Using Dynamic Geometry Software to Enhance Student Understanding of the Concept of Speed

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Abstract
The authors of this paper have conceptualized and constructed a tool, using dynamic geometry software, to simulate scenarios of word problems involving speed, and used it to help students acquire the concept of speed and solve speed problems involving two objects moving toward each other. This paper demonstrates how such a tool can be created using GeoGebra, and describes how the use of this tool had improved understanding of speed concept, and ability to solve speed problems for a class of sixth grade students in Singapore.

1. Introduction

The application of the formula "distance traveled = speed × traveling time" is a common topic in mathematics and science curricula in many countries. In Singapore, this formula is first introduced at the upper primary school level (sixth grade). Due to the "spiral approach" of Singapore's mathematics curriculum, students have another opportunity to learn this topic in the secondary school mathematics syllabus (seventh to ninth grade), which includes speed–time and distance–time graphs (see [4] and [5]). In comparison, in the United States, mathematical concepts pertaining to speed are covered by the ninth grade. As speed involves a proportional relationship, and proportional reasoning is at the heart of middle grade mathematics, we believe that teaching of concepts involving speed is also relevant to middle school educators.

Mathematics problems involving speed are usually put in the context of real-world situations to render them more familiar and authentic to students. However, evidence gathered from classroom observations by us, the authors, suggests that students are in general not confident and successful in solving these problems. This raises a very natural question for teachers who are concerned about the performance of students in solving problems involving speed: *What pedagogical approach could lead students to construct effective strategies for solving problems involving speed which are put in the context of real-world situations?* We suggest the use of technology to address the above-mentioned concern.

Advances in information and communications technology and its wide-spread accessibility for teachers and students have affected the roles played by a teacher in the classroom and the way in which students learn, collaborate and construct their knowledge. New pedagogy that uses or integrates technology has become a key approach for engaging students in learning. In particular, in the teaching and learning of mathematics, dynamic geometry software allows teachers to craft problem situations that can be modeled, manipulated and reasoned through with students in class.

GeoGebra is a free dynamic geometry software that has been gaining popularity and becoming wide-spread in both secondary and primary school mathematics in many countries. Being user-friendly freeware and having a very large community of users contributing teaching and learning resources via online learning platforms such as GeoGebraTube, GeoGebra has gradually become one of the "must-have" software tools for mathematics teachers. However, anecdotal evidence from classroom observations in Singapore has shown that despite an increasing number of secondary mathematics teachers developing resources for GeoGebra and using them in classroom teaching, GeoGebra is rarely used at the primary school level.

One objective of this paper is thus to share with teachers how a GeoGebra construction, conceptualized and constructed by us, the authors, can be used to help students to acquire the concept of speed and then use it to solve speed problems involving two objects moving toward each other. We shall also demonstrate how the use of this technology had improved student understanding of speed concept for a class of sixth grade students in Singapore.

2. GeoGebra Construction for Two Objects Moving in Opposite Directions

Problem scenarios involving one or two moving objects are commonly used to study rate and speed in mathematics and science. Instructional pedagogy based on simulation provides students with useful visualizations, enabling them to observe how objects move toward or away from each other —this observation is useful for the students to understand the various concepts involved in solving speed problems, such as the "chasing" problem, in which one moving object overtakes another, or the "meeting" of two objects moving in opposite directions. In this section, we briefly describe three key GeoGebra ideas we use for constructing a simulation of two cars moving in opposite directions. The ideas may be useful to teachers who wish to construct animations for teaching similar or related mathematical concepts. Figure 1 shows the completed GeoGebra construction, which is made available by us, the authors, at GeoGebraTube (http://www.geogebratube.org/student/mK6mLTbku).

