Challenges in Integrating Technology in Teaching and Learning Mathematics in Basic Education

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Abstract
This paper presents considerations about the role of technology in teaching and learning mathematics from the perspective of teacher education programs that is required to prepare new generation of teachers for the potentialities of diverse uses of technology in the teaching mathematics content on one side, and in the learning new classroom dynamics on other side. The reflections retrieve some insightful initiatives and foresights of pioneering researchers about the educational possibilities of technology tools from late 90´s and 2000´s, bring frameworks from researches in mathematics education, and are illustrated with examples from the author´s work about development of teaching materials for prospective and in-service teachers, regarding the contemporary needs of mathematics education. The paper discusses the concept of experimental mathematics as a core concept that connects the learning of mathematics content, at student level, with the learning by teachers of new methods of teaching, at professional level. The examples, not exhaustive, will be given with CAS, DGS, Calculators. The challenges to integrate effectively the technology in educational context are commented considering the presence and advantages of information and communication technologies that imply necessarily the rethinking of teacher education. The reflections of this paper take into account the power of technology towards the educational needs of developing countries that strive for a quality education.

1. Introduction

In the ATCM-2014 paper [20], the author raises the issue that, despite the rapid advances of technological tools in last decades, the adoption of exploration activities has not been implemented in many school curricula yet, and discusses the importance of supporting materials for teachers to contribute to the situation. In fact, since past decades, many conferences, studies and renown researchers have been working hard to bring into practice the educational potentialities of diverse technology to improve the mathematics teaching and learning, and the key actors in this process are the well prepared (and supported) teachers with specific knowledge of the potential of technology for both, enhanced content knowledge as well as new pedagogy for the technology-rich mathematics lessons. Since 1995, ATCM papers have been contributing to the understanding of the potentialities but also of the challenging difficulties of implementation of technology in school environment. Recent ATCM-2014 papers ([10], [14], [12]), are good examples of different perspectives on this issue, though sharing a common topic of integrating the technology into teaching and learning mathematics underlined by the aspect of interactivity that calls for experimental activities in the classroom practice and in the preparation courses for teachers.

In 1999 paper, Waits & Demana ([19]) claim, as they analyze the introduction of electronic calculators equipped with computer algebra system (CAS) and dynamic geometry system (DGS) in the turn of the century, the expansion of the use of scientific calculators to more powerful educational context, based on their previous research as well as on investigations already being carried out by developers and educators in last decades. They have foreseen a new era of transformation in teaching and learning of mathematics. The hand-held technology with powerful and computer-like capabilities would actually transform the classrooms in laboratory that allowed teaching and learning by experimentation, exploration, discovery and development of investigative thinking skills. In [19], the authors offer an overview of the accumulated knowledge about the educational possibilities of calculators with insightful reflections about the constraints the changes would bring into the classrooms. They argue rightfully that the CAS technology made the major change in the attitude of students and teachers in mathematics lessons, either with desktop computers or hand-held technology. The key issue is that the graphing capability that permits visualization and dynamic manipulation is definitive in promoting interesting experimental activities. Still, the activities and experiments with CAS or graphing calculators were mainly designed to higher grades of schooling and focused obviously on higher mathematics. From 80´s, the development of dynamic geometry systems (DGS) and the calculators with built-in DGS has brought more possibilities to understand the role of technology in teaching and learning of mathematics in basic education (see [waits]). The reference [19] is part of [11], a collection of papers that offer an overview about the impact that hand-held technology of calculators has caused in teaching and learning of sciences and mathematics, in last decades of 20th century.

