagorean Theorem as an example, we let the trainees in the program raise conjectures to examine how they discover and pose problems. Their conjectures, such as "the adjacent sides of the rectangle and the diagonal line satisfy "$a^2+b^2=c^2$", "for an acute triangle, $a^2+b^2 \neq c^2$", are mainly closed or known problems. This is consistent with our classroom observations: the inquiry activities they designed are more operations under the teachers' instructions than the students' independent thinking.

Before this lesson study, we investigated on the trainees: any quantitative conjectures about the altitudes of a triangle? The result is almost blank, they don't know that "Orthocenter partition altitudes into two parts, and the product of two parts are equal" or "CEVA Theorem", but they should know the quantitative relations of the triangle's medians, the question is why they failed to transfer the relation analogy to the conjecture concerning the quantitative relations of the triangle's altitudes? We realized the importance of the basic understanding of mathematics and its teaching again.

5.2 The understanding and application level of information technology limits the development of teachers' high-quality inquiry activities

Through our investigation，the level of information technology of junior middle school mathematics teachers is not high, especially non-Beijing teachers, this greatly hinders the using of the advantages of information technology. Teachers who are more skilled in dynamic software are basically using it in lesson teaching, the software is rarely used as a powerful tool for students in their mathematical inquiry [5]. These means that teaching activities developed upon information technology fall into the first three categories of closed or semi-open problems mainly, according to the five types from the Continuum Theory (1991) of J. Maker et al [6].

5.3 "Motivation-Understanding-Practice-Reflection" promotes the deep integration of information technology and mathematics teaching

By investigating the students of the two classes, students were interested in exploring math problems with GeoGebra or *geometer's sketchpad*, they were able to use the dynamic functions of the software to find the exploratory direction and build experiences in mathematics activities. Moreover, students can discover and pose problems under the appropriate guidance of teachers. The performance of the students inspired the trainees of the program, especially, it changed the belief of assimilating information technology into their teaching styles that was previously held by the teachers, deepened the understanding of mathematics teaching contents, and affected their understanding of students' mathematics learning ability.

Our research shows that there are some points for an effective lesson study training program concerning mathematical inquiry based on information technology: negotiation with the trainees who participate in the teaching to a consensus (motivation); discussion of teaching strategies according the content of mathematics, analysis of the students and the characteristics of technology (understanding); teaching practice using technology (practice); reflection process based evidence (reflection).

References

[1] Ministry of Education of People's Republic of China. *National Mathematics Curriculum Standard for Compulsory Education (Version 2011)* [S]：Beijing Normal University Publishing Group, 2012.

[2] http://www.moe.gov.cn/srcsite/A16/s3342/201804/t20180425_334188.html

[3] Peoples Education Press. *Mathematic textbook for compulsory education, grade8(first)* [M]Beijing：Peoples Education Press，2013:10.

[4] Shulman. L. *How and What Teacher Learn: a Shifting Perspective* [J]. Curriculum Studies,2004, (2).

[5] Chunlan Wu. *Analysis and Suggestions about information technology and the present situation of middle school mathematics classroom teaching integration* [J]. Journal of Mathematics (China):2012, (3):1-5, 9.

[6] Shaoying Xing, Shujuan Zhang. *The current teaching reform from the perspective of problem continuum and Multiple Intelligence Theory* [J]. Theory and Practice of Education, 2006, (8).