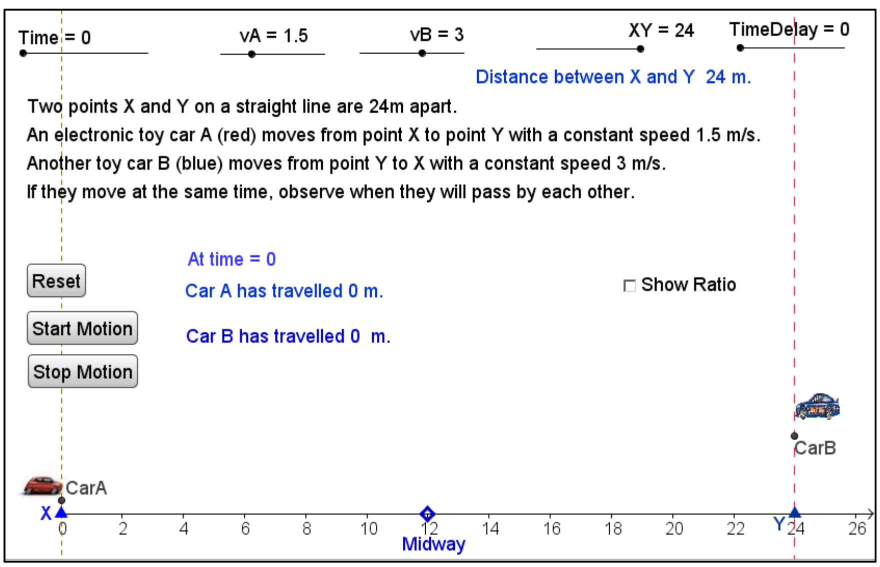

Figure 1: The opening screen of the GeoGebra construction.

Idea 1: Suitable pictures of toy cars can be inserted into a GeoGebra page. These pictures can be made to "move" according to the coordinates of their corner points. In GeoGebra, the size and position of a picture is determined by three corner points, as shown in Figure 2 for car A.

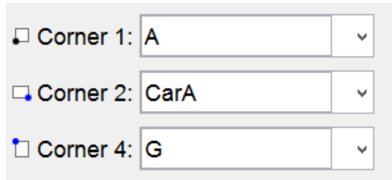

Figure 2: The Three Corner Points Determining the Position and Size of a Picture.

For car A in Figure 1 (which moves from left to right), the corner point labeled car A has the coordinates (Time × vA, 0.2) where Time and vA are the traveling time and the speed of car A respectively. To avoid having too many labels on the page, the corner points A and G, with the coordinates A = (x(B) – 1.5, 0.2) and G = (x(A), 0.8), are hidden. Note that x(A) is a GeoGebra command that returns the *x*-coordinate of point A. Similarly, we use the Cartesian coordinates to position car B according to the distance it has moved.

Idea 2: One way to make an object move in GeoGebra is by using a slider. We create a slider (named Time) to simulate the traveling time (in seconds) and to change the coordinates of the two cars, thus creating a moving effect. The maximum value of this slider is set to the time taken for the slower car to complete the full journey given by the initial distance between the cars (given by the XY slider). Three other sliders, vA, vB and TimeDelay, allow us to change the speed of car A, the speed of car B and the time before car B begins its motion, respectively, giving students the freedom to change the parameters of the problem.

Idea 3: An animation button named "Start Motion" allows the movement to be more natural and authentic. Clicking on it executes the command line StartAnimation(Time), which needs to have been entered under "Scripting and On Click." When the "Stop Motion" button is clicked, the command line StartAnimation[Time, false] allows students to pause the motion of the cars. The "Reset" button moves the two cars back to their original positions by having Time = 0 as a common line. The students can also observe the cars in "slow motion" by dragging the Time slider manually, allowing them to take note of the respective distances the cars have traveled; these distances and the time elapsed are displayed on the GeoGebra page for the students to record on an accompanying worksheet (see Appendix).

3. Classroom Implementation

In a recent study conducted by the first author of this paper on a primary school class of 38 Grade 6 students taught by an experienced teacher, Lynette, only three students were able to solve the following problem correctly after the class of 38 students had learnt this topic by direct instruction.

"Car A was traveling from Town X to Town Y at a constant speed of 50 km/h and at the same time Car B was traveling from Town Y to Town X at a constant speed of 58 km/h. They met at a place which was 20 km away from the midpoint of Town X and Town Y. Find the distance between Town X and Town Y."

(Here we use the abbreviations km and h for the units of measurement kilometer and hour respectively.) Note that the distance traveled by car B (the faster vehicle) was 40 km more than that traveled by car A in the same time, and the difference in speed between car A and car B is 58 – 50 = 8 km/h. As they began traveling at the same time, both of them should have traveled for 40 ÷ 8 = 5 h before they met. Thus, the distance between the two towns is 5 × (58 + 50) = 540 km.