The reference [19] is an eye-opening introduction to the state of art of different technology being developed in the beginning of 21st century which allows renewed perspectives about the educational role of software and instruments in the teaching and learning Mathematics for the classroom practices. It analyses the role of calculators for the future of mathematics teaching and learning, in addition to the possibilities of spreadsheets, DGS and CAS already being developed and studied as teaching/learning aid from the previous decades, especially in 90´s. The portability of calculators equipped with CAS and graphing devices was considered then as a determinant factor to speed up the use of technology by teachers and students to transform the classroom practices, implying a necessary quality shift in learning mathematics and the content of mathematics to be taught. The summary in [19] of main aspects of teaching mathematics that would be influenced by calculators offers still today a clear vision of the potential impact of the educational use of technology and its role in the hands of students and teachers. However, knowing the different possibilities of mathematics content that can be taught is not enough to the effective implementation of technology based lessons. In the same period of time, important researches in Mathematics Education were producing as well a large amount of knowledge about diverse dimensions of the technology in education context, especially of the teacher education. Still it has been observed the access to technology does not guarantee its use by teachers.

The reference [7] of 2006, analyses the issue from mathematics education perspective. It consisted in a revisited study about the role of technology in mathematics education that included the situation of developing countries, after the previous Study done as early as 1986, when the graphing calculators were not developed yet. The main themes of the 2006 Study were: Implementation of curricula-issues of access and equity; Teachers and teaching; Learning and assessing mathematics with and through digital technologies; Design of learning environments and curricula. From the description of these subjects ([7]) it is possible to realize the importance and the interest worldwide in mathematics education research on this subject. This Study, in analyzing the role of the teacher in the educational use of technology, mentions that “earlier studies did not distinguish between the use of technology and the context of teacher´s professional learning”. This aspect is the main motivation for the research that originated the reflections of this present paper.

In this paper, we articulate the concept of “exploration activities” into the concept of Mathematics Knowledge for Teaching-MKT ([5]) that is a modern revision of Pedagogical Content Knowledge-PCK of Shulman ([17]), which frames theoretically the major domains of the knowledge of teachers to teach. We refer to [17], [5] for these concepts, but we will return to discuss them later.

The research of the technology as educational instrument for the learning of mathematics has gained increasing importance and attention since the last decades of 20th century, and nowadays consists in very important area of research of mathematics education, as can be seen in the survey reference [7], besides the large bibliography accumulated in the literature showing the importance of the subject. As well, the keen attention of educational policy makers can be perceived, as in the UNESCO document of 2002 [18].

> *“Educational systems around the world are under increasing pressure to use the new information and communication technologies (ICTs) to teach students the knowledge and skills they need in the 21st century. The 1998 UNESCO World Education Report, Teachers and Teaching in a Changing World, describes the radical implications ICTs have for conventional teaching and learning …..the teaching profession is evolving from an emphasis on teacher-centred lecture-based instruction to …interactive learning environment… implementing successful ICT-enabled teacher education is the key to fundamental, wide-ranging educational reforms”. [18] p.3*

We will not do a thorough revision of the literature on the role of technology in teaching and learning mathematics, for we want to focus on the most influential aspect of the research findings of last decades that address to the challenges still faced by schools, students and teachers in most countries of the world, especially the developing countries with needs of rapid qualification of citizens, before the higher studies at university level.

Especially, we delimit the focus of discussion to the role of technology in the professional development of teachers of Basic Education (k-12 education) that promotes the competency in exploring the experimental mathematics enabled by technology tools, CAS, DGS or Calculators.

The paper is divided in sections:

2. An interpretation of the term “experimental mathematics” for teacher education;
3. The problem solving methodology in participative learning environment
4. Examples
5. Concluding remarks.

The objective of the paper is to distinguish the research findings of mathematics education about the challenges of integration of technology in educational context to make the connections to the knowledge for teaching of teachers in the era of technology, enhancing the mathematics content knowledge needed to the development of sciences and technology.

2. An Interpretation of the Term “Experimental Mathematics” for Teacher Education

The true meaning of the original term “experimental mathematics” of Borwein & Bailey corresponds to “the utilization of modern computer technology as an active tool in mathematical research” ([6]). With the possibility of doing complex calculations in short time, allowing to explore properties and patterns or to test conjectures, the technology of robust symbolic mathematics software has brought alternatives to the methodology of doing mathematics investigations, and actually, the mathematicians have been able to discover and prove new results aided by technology. See [6] for details of this concept with examples, methods and implications.

