

Enhancement of Plotting Environment of STACK with Gnuplot

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Abstract

In this paper, we explain our approach to building an enhanced plotting environment in which, for example, graphs of implicit functions and surfaces described by functions containing two variables could be visualized. This enhancement is achieved by creating Maxima functions, which produce the appropriate scripts of gnuplot, which is then called with these scripts. This approach enables us to make full use of the power of gnuplot, and can be applied to any plotting software other than gnuplot, such as matplotlib, and Pyxplot. Several examples of questions with enhanced plots are presented.

1 Introduction

Since the use of learning management systems (LMSs) has been popular for educational purposes in recent years, computer aided assessment (CAA) systems have often been used in universities and high schools. CAA consist of various types of questions: true or false questions, single- or multiple-choice questions, questions requiring the input of a numerical value, questions requiring a simple description, and so on. Among these questions, tests based on multiple-choice questions (MCQs) are most commonly used to evaluate students learning results. A well-constructed MCQ has a list of plausible distracters that is constructed based on the knowledge and understanding of common student errors. Since computers process the responses automatically, CAA is convenient to assess students answers and evaluate students understanding especially in a large class. However, the MCQ format is problematic in some

respects. This format requires students to “choose” an answer, regardless of whether he/she understands the work correctly. In other words, he/she might obtain the correct answer by guesswork. When there are a large number of questions students have to solve, it is possible to evaluate students overall comprehension, but it remains unclear as to whether a student understands each question correctly.

We propose avoiding the problems associated with the MCQ type by strongly recommending a system that evaluates answers provided by the students themselves. Especially for scientific subjects, it is more appropriate to assess mathematical expressions as answers submitted by students. As examples of such systems, there are Maple T.A.[1] and STACK[2, 3], which can evaluate true or false mathematical expressions algebraically. Maple T.A. is commercial software and can be accessed from a wide range of LMSs such as Blackboard and Moodle. Unlike Maple T.A., STACK is integrated with Moodle and open source software.

Although STACK is preferable on account of being free of charge, the use of this system presents some difficulties. One of them is that it only provides a poor plotting environment. Since STACK makes use of various Maxima plotting functions to draw graphs, it should be possible to produce a greater variety of plotting types on STACK. However, STACK can show only be used to graphically visualize two-dimensional graph of explicit functions as a practical matter. Therefore it is significant to provide a more enhanced plotting environment, such as drawings of implicit functions and surfaces, in STACK. The enhancement of plotting function of STACK was done before[4]. Nakamura et al. integrate MapleNET with STACK in order to use rich plotting function of Maple. The integration was successful but STACK should be customized to call Maple’s Plotting function through MapleNET. Furthermore, MapleNET that is commercial software should be prepared together with STACK. Then the integration was not widely used.

In this paper, we would like to introduce an easier way of enhancement of plotting function of STACK. It is not necessary to prepare new software nor to customize STACK itself. We explain how we can build such environment and present several examples.

2 Brief review of STACK

STACK, i.e., the System for Teaching and Assessment using a Computer algebra Kernel, is a computer-aided assessment package for mathematics that was developed by Sangwin[2]. Since version 3, STACK has been fully integrated with Moodle in the form of a special question type in the quiz module that accepts mathematical expressions as answers.

Examples of mathematical expressions as answers are polynomial expressions, matrix expressions, and functions. STACK evaluates these answers and generates outcomes that do not only produce a true or false result but also some feedback. Maxima, a computer algebra system, is utilized to algebraically assess answers in the form of mathematical expressions submitted by students.

Figure 1 presents an example of a question requiring the solution of a simple differential equation, which is denoted by

$$\frac{dy(x)}{dx} + 2y(x) = 0.$$

The question is quite simple and the correct answer is easily found to be $y(x) = C \exp(-2x)$ where C denotes an arbitrary constant. In this case, there are two elements that we should

check in order to confirm the students level of understanding: one is whether students performed the calculation correctly and the other is whether an arbitrary constant is added to the answer. Although the correct answer is $y(x) = C \exp(-2x)$, there are two typical incorrect answers. One is $y(x) = C \exp(2x)$, which is an example of miscalculation and the other is the answer $y(x) = \exp(-2x)$ that does not contain an arbitrary constant C . We can design questions that can provide the feedback “**Incorrect answer. Your answer should satisfy the ODE, but does not.**” for the first type of the incorrect answer (see above panel of Figure 2) and the feedback “**Your answer is partially correct. The solution should contain a constant but your answer does not.**” for the second type of incorrect answer (see below panel of Figure 2).

Figure 1: Example of a question requiring a simple differential equation to be solved.

The first example in Figure 2 (above) allows students to understand why their solutions are incorrect by referring to the feedback. They understand that the solution of the differential equation should satisfy this equation. This concept involves a very basic point of view about a differential equation, but more than a few students do not recognize the concept. Another important concept that students often forget is that the general solution of a differential equation should contain arbitrary constants in response to the order of the differential equation and should be also checked. The feedback shown in the second example of Figure 2 (below) is designed to call students attention to their mistake. Suitable feedback can be given to students by analyzing answers with a mechanism known as a “**potential response tree**”, which is one of the most outstanding features of STACK.

When students complete an assignment, we have to assess their reports and return them to the students to improve the value they obtain from their educational experience. However, this is demanding in terms of time and effort, which is why the CAA system could help teachers. If we design a question carefully to provide feedback as in Figure 2, this would produce significant educational benefits.

3 Enhancement of Plotting Environment of STACK

As we have shown in the previous section, STACK is an effective e-learning system for scientific subjects, even though it only has a poor plotting environment. We consider an improved plotting ability to be desirable for a more effective e-learning system. In this section, we describe our approach to enhance the plotting environment with the power of gnuplot, and present several examples of questions and their feedback with more attractive figures.

Question 1

Incorrect

Mark 0.00 out of 1.00

Flag question

Find a general solution of the following ODE. Tidy question | Question tests & deployed versions

$$\frac{dy(x)}{dx} + 2 \cdot y(x) = 0$$

y(x) =

Your last answer was interpreted as follows:

$$C \cdot \exp(2 \cdot x)$$

The variables found in your answer were: [x, C]

Incorrect answer.

Your answer should satisfy the ODE, but does not. If your answer is substituted into the ODE, the LHS is $4 \cdot e^{-2 \cdot x} \cdot C$

A correct answer is $e^{-2 \cdot x} \cdot C$, which can be typed in as follows: `%e^(-(2*x))*C`

Question 1

Partially correct

Mark 0.50 out of 1.00

Flag question

Find a general solution of the following ODE. Tidy question | Question tests & deployed versions

$$\frac{dy(x)}{dx} + 2 \cdot y(x) = 0$$

y(x) =

Your last answer was interpreted as follows:

$$\exp(-2 \cdot x)$$

The variables found in your answer were: [x]

Your answer is partially correct

The solution should contain a constant but your answer does not.

A correct answer is $e^{-2 \cdot x} \cdot C$, which can be typed in as follows: `%e^(-(2*x))*C`

Figure 2: Examples of feedback given to incorrect (above) or partially correct (below) answers.

The strategy for achieving the enhancement is quite simple. The present version of STACK draws graphs of functions with the help of Maxima plotting functions, which call gnuplot eventually. However, for some reason it is not possible to call some of these functions. This failure gave rise to the idea that it would be more effective to call gnuplot directly without the use of Maxima plotting functions. Then our enhancement consists of the following two steps:

- (1) Define the Maxima functions, which call gnuplot to plot the desired graphs.
- (2) Set up STACK to run the above new Maxima functions.

The Maxima functions of (1) should be created using the five steps describing the following content:

- (i) Determine the two filenames by using the Maxima function “`absolute_real_time()`” and the “`rand()`” function defined in STACK, which generates a random number less than 10^8 . One of these filenames is used for the script file of gnuplot to be created, and the other is used for the image file created by gnuplot.

- (ii) Check the arguments passed to these Maxima functions.
- (iii) Determine the options to be passed to gnuplot, which are fixed on the basis of the checked arguments of (ii).
- (iv) Create the gnuplot script to produce the desired plotting image file in png format.
- (v) Call gnuplot and run the script in (iv) to obtain the final result.

Note that it is important to pay attention to the difference between the mathematical expressions of Maxima and gnuplot when generating the gnuplot script in (iv). For example, x^2 is expressed as `x^2` in Maxima, whereas it is expressed as `x**2` in gnuplot. On the account of this difference, it is necessary to take care when transforming the mathematical expressions of functions in Maxima to the corresponding ones in gnuplot.

The setup of STACK for running these Maxima functions consists of the following three steps:

- (a) Add the new function names to the variable '`$studentallow`', which are defined in the file '`casstring.class.php`' in the directory

(Moodle Top Directory)/question/type/stack/stack/cas.

- (b) Copy the file in which the new Maxima functions are defined to the directory

(Moodle Data Directory)/stack.

- (c) Ensure that Maxima loads the file in (b) by appending the two lines below to the file '`stackmaxima.mac`', which is in the directory

(Moodle Top Directory)/question/type/stack/stack/maxima,

```
PKG_DIR: "(Moodle Data Directory)/stack/"
load(concat(PKG_DIR, "<filename>"))$
```

where `<filename>` denotes the file in which the new Maxima functions are defined.

After building the enhanced plotting environment, we can plot implicit functions and surfaces described by functions containing two variables. Here are two examples of questions displaying such enhanced plots. The first example is shown in Figure 3 and the question asks students to find the area of the region surrounded by the implicit function

$$(x^2 + y^2)^3 = (2x^2 + 3xy + 2y^2)^2.$$

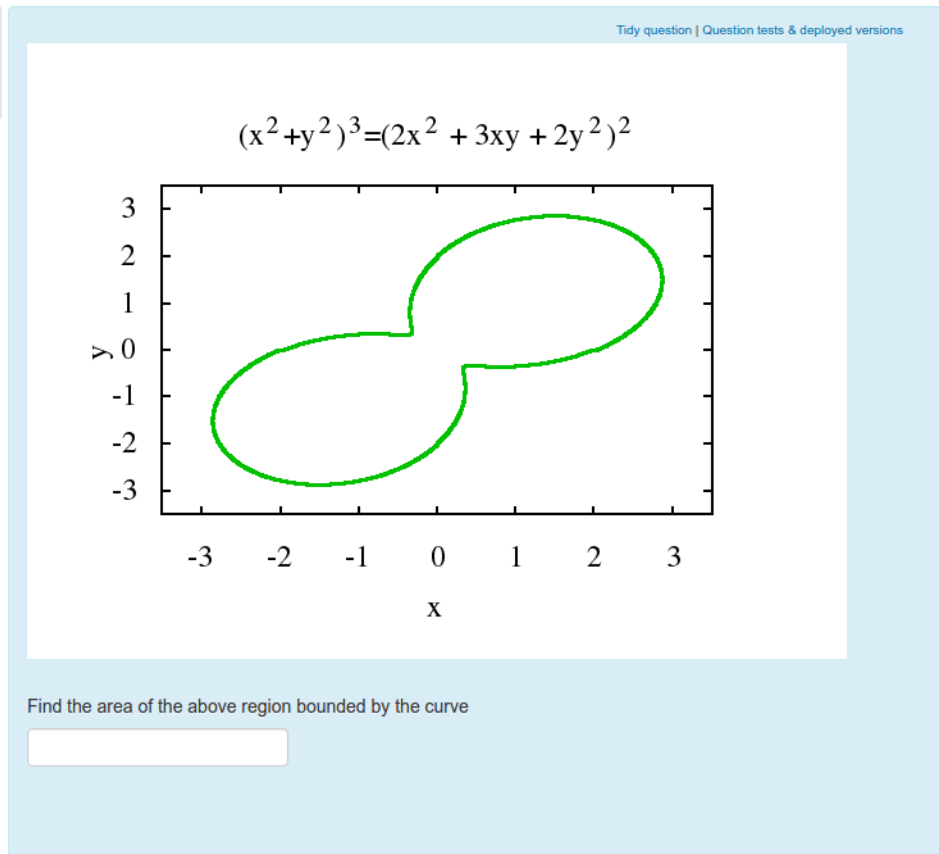
The plot shown in Figure 3 is expected to help students with the calculation of the area.

Preview question: Example of the question with implicit plotting

Question 1

Not yet answered

Marked out of 1.00



Start again

Save

Fill in correct responses

Submit and finish

Close preview

Figure 3: A question with the graph of $(x^2 + y^2)^3 = (2x^2 + 3xy + 2y^2)^2$.

The second example is shown in Figure 4. The question asks the student to find the coordinates (x, y) to minimize the function

$$f(x, y) = x^4 + 2x^2y^2 + y^4 - 7x^2 - 2\sqrt{3}xy - 5y^2 + 1.$$

The ordinary procedure to find the answer to the question is the following:

- (1) express the function in polar coordinates r, θ .
- (2) differentiate the function by r and θ respectively, and set the expressions to be 0.
- (3) solve the simultaneous equations of (2) and find the answer.

There are two points to minimize the function and the correct answer is $(x, y) = (\sqrt{3}, 1)$, $(-\sqrt{3}, -1)$. In finding the correct answer among the solution of the equations of (2), the

contour map of the function $f(x, y)$, apart from that in Figure 4, is also helpful. We display this map in the example of the feedback of Figure 5 . In this feedback, which is displayed when the answer is incorrect, we explain the ordinary procedure to find the answer and let the student find in which step he/she made a mistake.

4 Summary

In this paper, we explained our approach to building an enhanced plotting environment for STACK, such as drawing graphs of implicit functions and surfaces described by functions consisting of two variables. This enhancement is achieved by creating the Maxima functions, which produce the appropriate scripts of gnuplot and then call gnuplot with these scripts. This approach enables us to make full use of the power of gnuplot, and allows us to apply it to any plotting software other than gnuplot, such as matplotlib and Pyxplot. Two examples of questions with enhanced plots were presented. In one of these two examples, we showed that enhanced plots can also be used in the feedback provided by question authors.

In the process of generating the gnuplot script, one has to pay attention to the difference between the mathematical expressions of Maxima and those of gnuplot. This difference means that it is necessary to convert one expression to the corresponding one in the other system. The origin of this conversion lies in the fact that gnuplot has to be called through Maxima. If gnuplot can be called directly by circumventing Maxima, there is no need for conversion, which is more favorable.

Teachers may use STACK when giving their students homework that requires them to solve problems involving differentiation, integration, and so on. We feel that a teacher who carefully designs questions and feedback with the use of the enhanced plotting environment would be able to generate significant educational benefit.

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References

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- [2] Sangwin, C., Computer Aided Assessment of Mathematics, Oxford University Press, 2013.
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Preview question: Example of the question with surface plotting

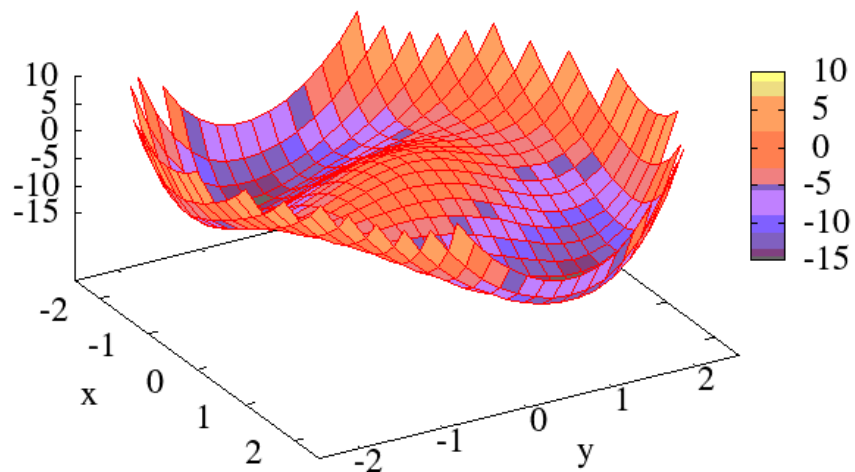
Question 1

Not yet answered

Marked out of 1.00

Tidy question | Question tests & deployed versions

$$f(x, y) = x^4 + 2x^2y^2 + y^4 - 7x^2 - 2\sqrt{3}xy - 5y^2 + 1$$



Find the coordinates to minimize the function

$$f(x, y) = x^4 + 2x^2y^2 + y^4 - 7x^2 - 2\sqrt{3}xy - 5y^2 + 1$$

Figure 4: Question relating to the graph of $f(x, y) = x^4 + 2x^2y^2 + y^4 - 7x^2 - 2\sqrt{3}xy - 5y^2 + 1$.

Your last answer was interpreted as follows:
 $[-\sqrt{3}, -1]$

Your last answer was interpreted as follows:
 $[\sqrt{3}, -1]$

The entries underlined in red below are those that are incorrect.

$[[-\sqrt{3}, -1], [\sqrt{3}, \underline{-1}]]$

Your answer is incorrect.

The ordinary procedure to find the answer of the question is following:

1. express the function in a polar coordinates r, θ .
2. differentiate the function by r and θ respectively, and set the expressions to be 0.
3. solve the simultaneous equations of (2) and find the answer.

See the contour map of the function $f(x, y)$ below.

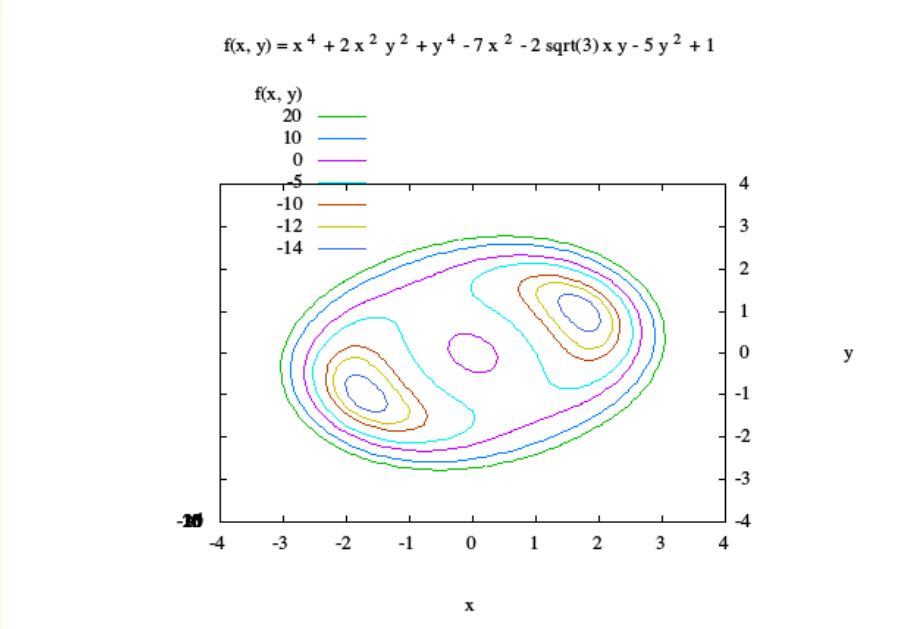


Figure 5: Example of the feedback with the contour map of $f(x, y)$.