The figures below show the responses of two students who did not solve the problem correctly.

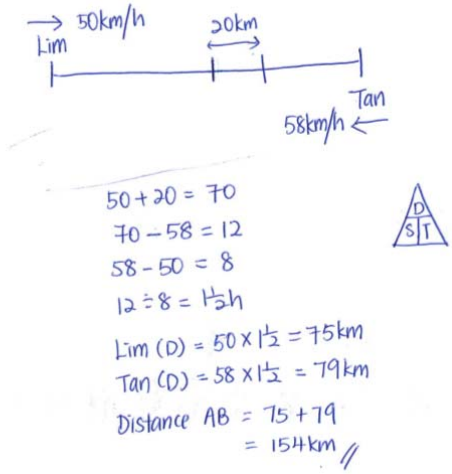

Figure 3: June's Solution

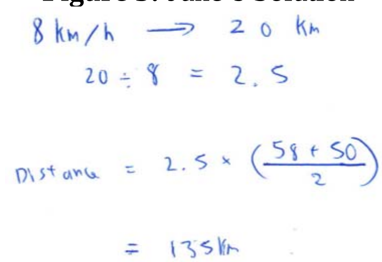

Figure 4: Sam's Solution

One of the errors in June's solution is that the location where the two cars met was wrongly identified to be nearer Town Y than Town X (see Figure 3) whereas the main error in Sam's solution is that the difference in the distances traveled by the two cars was wrongly identified to be 20 km; car A traveled 40 km more than car B (see Figure 4).

The GeoGebra construction which we, the authors, had constructed was then used by teacher Lynette in an attempt to help the class of sixth grade students, hereafter the participating students, achieve conceptual understanding in solving speed problems. To assist student learning, Lynette's instructional sequence began with the scenario in which both cars are traveling at the same speed. Although the outcome of this scenario should be intuitive for many students — the two cars will pass by (or "meet") each other at the midway point — the participating students were directed to observe how the distance traveled by the two cars changes with respect to time before and at the moment of meeting each other and, most importantly, to observe that the total distance traveled by the two cars is equal to the distance they are apart initially.

To verify and reinforce the participating students' initial understanding of the condition for the two cars to "meet," the students were guided to explore a few speeds, for example by setting a new speed for car B, say vB = 2.5 m/s. (Here we use the abbreviations m and s for the units of measurement meter and second respectively.) They were instructed to stop the motion at the moment when two cars were passing each other and record the distance traveled by both cars. This had helped them to discover that at the instant when the cars "meet," the total distance traveled by them again adds up to the distance between the two starting points.

The TimeDelay slider provided the participating students with a flexible "tool" for extending the scope of the original question by making car B begin its motion later. The GeoGebra construction was used to show them that if car B starts 1 s later, the two cars "meet" when car A has traveled 9 m and car B has traveled 15 m, giving a total distance traveled of 24 m. By varying the speeds of the cars and the value of the time delay, the participating students were able to visualize the condition for the two cars to meet when traveling in opposite directions, regardless of whether they begin their motion at the same time or not.

An effective instructional practice is to provide opportunities for students to learn beyond the boundaries of a problem. Using this GeoGebra simulation and an accompanying worksheet (see Appendix), other teaching points were delivered to the students through a self-discovery approach. For example, for the case of both cars beginning to move at the same time (slider TimeDelay = 0), the participating students were asked to investigate whether a relationship exists between where (in terms of the distance from the midway point) the cars "meet" and the distance they have traveled. The GeoGebra construction was used to show that the cars "meet" at a point 3 m away from the midpoint, which is half the difference between the distances they have traveled (15 – 9 = 6 m). Is this always the case? Through further exploration the participating students were able to ascertain that this was indeed true. If the lesson were conducted in a non-ICT environment, teachers may either fail to deliver this teaching point or find it hard to illustrate it clearly.

Students' learning can be further consolidated if they are given opportunities to apply what they have learnt. Thus, the accompanying worksheet includes problems for the students to solve without using the GeoGebra construction. For example, the participating students were asked to solve the following problem.

