

# Improvement of the Usability of Figures in STACK by Use of CindyScript and CindyJS

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## Abstract

Last year at ATCM2017, we reported the enhancement of figures by appending the capability of interactive manipulations in STACK, which is the e-learning system for mathematics. In this enhancement, we fix the parameters and functions to create figures that can be manipulated interactively, but it is favorable that parameters and functions are selected randomly when a question is presented to students. In this paper, we explain how randomly selected parameters and functions can be used to create figures with interactivity by use of CindyScript and CindyJS. Finally, several examples are presented.

## 1 Introduction

Learning management systems (LMSs) have been popular for educational purposes in recent years, and computer-aided assessment (CAA) systems have often been used in universities and high schools. Because computers process the responses automatically, CAA is convenient to assess students answers and evaluate students understanding, especially in a large class. However, the multiple-choice question (MCQ) format is problematic in some respects. This format requires students to “choose” an answer, regardless of whether he or she understands the work correctly. When students answer a large number of questions, teachers might obtain an evaluation of their overall comprehension, but it remains unclear as to whether a student understands each question correctly.

We believe that it is more appropriate to assess mathematical expressions as answers submitted by students, especially for scientific subjects. As examples of such systems, there are Maple T.A. [1], MATH ON WEB [2], WeBWork [3], NUMBAS [4], and STACK [5, 6], which

can evaluate true or false mathematical expressions algebraically. Maple T.A. is commercial software and can be accessed from a wide range of LMSs such as Blackboard and Moodle. MATH ON WEB also utilizes commercial software: webMathematica. Unlike Maple T.A. and MATH ON WEB, WeBWork, NUMBAS, and STACK are open source systems and anyone can edit or adapt them to their needs. They all have plotting functions for editing questions and NUMBAS especially makes full use of modern Web technology, such as JXGraph, for interactive diagrams. STACK can be used by being integrated with Moodle, one of the most popular LMSs, and teachers can use STACK seamlessly with other subjects registered to the Moodle system, which is an advantage of STACK.

Although STACK is preferable on account of it being used in connection with Moodle, the use of this system presents some difficulties. One of them is that it only provides a poor plotting environment compared to the other systems. Because STACK makes use of various Maxima plotting functions to draw graphs, it should be possible to produce a greater variety of plotting types on STACK. However, STACK can only be used to graphically visualize two-dimensional graphs of explicit functions as a practical matter. Therefore, it is important to provide a more enhanced plotting environment, capable of drawing implicit functions and surfaces, in STACK. We have already performed this enhancement of the plotting environment of STACK and have reported our results at ATCM 2016 [7].

Last year at ATCM 2017, we reported the enhancement of figures by appending the capability of interactive manipulations in STACK. In this enhancement, we fix the parameters and functions to create figures that can be manipulated interactively, but it is favorable that parameters and functions are selected randomly when a question is presented to students.

In this paper, we explain how randomly selected parameters and functions can be used to create figures with interactivity by use of CindyScript, which is a scripting language similar to Javascript [8], and CindyJS, which is a framework in which to create interactive (mathematical) content for the Web [9]. Finally, several examples are presented.

## 2 Brief Review of STACK

STACK is a computer-aided assessment package for mathematics that was developed by Sangwin [5]. Since version 3, STACK has been fully integrated with Moodle in the form of a special question type in the quiz module that accepts mathematical expressions as answers. These mathematical expressions are polynomial expressions, matrix expressions, and functions. STACK evaluates these answers and generates outcomes that only produce a true or false result but also provide some feedback. In assessing answers algebraically, STACK utilizes Maxima, a computer algebra system.

Figure 1 presents an example of a question requiring the solution of the following simple differential equation:

$$\frac{dy(x)}{dx} + 2y(x) = 0.$$

The question is quite simple and the correct answer is easily found to be  $y(x) = C \exp(-2x)$ , where  $C$  denotes an arbitrary constant. In this case, there are two elements that we should check to confirm the students' level of understanding: One is whether students performed the calculation correctly and the other is whether an arbitrary constant is included in the answer. Therefore, two typical incorrect answers are the following:  $y(x) = C \exp(2x)$  and

$y(x) = \exp(-2x)$ . The first incorrect answer is an example of miscalculation and the second is an answer that does not contain an arbitrary constant  $C$ . In this case, we can provide feedback as follows: **“Incorrect answer. Your answer should satisfy the ODE, but does not.”** for the first type (see the top panel of Figure 2) and **“Your answer is partially correct. The solution should contain a constant but your answer does not.”** for the second (see the bottom panel of Figure 2).