Therefore, apparently this term does not apply to the educational context of the use of technology. However, the objective and the uses that characterize the "experimental mathematics" are very suitable to describe what we would want to follow in teacher education courses, in order to change the paradigm of teaching and learning mathematics in the presence of technology.
More precisely, the objective of "experimental mathematics" is "to generate understanding and insight; to generate and confirm or confront conjectures and generally to make mathematics more tangible". ([6]),

As well, the "experimental mathematics" is a method of doing mathematics that include the use of computation for [6]):

1. Gaining insight and intuition;
2. Discovering new patterns and relationships;
3. Using graphical displays to suggest underlying mathematical principles;
4. Testing and especially falsifying conjectures;
5. Exploring a possible result to see if it is worth formal proof;
6. Suggesting approaches for a formal proof;
7. Replacing lengthy hand derivations with computer-based derivations;
8. Confirming analytically derived results.

With minor and convenient substitution of some terms, we can recognize in the list above the desired process of developing mathematical thinking skills of students in basic school curricula, through Problem Solving lessons.

Indeed, the steps of the widely known methodology of problem solving - MPS, as stated by Polya ([16]) and many other mathematicians/educators, can be put in direct correspondence with the list above with clear interpretation of terms.

The problem solving steps as usual in the school curricula consist basically of:

a) Reading and understanding the text of the problem, sorting the given data and the actual question posed to solve;
b) Figuring out a strategy of resolution, modelling it with previously known mathematics concepts and techniques;
c) Carrying out the strategy, computing the necessary calculations or developing algebraic computations and giving an answer to the question;
d) Investigating the answer and its validity in the context of the problem, exploring possibilities of other answers or alternatives of strategy/approach, exploring possible generalizations.

The item 1 corresponds to the step a), when the potential of educational software or technological device helps to interact with the given data to gain insight and intuition about the properties of the hypothesis and isolating the question to answer. The phase of manipulation or exploring the data, as in steps 2, 3 and 4, correspond to the step b) of using the previous content knowledge to elaborate a strategy of resolution. At the level of early grades of the basic education, instead of formal proof as in 5 and 6, the student will find out the formula or equation to be worth solving. Clearly, the confirmation of the results in 7 corresponds to the analysis of the solution and investigation in the context, as in the step d) of problem solving.

Therefore, we recognize a similarity between the scientific methodology of discovery through experiments by research mathematicians and the development of mathematical thinking through problem solving activity by students, in educational context. This finding contributes to the achievement of mathematics literacy in basic education, as sought in curriculum standards of many countries worldwide, especially those undergoing curriculum reform process. See, for instance,[4], [12].

In this scenario, we argue that teacher education programs, especially the professional development programs for in-service teachers, have a concrete proposal to support the learning to teach with technology. Regarding this aspect, we bring back to the discussion the already mentioned concept of Pedagogical Content Knowledge -PCK of Shulman ([17]). Essentially, the PCK consists of the special knowledge required for effective teaching, blending the knowledge of content with the pedagogical knowledge about teaching and learning methodologies and curricular knowledge. The central idea of PCK is that learning to teach specific subject requires, besides the understanding the content itself, developing adequate instructional strategies and skills for students. The PCK has provided main framework to educational research in teacher education, being modernized and completed as MKT, as already mentioned.

With increasing presence of technology in educational context, research on the instructional uses of technology has revealed that teachers often fail in taking profit of technology as a tool to enhance the learning of subjects in depth [8]. The same way MKT resulted as the evolution of PCK, as framework for the research in teacher education, the authors in [8] generalized the concept of PCK to the Technological Pedagogical Knowledge - TPACK as framework to support the research in teacher education with the kinds of knowledge needed by a teacher for effective integration of technology in teaching practices. The TPACK has three major domains of knowledge in its foundation, the knowledge of content, of pedagogy and of technology. We refer to [8] for more details.