"Two points X and Y are 24 m apart. Car A starts its motion from point X to Y with a constant speed of 1.5 m/s and car B starts its motion from Y to X a few seconds later but with a faster speed of 3 m/s. If both cars "meet" at a point which is 11 m away from Point X, how many seconds later than car A will car B begin its motion?"

The participating students were able to understand that the total distance traveled by both cars when they meet is 24 m. Car A has traveled 11 m, so car B must have traveled 13 m. Thus, car A has traveled for $11 \div 1.5 = 7\frac{1}{3}$ s before meeting car B, which has traveled for $13 \div 3 = 4\frac{1}{3}$ s. Thus, car B begins its motion $7\frac{1}{3} - 4\frac{1}{3} = 3$ s later than car A. To encourage self-directed learning, we allowed the students to verify the answer using the GeoGebra construction. With the aid of the GeoGebra construction, the participating students were convinced by seeing the "live motion" of the two cars approaching and meeting each other.

Although students in Singapore may have learnt and applied the formula "distance = speed × time" to solve word problems involving constant speed at the upper primary and lower secondary levels, they tend to overlook a useful concept "hidden" in the above formula: if two moving objects travel for the same amount of time, the ratio of the distances traveled by these objects is equal to the ratio of their speeds. To avoid having too much information on the GeoGebra page, a checkbox named "Show Ratio" is created. If this box is checked, the ratio of the distances traveled by the two cars will be shown as the Time slider is manually dragged. For the case of both cars beginning to move at the same time, this ratio remains unchanged for as long as they are in motion. By changing the speed of either car, the participating students discovered that the ratio of the distances traveled by the two cars is the same as the ratio of their speeds.

Last but not least, the participating students were asked to find the additional time required for car A to reach its destination (point Y) when the faster car (car B) has reached point X. The visualization and the information shown on the GeoGebra page readily provided the students with the answer, which is given by $\frac{24-14.4}{1.8} = 5\frac{1}{3}$ s. However, to encourage the participating students to think and reason, they were challenged to explain how the difference in the distance traveled has arisen. The aim was for the participating students to explain that for every 1 s, car A travels 3 – 1.8 = 1.2 m less than car B. As car B travels a total of 24 m in 8 s, car A has another 8 × 1.2 = 9.6 m to travel to reach its destination, which takes it a further $\frac{9.6}{1.8} = 5\frac{1}{3}$ s.

The classroom implementation of the use of our GeoGebra construction as described above took about an hour to complete, after which the participating students were then asked to solve the initial problem again. Thirty-two out of 38 students were able to obtain the correct answer. June and Sam were among those who could obtain the correct answer and they expressed that they were able to solve the problem correctly because the GeoGebra animation had helped them develop a deeper understanding of speed concepts.

4. Extension to Secondary School Mathematics

GeoGebra provides two Graphic View screens. The additional screen, named Graphics 2, allows the motion of the two cars to be presented on a distance (from a fixed point) versus time graph. In the Singapore mathematics syllabus for secondary school students, graphical representations and interpretations of motion in a straight line are explored in the sub-topic "Application of Graphs in Practical Situations." Student misconceptions arising from the gradient and the shape of a distance–time curve have been reported in a number of studies (see [1], [2] and [3]). We believe that a "real-time" distance–time graph sketched concurrently with the motion of the cars will help to dispel such misconceptions. For example, the students will be able to see how the two straight lines are traced out in real time in relation to the motion of the two cars (Figure 5).