Find a general solution of the following ODE.

$$\frac{d}{d \cdot x} \cdot y(x) + 2 \cdot y(x) = 0$$

$y(x) =$

Figure 1: Example of a question requiring a simple differential equation to be solved.

The first example in Figure 2 (top) allows students to understand why their solutions are incorrect by referring to the feedback. They understand that the solution of the differential equation should satisfy this equation. This concept involves a very basic point of view about a differential equation, but more than a few students do not recognize the concept. Another important concept that students often forget is that the general solution of a differential equation should contain arbitrary constants in response to the order of the differential equation and should be also checked. The feedback shown in the second example of Figure 2 (bottom) is designed to call students’ attention to their mistake. We think that a careful design of questions and feedback would produce significant educational benefits. Suitable feedback can be given to students by analyzing answers with a mechanism known as a “potential response tree,” which is one of the most outstanding features of STACK.

### 3 Enhancement of the Usability of Figures in STACK

As we have shown in the previous section, STACK is an effective e-learning system for scientific subjects, even though it has a poor plotting environment. In this paper, we consider the enhancement of usability of figures in STACK. Here we describe our approach to enhance usability with the power of Cinderella, CindyScript, and CindyJS, and we present two examples of questions with more attractive figures.

Cinderella is a dynamical mathematical software, CindyScript is an interpreted scripting language designed to manipulate the mathematical contents in Cinderella, and CindyJS is the framework in which to create interactive (mathematical) content for the Web. Then the enhancement consists of the following four steps:

- (1) First, in Cinderella, create a figure that can be manipulated interactively by dragging points with the mouse and/or sliders.
- (2) Export the figure to CindyJS, which generates one html file.

**Question 1**

Incorrect

Mark 0.00 out of 1.00

Flag question

Find a general solution of the following ODE. Tidy question | Question tests & deployed versions

$$\frac{dy(x)}{dx} + 2 \cdot y(x) = 0$$

y(x) =

Your last answer was interpreted as follows:

$$C \cdot \exp(2 \cdot x)$$

The variables found in your answer were: [x, C]

**Incorrect answer.**

Your answer should satisfy the ODE, but does not. If your answer is substituted into the ODE, the LHS is  $4 \cdot e^{-2 \cdot x} \cdot C$

---

A correct answer is  $e^{-2 \cdot x} \cdot C$ , which can be typed in as follows: `%e^(-(2*x))*C`

**Question 1**

Partially correct

Mark 0.50 out of 1.00

Flag question

Find a general solution of the following ODE. Tidy question | Question tests & deployed versions

$$\frac{dy(x)}{dx} + 2 \cdot y(x) = 0$$

y(x) =

Your last answer was interpreted as follows:

$$\exp(-2 \cdot x)$$

The variables found in your answer were: [x]

**Your answer is partially correct**

The solution should contain a constant but your answer does not.

---

A correct answer is  $e^{-2 \cdot x} \cdot C$ , which can be typed in as follows: `%e^(-(2*x))*C`

Figure 2: Examples of feedback given to incorrect (top) or partially correct (bottom) answers.

- (3) Define the Maxima function that reproduces the html file of (2), except that some of functions and/or parameters are replaced by randomly selected ones.
- (4) Append the following code to a question in html format:

```
<a href="URL of the file">
  Click here to open the figure window</a>
```

which produces a link to open the window representing the objective figure.

The export to CindyJS in step (2) is made by selecting the ‘Export to CindyJS...’ item in the file menu in Cinderella. In the Maxima functions of step (3), the randomly selected functions and parameters are given through the arguments of this function, and the html file to be generated will be put in the appropriate folder (or directory). Its file name is attached uniquely and is also passed to the Maxima function.

The setup of STACK for running the Maxima function consists of the following three steps:

- (a) Add the new function name to the variable ‘\$studentallow,’ which is defined in the file ‘casstring.class.php’ in the directory

(Moodle Top Directory)/question/type/stack/stack/cas.

- (b) Copy the file, in which the new Maxima function of step (3) is defined, to the directory

(Moodle Data Directory)/stack.