In our research on the professional development of in-service teachers of basic education we focus the development of learning material for teachers to understand the integration of technology in experimental activities through problem solving lessons, as explained above, grounded in the PCK framework that lately has included the evolution as TPACK. The teaching is oriented to be performed in a participative learning environment to enable a student centered classroom dynamics. In the next section, we will comment about this approach.

3. The Problem Solving Methodology in Participative Learning Environment

This paper has developed the ideas behind the necessary transformation of teaching and learning practices, centered around the key concept of “experimental mathematics”, interpreted in educational context. When an “experiment” is carried out as learning activity inside a classroom or a computer laboratory, the student becomes the main actor of the experiment, and this demands a preparation of the teacher to exchange the roles with the students, meaning a change of teaching paradigm as well as the attitude of students from passive listener of traditional lecture-like lessons to active participant of the learning process.

The change of traditional expository lessons to this demanding new style of conducting lessons is often the main reason of the resistance to the successful integration of technology in classrooms.
The steps of problem solving methodology, as described and analyzed before, call for teaching style with inquiries to stimulate the development of mathematical thinking of students through the experiments of exploration, mediated by technology.

The ATCM2014 paper of Kissane & Kemp [9] offers a nice perspective of the educational role of calculators, offering an organized four-part model to help teachers and technology users to understand the potential of technology in developing problem solving skills.
The parts of this model consists of “Representation”, in which the symbolic representation of mathematics language on the screen is closely related to the first step of comprehension of the problem and the interpretation of given data; “Computation” that corresponds to the experimentation and execution of a strategy of resolution; “Exploration” that corresponds to the validation and investigation of solutions, also to a form of experimental mathematics, as the authors mention in the paper. See [9] for nice examples of each part of this model.

As we recognized in this model a correspondence to our interpretation of the problem solving steps assisted by technology, a research project with prospective teacher, to develop teaching material to introduce Casio fx-991 in mathematics lessons for 5th grade classroom, has been carried out in 2014 ([2]) with a careful analysis of the model inside the lesson plan to enhance the understanding of teacher about the pedagogical features of technology

Therefore, we argue that the integrated view of the role of technology as catalyst of diverse actions to perform a lesson based in the steps of problem solving, in which the use of technology is fundamental to explore the strategies of resolution, is aligned with the contemporary needs of school curriculum, as indicated in the standards for mathematics literacy of students as well as for the competency of teachers.

The participative learning environment is determinant in educating students with autonomy in making decisions and solving problems. Large scale comparative assessment like OECD-PISA ([15]) is a reference for developing countries to measure the efficiency of their school system. It is noted that the PISA started recently to assess the mathematics literacy of students with problems that can use either the informatics platform or the built-in calculator to get the answers. See Figure 1.

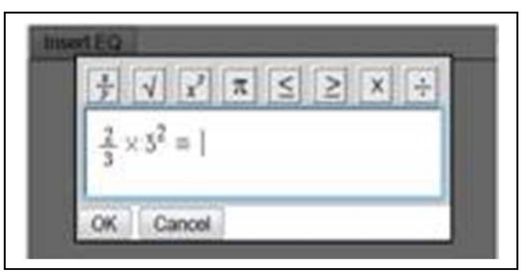

Figure 1. Built in editor tool in PISA 2015

This indicates that actually the technology literacy is considered much important to measure the competency of young generation of citizens prepared to technology based society. Therefore, it is more than timely that the teacher educators take the research findings into the classroom practices to effectively contribute to the improvement of mathematics education with content in depth and technology.

In next section we present some examples from our research with in-service teachers in Brazil and comment them with the perspectives discussed in this paper.

4. Some Examples

4.1 The case of DGS to connect geometry problem with modelling functions.

The following problem was proposed in a professional development project for lower secondary school teachers, with the objective to connect the perception of geometric properties to the activity of modelling functions.