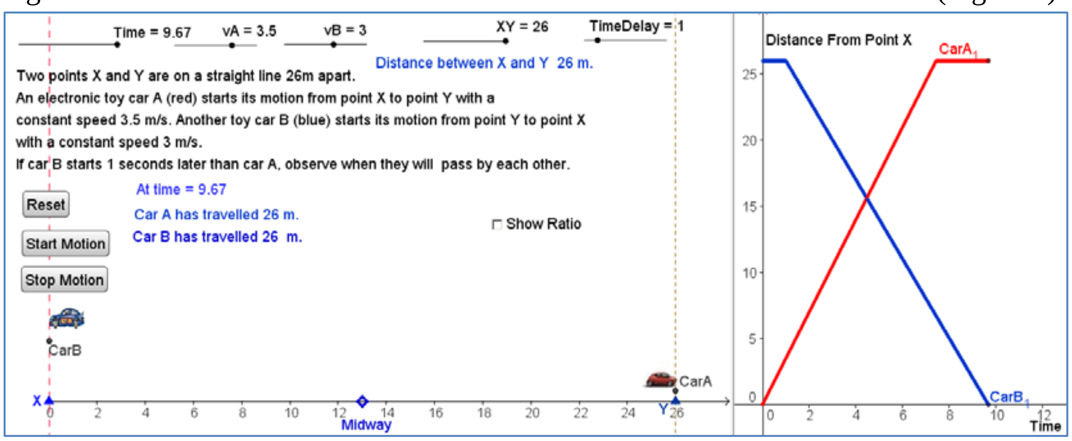

Figure 5: Graphical representation of the motion of two cars.

The downward-sloping line, which represents the motion of car B, will help the students to understand that the car is moving toward point X and not moving down a slope or having a " negative speed," reinforcing the concept that in this case the negative sign of the gradient refers to the direction of motion. In addition, the students can be directed to observe how the "flat" portion of the downward-sloping line arises from the motion of the car.

5. Conclusion

In this paper, we demonstrate how a GeoGebra construction can be used to simulate the motion of two cars moving toward each other. We also illustrate how the functionalities of GeoGebra provide opportunities to extend students' learning through varying the parameters associated with this type of problem. The animated and interactive GeoGebra constructions, supported by the display of key information on the GeoGebra page, allow the students to visualize the relative positions of the two cars during the journey, giving them a complete picture of how these two cars are moving toward each other. Appropriate use of technology allows teachers to focus more on facilitating student learning and less on a teacher-centered approach. We believe that an exploratory student-centered pedagogy based on appropriate technology is useful for engaging students and empowering them to be self-directed learners.

Development of proportional thinking and reasoning is a cornerstone of middle school mathematics and we have illustrated in this paper that dynamic geometry software such as GeoGebra could potentially assist middle school practitioners in teaching the concept of speed which involves a proportional or multiplicative relationship. We suggest that middle school mathematics teachers explore how the functionalities, features and strengths of GeoGebra could be used to facilitate student learning of ratio, rate and speed, all of which involve proportional thinking and reasoning.

References

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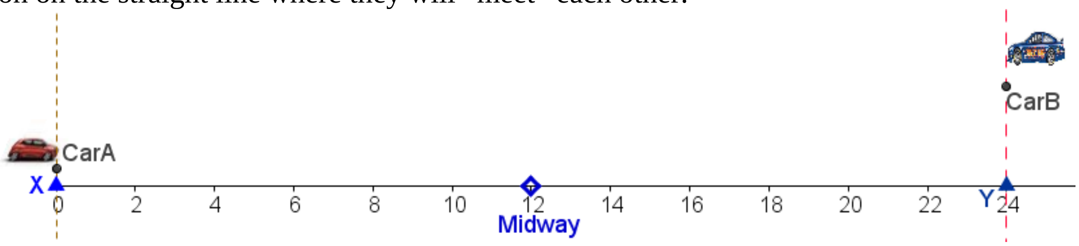
[5] Ministry of Education, Singapore. (2013). Primary mathematics teaching and learning syllabus. Retrieved July 23, 2015, from http://www.moe.edu.sg/education/syllabuses/sciences/files/maths-primary-2013.pdf