- (c) Ensure that Maxima loads the file in (b) by appending the two lines below to the file ‘stackmaxima.mac,’ which is in the directory

(Moodle Top Directory)/question/type/stack/stack/maxima,

```
PKG_DIR: "(Moodle Data Directory)/stack/"
load(concat(PKG_DIR, "<filename>"))$
```

Here <filename> denotes the file in which the new Maxima function is defined.

Here we illustrate two examples of questions with the window displaying figures with enhanced usability. The first example is shown in Figure 3 and the question asks students to find the absolute value of the difference of two values of  $k$  when the equation

$$x^3 + ax^2 + bx = kx \tag{1}$$

has just two solutions. In this example, the values of the parameters  $a$  and  $b$  are selected randomly when this question is presented to students (in Figure 3,  $a = -3$ ,  $b = 2$ ). Students can change the slope of the line by moving the slider to examine the number of points of intersections interactively. This manipulation helps students to investigate the number of the solutions by watching the number of intersections.

In the second example, students are asked to find the differential equation

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = e^{cx}, \tag{2}$$

with the initial condition

$$y(0) = 1, \quad y'(0) = 0, \tag{3}$$

where  $a$ ,  $b$ , and  $c$  are selected randomly when the question is presented to students.

When the student gives an incorrect answer, Figure 4 is presented as feedback, which explains the solution of the differential equation

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0, \tag{4}$$

with the initial condition (3), and the solutions of its related characteristic equation

$$\lambda^2 + a\lambda + b = 0. \quad (5)$$

In this figure, the parameters  $a$  and  $b$  are controlled by the sliders. The graph in the upper left shows the left-hand side of the characteristic equation (5). The graph in the upper right shows the discriminant of the characteristic equation (5). The graph in the lower left shows the solution of the characteristic equation by two points. The graph in the lower right part shows the solution of the differential equation (4) with (3), and one finds that the shape of the graph varies according to the sign of the discriminant of the characteristic equation. One can also observe that, as the values of  $a$  and  $b$  vary, graphs and points in the four parts move simultaneously. We think that this manipulation helps students to understand the solution of the second-order linear differential equation.

## 4 Summary

In this paper, we explained our approach to improve the usability of figures in STACK, in the sense that figures can be manipulated and, at the same time, functions and parameters in questions can be selected randomly so that they would vary each time a student tries an answer. The improvement is achieved by making use of Cinderella, CindyScript, and CindyJS and by creating the appropriate Maxima function, which generates the html file to be used to show the objective figure.

Teachers may use STACK when giving their students homework that requires them to solve problems involving differentiation, integration, and so on. We think that the careful design of questions and feedback with the use of figures to be manipulated would be able to generate significant educational benefit.

There are several methods to draw three-dimensional graphics in Cinderella, that is, to use Cindy3D plugins, SpaceCindy, SpaceCindyRay, and SpaceCindy2KeTpic. However, these methods do not seem to be usable for drawing three-dimensional graphics in a Web browser. When we need to draw space curves, surfaces,  $\dots$  that can be interactively manipulated, we may need to find other tools or methods. This is our future work to be done.

## Acknowledgment

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Preview question: Test1 - Opera

localhost/moodle/question/preview.php

Question 1  
Not yet answered  
Marked out of 1.00

Click here to open the figure window Tidy question | Question tests & deployed versions

Find the absolute value of the difference of two  $k$  values when the equation

$$x^3 - 3 \cdot x^2 + 2 \cdot x = kx$$

has just two solutions.

Your Answer=

Your last answer was interpreted as follows:

$$\frac{9}{4}$$

Start again Save Fill in correct responses Submit and finish Close preview

Technical information ? ▶

▶ Expand all

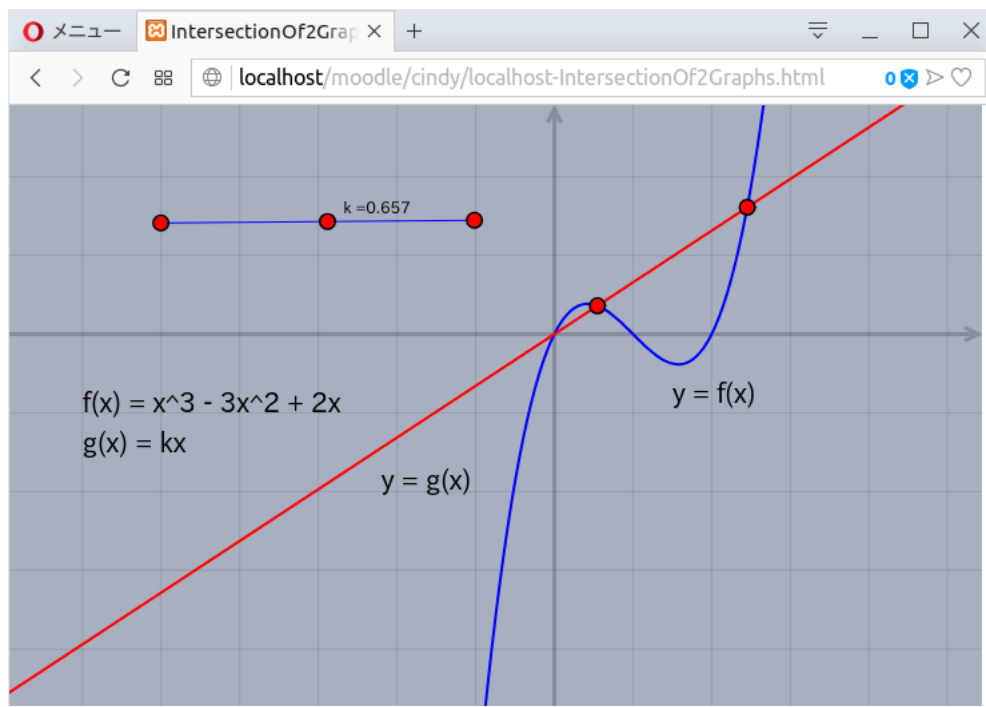


Figure 3: First example of the figure to be manipulated with a slider (bottom) and its question (top).



Preview question: Test2 - Opera

localhost/moodle/question/preview.php

Question 1  
Incorrect  
Mark 0.00 out of 1.00

Find the solution of the following differential equation with the initial condition:

$$\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = e^x,$$

$$y(0) = 1, \quad y'(0) = 0$$

Your Answer=

Your last answer was interpreted as follows:  
 $(2 - x) \cdot \exp(x)$

The variables found in your answer were: [x]

Your answer is incorrect.  
[Click here to open the figure window](#)

A correct answer is  $(1 - x) \cdot e^x$ , which can be typed in as follows: `(1-x)*%e^x`

Start again Save Fill in correct responses Submit and finish

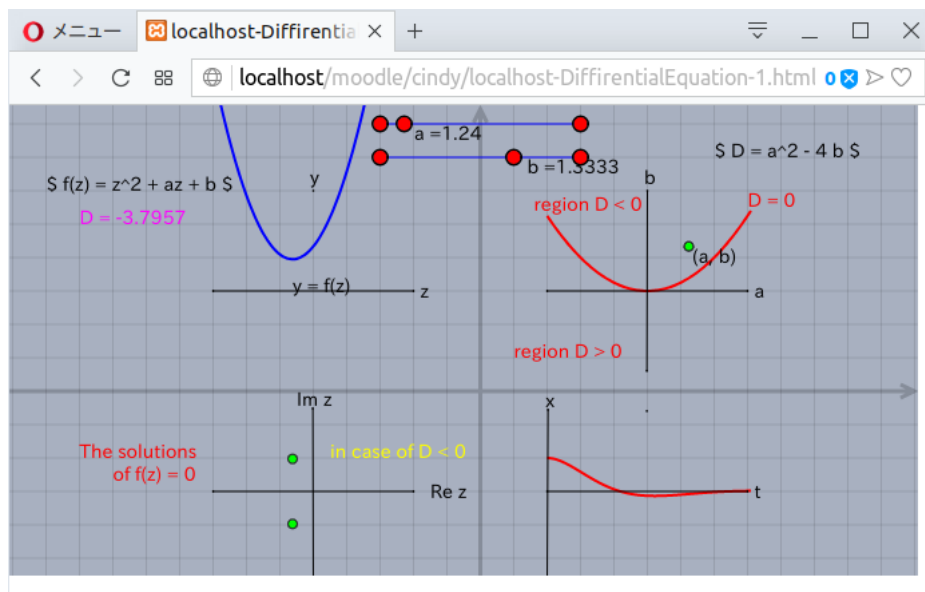


Figure 4: Second example of the figure to be manipulated with a slider (bottom) and its question (top).