Problem: A square with side 2 cm is cut by its diagonal, and its parts are put in different positions, with overlapping as in the Figure. Calculate the area of figures of the overlapped regions. Study the variation of the area of overlapping regions as a function, giving a graph interpretation.

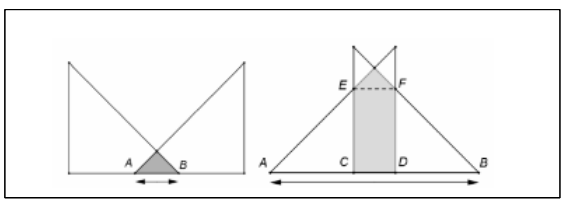

Figure 2: Overlapping triangles [3]

Before taking the problem to their classrooms, teachers studied the steps of problem solving methodology, planning a possible lesson through inquiries that would promote autonomous and spontaneous responses of students to explore the problem. The process passed from manipulating concrete material to figure out the items given by the problem to recognizing the dynamic phenomenon of continuous movement that called for a DGS to extend the experimental activity. See Figure 3 for three moments of a sequence.

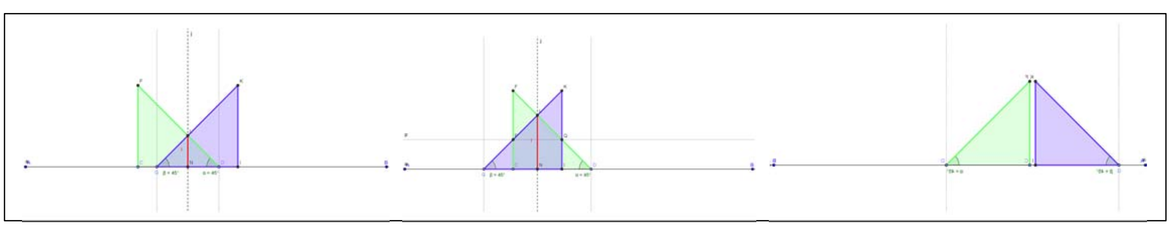

Figure 3. Dynamic experiment to explore geometric properties. [3]

At this stage of interpreting the problem, teachers perceived the pedagogical dimension of this problem that could be worked out from as earlier as 5^{th} grade classrooms, in which some fixed measures of the distance between the vertices of two triangles could be given to exercise the visualization, recognition of figures in the overlapping region and calculating the area as sum of its components. The dynamic experimentation with DGS facilitates to realize the relationship between varying elements of the experiment.

Understanding the independent variable in the movement permits to model algebraically the area function of the overlapped region defined in a domain split in parts, yet as a continuous function. To derive an algebraic formula to express the function brings out the necessary algebraic skills connected to the interpretation of geometric properties of figures, including the recognition of a characteristic of a square by means of the perpendicularity between its diagonals, a property often forgotten in classroom practices.

The interpretation of the analytic behavior of the area function is facilitated by analysis of the graph, accurate thanks to the technology and dynamic exploration. The concept of the domain of a function has gained new meanings with the mediation of technology, as well as the idea of continuity property of the function. See Figure 4.

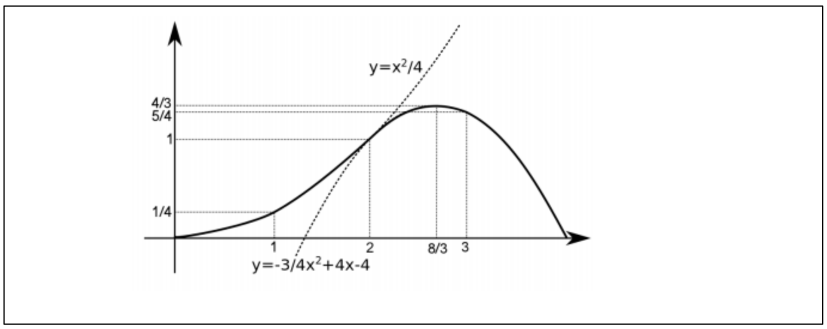

Figure 4. Graph of the area function of overlapped region, defined in parts. [13]

The exploration of the maximum value of this function was a surprising discovery for teachers to deconstruct the first "intuitive" guess about when this maximum value would be attained. Actually, teachers retrieved the previous algebra knowledge about the property of the vertex of a parabola as graph of a quadratic function, and interpreted in the initial geometric setting of the problem, so accomplishing a meaningful teaching and learning activity, inside the school curriculum content.

This example presents the conditions to illustrate the process of learning with "experimental mathematics" at the level of basic education, for both students and teachers.

4.2 An integrated use of technology that enhances the power of technology

The combined use of CAS and DGS was used in a project aimed to attribute meanings to a plane geometry approach of conics, traditionally studied with algebraic equations detached from their original spatial setting in secondary school classrooms. The graphing and animation capabilities of software allowed the "experimental mathematics" at the level of secondary level classrooms, impossible in traditional lessons. The graphing feature of CAS with animation brought attractive setting to teachers and students to explore the definitions. ([1])

Figure 5 The original meaning of conic sections; case of a parabola (CAS) [1]

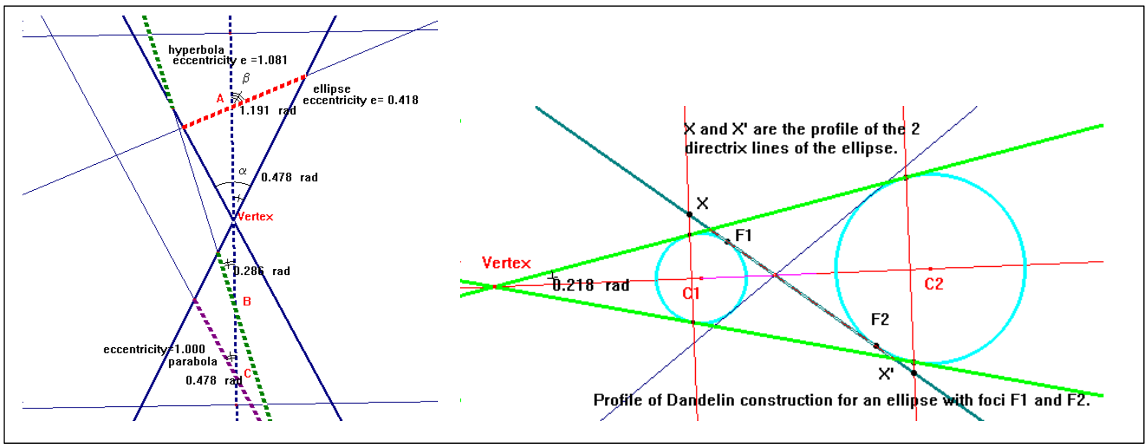

Figure 6. DGS to interpret geometrically the elements of conics in a profile plane section.[1]

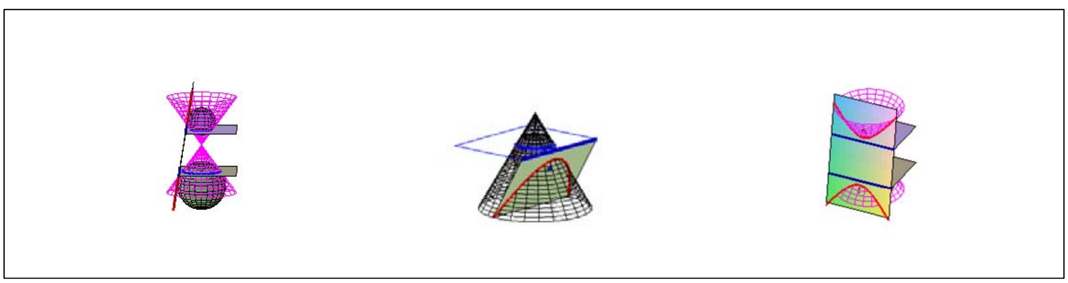

Figure 7. CAS mediated study of Dandelin spheres and conic sections. [1]

4.3 Calculator as didactical aid in problem solving activity in a Physics lesson at secondary level.

In an ongoing project of professional development program in a learning community of in-service teachers in Brazil, the innovation to teaching methodologies are much feared by teachers used to the traditional lecture-like lessons. The introduction of scientific calculators as didactical aid was proposed first to be studied previous to classroom activity, and planned carefully in a collective participative environment, in which teachers themselves experienced the learning supported by inquiries and executing the problem solving methodology steps. The integration of hand-held technology in lesson plan constitutes a real challenge for teachers. Still, the Brazilian curriculum standards of basic education as well as of teacher competencies require the technology literacy.

The following problem is being currently worked out in the classrooms of some participants of the community, including the practice of inquiry based problem solving, enabling the meaningful learning by students. The outcome of this project will be analyzed in future publication.

Problem: In a roller coaster a car (weight 450 kg) is on the top of a hill-track (80 m above the ground) about to race down the download slope towards the ground level. Considering that at the top of the hill the car is practically stopped, calculate its speed when it is 60 m above the ground level. Calculate its speed when it reaches the ground level at the end of hill.

This problem was planned to study the concepts of potential and kinetic energies, the conversion of units in measuring physics concepts. The scientific calculator (no graph calculator) was introduced to explore the meanings of the concepts as well as the algebraic skills associated to the interpretation of the relations between variables to model the formulas to input into the calculator. The exploration of Tables to analyze the dependency between variables is part of strategy. The careful planning of inquiries to stimulate the participation of students in the discussions is crucial

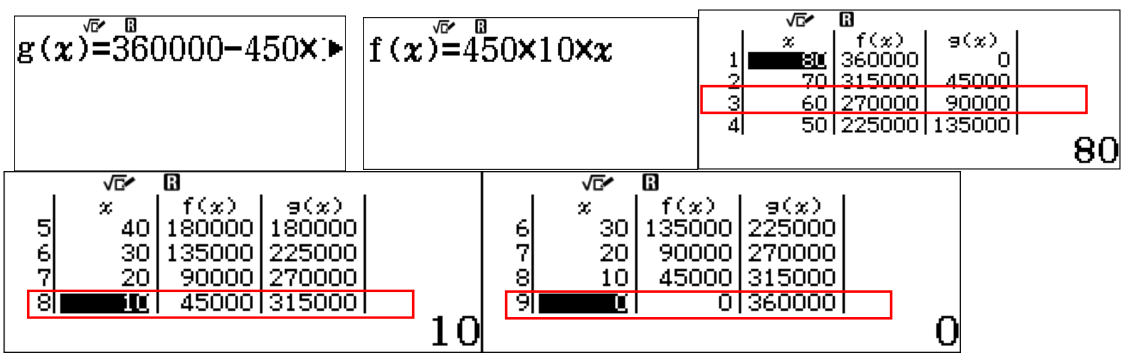

Figure 6. Screens to explore the algebraic modelling of kinetic and potential energies.

After these activities, the resolution proceeds with the computations needed to answer the questions about the speed posed by the problem.

Concluding Remarks

In this paper we did not discuss the role of communication technologies in teaching and learning mathematics, because the aim of the paper was delimited to the improvement of teachers´ competencies in providing appropriate learning environment to their students in actual classrooms, so that the students could attain a mathematics literacy, supported by the power of technology. The professional development of teachers to acquire the pedagogical content knowledge for teaching, enlarged with TPACK framework, is the first main challenge of teacher education of most countries, especially of developing countries as Brazil.

As another aspect, the importance of communication technology is very large, indeed, to support distance education for equitable access to knowledge, and actually it has a distinguished educational dimension concerning the assessment. The discussion of this aspect is out of the scope of this paper, and reflections on this regard will be disclosed in another opportunity.

We hope that the efforts and research findings of the community of mathematicians, mathematics education researchers, educators and teachers continue to contribute to a better education faster than predicted 20 years ago. The challenges are at the present moment.

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