Appendix

Speed Problems Involving Two Objects Moving in Opposite Directions

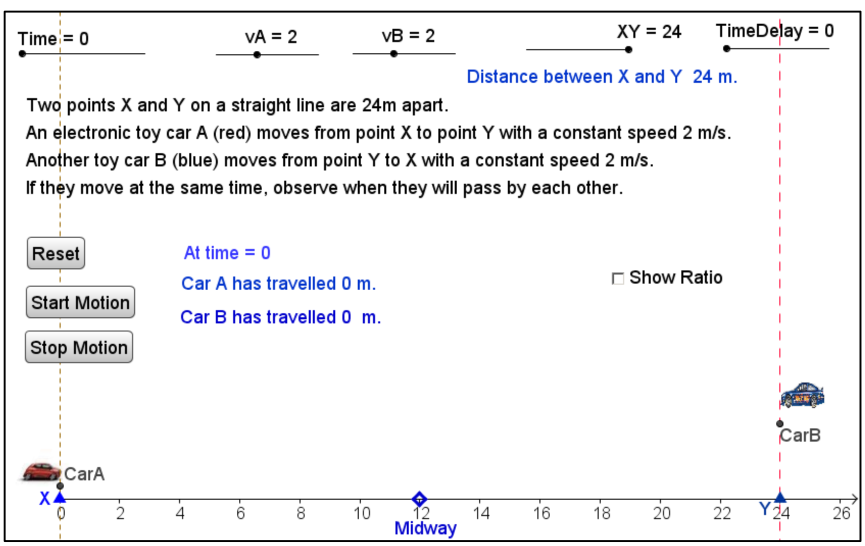
Activity 1

(a) Two points X and Y on a straight line are 24 m apart.
An electronic toy car A (red) moves from point X to point Y at a constant speed of 2 m/s when another toy car B (blue) moves from point Y to X at the same constant speed of 2 m/s. Find the location on the straight line where they will "meet" each other.



Answer:
If the cars start their motion at the same time and move at the same speed, they will "meet" _______ m from X.

(b) Open the file Speed.ggb
You will see a screen as follows. The values of *vA* and *vB* are the speeds of the toy cars A and B respectively.



General Instructions:

(1) You can either use the *Start Motion* and *Stop Motion* buttons or the *Time* slider to find out when and where the two cars "meet".

(2) Use the sliders *vA* and *vB* to vary the speeds of the cars, the slider *XY* to vary the distance between X and Y, and the slider *TimeDelay* to change the time at which car B begins to move.
Complete the table below at the moment <u>when the two cars "meet"</u>.

Time Delay (s)	vA (Speed of Car A) m/s	vB (Speed of Car B) m/s	Distance XY (m)	Distance Travelled by Car A (m)	Distance Travelled by Car B (m)

My Observation:
When the two cars meet (whether or not they start at the same time), I observe that ____

Exercise (Applying what you have learnt)
1. Two points X and Y are 24 m apart. Car A starts its motion from points X to Y at a constant speed of 1.5 m/s. Car B starts its motion from Y to X a few seconds later at a higher speed of 3 m/s. If the cars "meet" at a point which is 11 m away from Point X, how many seconds later than Car A will Car B begin its motion? Show your solution clearly and use the GeoGebra file to verify your answer.

2. The distance between Town A and Town B was 450 km. A car started from Town A and travelled towards Town B at an average speed of 45 km/h. At the same time, a bus started from Town B and travelled towards Town A at a speed of 30 km/h. What distance had they each travelled when they passed each other on the way?

Activity 2:
Instruction: Set *TimeDelay* = 0
Complete the table below at the moment when the two cars "meet".

vA (Speed of Car A) m/s	vB (Speed of Car B) m/s	Distance XY (m)	Distance Travelled by Car A (m)	Distance Travelled by Car B (m)	Distance from the Midway Point (m)

My Observation:
If the two cars start at the same time and when they meet, the distance of their meeting point (away from the midway point) is equal to _______________ m.

Exercise (Apply what you have learnt)
Toy car A starts its motion and moves from point X to point Y at a constant speed of 4.5 m/s when toy car B starts its motion and moves from Y to X at a constant speed of 3 m/s. If the two toy cars "meet" at a point which is 2.6 m away from the midway point between X and Y, find the distance between X and Y? Show your solution clearly and use the GeoGebra file to verify your answer.

<u>Activity 3</u>
Instructions:
(a) Set the sliders *TimeDelay* = 0, *vA* = 1.5 and *vB* = 4.5
(b) Click the *Show Ratio Box*. Drag the slider *Time* manually. Observe the ratio of the distance travelled by Car A and Car B.
(c) Complete the table below as you drag the *Time* slider.

Time Delay (s)	***vA* (Speed of Car A) m/s**	***vB* (Speed of Car B) m/s**	**Speed Ratio *vA* : *vB***	**Time (Value of the Time Slider) (s)**	**Distance Travelled by Car A (m)**	**Distance Travelled by Car B (m)**	**Ratio of Distance Travelled by Car A to that by Car B**

My Conclusion: