

# Technology in the Mathematics Classroom: A Perspective from India

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**Abstract:** This paper highlights the role of technology in mathematics education with examples from the Indian context. Mathematics forms the foundation of education in Indian society and is a compulsory subject up to grade 10. The last few years have witnessed extensive use of computer technology in school education. However in the vast majority of Indian classrooms, mathematics is taught in the traditional “chalk and board” manner with hardly any use of technology and the student has little opportunity for visualization and exploration. The National Curriculum Framework of India (NCF, 2005) in its position paper *Teaching of Mathematics* recommends the use of technology in the form of computer software and calculators. It states “Technology can greatly aid the process of mathematical explorations,....If ordinary calculators can offer such possibilities, the potential of graphing calculators and computers for mathematical exploration is far higher”. The recommendations of the NCF, has been a motivating factor for conducting studies related to the use of technology in the mathematics classroom. In this paper, we shall describe two research studies conducted with students in higher secondary schools where technology in the form of graphics calculators and spreadsheets played a key role in developing their mathematical thinking. Through these studies, we will illustrate that the considered use of technology in a traditional mathematics classroom can help to create a rich and motivating environment for exploring mathematical ideas.

## 1. Introduction

The last few decades have witnessed a major shift of paradigm as far as mathematics teaching and learning is concerned, in many countries across the world. Mathematics education is being revolutionized with the advent of new and powerful technological tools in the form of dynamic geometry software (DGS), computer algebra systems (CAS), spreadsheets and graphic calculators which enable students to focus on exploring, conjecturing, reasoning and problem solving and not be weighed down by rote memorization of procedures, paper-pencil-drills and symbol manipulation. Some research studies have shown that the appropriate use of technology can lead to better conceptual understanding and make higher level processes accessible to students without compromising on paper – pencil skills. One of the first leading studies in the use of technology in mathematics education is the one reported by Heid [2] in which computer algebra and some graphing tools were integrated into a first year university calculus course. The technology provided scope for a “concept first” approach, in which, calculus concepts were extensively taught using the tools and computational aspects were dealt with at the end of the course. The results showed that technology helped to develop conceptual understanding without hampering procedural skills. In another study, Heid [3] describes CAS as a cognitive technology which makes higher level mathematical processes accessible to students. Kissane [4] describes some “pedagogical affordances” of the graphics calculator in visualizing and exploring concepts in calculus. According to him, “procedural competence can be best developed when students understand the underlying ideas well” (p. 16). Laborde [5] suggests that learning geometry with a Dynamic Geometry Environment (DGE) might offer students possibilities to construct and manipulate geometrical figures and do empirical investigations. Leung [6,7] describes that “a key feature of DGE is its ability to visually represent geometrical invariants amidst simultaneous variations induced by

dragging activities” (p.2). Thus mathematics education researchers across the world have conducted many studies which support the use of technology in mathematics teaching and learning.

While many countries are increasingly using technology for mathematics instruction, in the vast majority of schools in India, mathematics continues to be taught in the traditional 'chalk and board' manner. Also very few research studies have been conducted to study the impact of integration of technology for mathematics teaching and learning. In this paper, we shall highlight the opportunities for using technology in mathematics classrooms in India by describing two research studies. In section 2 we shall create a context for these studies by highlighting some aspects of the senior secondary mathematics curriculum in India. Section 3 will describe a module in which grade 12 students used graphics calculators and spreadsheets to explore the topic matrices and determinants through mathematical models and applications. Section 4 will focus on the role of graphics calculators in enabling grade 11 students to explore various topics of their curriculum using a multi-representational approach.

## 2. The Senior Secondary Mathematics Curriculum in India

The senior secondary stage is a significant landmark in the life of a student. In fact the National Curriculum Framework (NCF) 2005 in its position paper on the 'Teaching of Mathematics' (National Council for Educational Research and Training [NCERT], [8]) describes the higher secondary stage as the “launching pad from which the student is guided towards career choices.” At this stage the student has to make a choice between the science, commerce or humanities stream and has to appear for the school leaving examination (known as the board examination) at the end of grade 12. This is a high stakes examination as her performance in this examination holds the key to her future in terms of career opportunities. Thus mathematics teaching in grades 11 and 12 is largely driven by preparation for the board examination. Topics are taught in the traditional 'chalk and board' manner, without any substantial reference to their applications to real life problems. The mastery of procedural knowledge is emphasised leaving very little opportunity for visualization, exploration or discovery. Though the last few years have witnessed extensive use of computer technology in school education, there is hardly any use of technology for teaching and learning of mathematics. Technology, if used for teaching mathematics, is primarily for demonstration purposes and does not involve the student actively.

The position paper on *teaching of mathematics* (NCF 2005) [8], recommends that mathematics teaching at all levels be made more 'activity oriented'. While this has been the primary driving force for revisiting and revamping the elementary school mathematics curriculum, it has had little impact on the senior secondary curriculum. At this level, the textbooks, which happen to be the only resource for teachers and students, are usually examination oriented and mathematics teaching is very pedantic. Topics such as Sets, Relations, Logic, Sequences and series, Linear inequalities, Combinatorics, Conic sections and Complex numbers are introduced in grade 11. The curriculum in grade 12 is dominated by differential and integral calculus accounting for almost half of the content. Other topics in grade 12 include Matrices and Determinants, Vector Algebra, Three Dimensional Geometry, Linear Programming and Probability. Manipulative and computational aspects of these topics, rather than visualization and exploration of concepts and ideas, dominate mathematics at this stage. To highlight this point here are two sample questions from the topic of Differentiation in the text book of grade 12.

1. If  $y = (\tan^{-1} x)^2$  then show that  $(x^2+1)^2 \frac{d^2y}{dx^2} + 2x(x^2+1) \frac{dy}{dx} = 2$

2. Differentiate  $\log \sqrt{\frac{1+\cos^2 x}{1-e^{2x}}}$  with respect to  $x$ .

As is evident, the emphasis is on developing procedural knowledge through drill and practice and on learning ‘tricks’ to solve problems. The NCF 2005 strongly recommends the inclusion of modelling activities in the curriculum and suggests that this can be done in the form of investigatory exercises or projects which will enable the student to see the relevance of the mathematics taught at school. Another important recommendation of the NCF 2005 is that of making a shift from ‘content to processes’. It states that “content areas of mathematics addressed in our schools do offer a solid foundation.....What can be levelled as major criticism against our extant curriculum and pedagogy is its failure with regard to mathematical processes.....formal problem solving, use of heuristics, estimation and approximation, optimisation, use of patterns, visualisation, representation, reasoning and proof, making connections, mathematical communication.....”. In order to achieve this shift, especially at the senior secondary level, the NCF also advocates the use of technology in the form of computer software and calculators, “Technology can greatly aid the process of mathematical explorations,....If ordinary calculators can offer such possibilities, the potential of graphing calculators and computers for mathematical exploration is far higher”. The recommendations of the NCF, has been a motivating factor for conducting the two research studies related to the use of technology in the mathematics classroom which are described in this paper.

### 3. Research Study I: The module on application of matrices

In this section we shall describe a study which aimed to explore how learning is impacted by integrating mathematical modelling and applications into the senior secondary mathematics curriculum. In this study, 30 grade 12 students selected from two schools in Delhi explored the topic *Matrices and Determinants* through mathematical models and applications. The module was spread over eight sessions of approximately two hours each, conducted on four school days. Graphics calculators and MS Excel were used as the vehicles of exploration by students. Worksheets were provided where they had to record their paper – pencil work and write their findings and observations. At the end of the module they were required to respond to a short questionnaire and give a written feedback describing their learning experience in the module. The objective of the study was to reflect upon the following research questions

- (i) How does the integration of applications and modelling activities impact student’s understanding of mathematical concepts?
- (ii) Does introducing mathematical modelling activities, enabled by technology, help students access higher level mathematical concepts?

#### **Educational Setting**

All 30 students who were a part of the study were from two Delhi schools which follow the curriculum prescribed by the Central Board of Secondary Education (CBSE), a national board for school education in India. The CBSE (CBSE, 2009) does not prescribe the use of technology for teaching mathematics or permit its use in examinations. Individual schools however have the freedom to integrate technology in their classrooms. The topic, *Matrices and Determinants* comprises of the subtopics on types of matrices, matrix operations, inverse of a matrix, computing

determinants and their properties and solutions of systems of equations. The emphasis is on developing computational skills in dealing with matrices and determinants. Two sample exercises from the textbook are

1. If  $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$  and  $f(x) = 2x^3 + 4x + 5$ , find  $f(A)$ .

2. Using properties of determinants prove that

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$$

The 30 students who were a part of the study had been taught matrices in a traditional manner in their regular class. However the manner in which the content was presented did not focus on the need and use of matrices in solving or modelling practical problems. In the module designed by the author (who was also their teacher), the emphasis was on integrating applications and mathematical models based on matrices to introduce concepts and procedures thereby highlighting the relevance of matrices as a tool for solving problems. The mathematical models were taken from varied topics such as Genetics, Forest management, Consumer behaviour and Cryptography. In all the sessions students were given worksheets which required them to explore a problem (mathematical model) and record their solutions or observations. They were given access to graphics calculators and MS Excel for computational work. Prior to the module, a short workshop session was conducted to familiarize the students with performing matrix algebra on the on the graphics calculator (Casio fx-CG 20) and on MS Excel. In the rest of this section we shall describe a part of the module where students explored an application of matrices to cryptography, namely the Hill cipher method. While exploring the method, students were encouraged to do certain procedures, such as, finding the product and determinants of  $2 \times 2$  and  $3 \times 3$  matrices by hand and only use technology for higher order matrices. The objective was to maintain a balance between technology enabled explorations and procedural skills.

Even though the students were familiar with the concept of matrices and basic matrix algebra, they could not cite any practical use of matrices. They described a matrix as a ‘rectangular arrangement of numbers in rows and columns’ and said that the utility of matrices had not been discussed in their regular class. This prompted the author to focus the first two sessions of the module on the use of matrices in modelling practical problems. Examples such as ranking teams in a tournament and modelling consumer purchasing behaviour using transition matrices were introduced. Matrix operations of addition, subtraction, scalar multiplication and multiplication were used to analyse the models in the third session. This created a context for using matrices in real life problems. In the fourth session the concept of inverse of a matrix was recapitulated. Students were aware of the methods for finding the inverse of  $2 \times 2$  and  $3 \times 3$  matrices. For example, a majority of

the students knew that if  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  then  $A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ . They had memorised the rule but were unable to justify it. So the author asked them to find the formula for the inverse by solving the matrix equation  $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , where  $x, y, z$  and  $w$  were to be expressed in terms of  $a, b, c$  and  $d$ .

This task led to much excitement as most students obtained the formula by solving the two pairs of linear equations, namely,  $ax + bz = 1$ ,  $cx + dz = 0$  and  $ay + bw = 0$ ,  $cy + dw = 1$ .

### The Hill Cipher

Many students were curious as to whether there is any practical use of the inverse of a matrix. At this point the author decided to introduce them to the Hill Cipher method (Eisenberg, 1999), an application of matrices to cryptography. Students were familiarised with some preliminary information related to ciphers. Ciphers are methods for transforming a given message, the plaintext, into a new set of characters which is unintelligible to anyone who does not know the key (the transformation used to convert the plaintext), known as the ciphertext. The secret rule, that is, the inverse key, is required to reverse the transformation in order to recover the original message. After this, students were familiarised with the following preliminary definitions related to modular arithmetic (a topic which is not a part of the syllabus).

**Definition 1:** Given an integer  $m > 1$ , called the *modulus*, the two integers  $a$  and  $b$  are said to be *congruent* to one another *modulo*  $m$  if the difference  $a - b$  is an integral multiple of  $m$  or  $a = b + km$  for some integer  $k$  (positive, negative or zero).

**Definition 2:** Let  $m$  be an integer with  $m > 1$ . For an arbitrary integer  $a$ , the *residue of  $a$  modulo  $m$*  is the unique integer  $r$  among  $0, 1, 2, \dots, m - 1$  to which  $a$  is congruent modulo  $m$ .

Thus  $23 \equiv 7 \pmod{8}$ , since  $23 = 2 \times 8 + 7$ . Here  $a = 23$ ,  $m = 8$  and  $r = 7$ . Also  $-18 = -3 \times 8 + 6$ . Thus  $r = 6$  and we can write  $-18 \equiv 6 \pmod{8}$ .

Students were given a worksheet to familiarize them with computations related to the above definitions. After this they were then introduced to the Hill Cipher method and were given 29 characters along with their numerical values as shown in Table 1.

Table 1. A substitution table of 29 characters for the Hill Cipher Method

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
P	Q	R	S	T	U	V	W	X	Y	Z	.	-	?	
15	16	17	18	19	20	21	22	23	24	25	26	27	28	

The encoding matrix or key was chosen as  $E = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$ . Students were then introduced to the encoding process (converting the plaintext to the ciphertext) through the following steps.

Step 1. Let the plaintext message be **HOW\_R\_U?**

Step 2. Convert it to its substitution values from the substitution table and group them in pairs

7 14 22 27 17 27 20 28

Each pair will form a column of the message matrix  $M$ .

Step 3. Compute the product  $EM$  and reduce it modulo 29.

$$EM = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} 7 & 22 & 17 & 20 \\ 14 & 27 & 27 & 28 \end{bmatrix} = \begin{bmatrix} 35 & 76 & 71 & 76 \\ 119 & 255 & 240 & 256 \end{bmatrix} = \begin{bmatrix} 6 & 18 & 13 & 18 \\ 3 & 23 & 8 & 24 \end{bmatrix} \pmod{29}$$

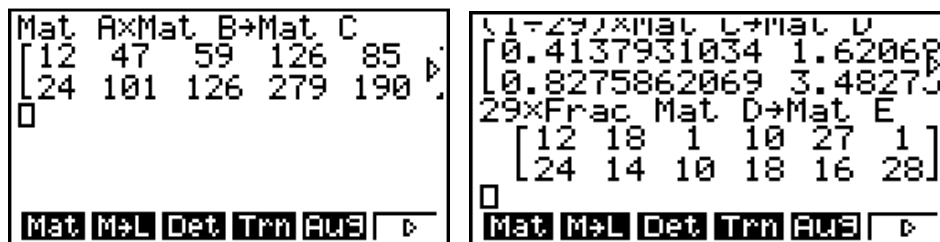
Step 4. The pairs of numbers 6 3 18 23 13 8 18 24

lead to the encrypted message or ciphertext is **GDSXNITY**.

Some students used the graphics calculator for the matrix computations while others used MMULT and MINVERSE commands in Excel. Figure 1 shows screenshots of the encoding process on Excel and figure 2 shows similar computations on the graphics calculator.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1														
2														
3		1	2		7	22	17	20		35	76	71	76	
4		3	7		14	27	27	28		119	255	240	256	
5														
6										6	18	13	18	
7										3	23	8	24	

**Figure 1** An Excel screenshot of the encoding process as done by the students.



**Figure 2** Screenshots of the encoding process as done by the students on the Casio 9860 G II

To decode the cipher text the inverse of the matrix  $E = \begin{bmatrix} 1 & 3 \\ 3 & 7 \end{bmatrix}$ , that is  $E^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$  is required.

The decoding process was introduced as follows

Step 1. The ciphertext message reaches the receiver as **GDSXNITY**

Step 2. Convert the characters to their respective Hill-2-cipher values and make pairs.

$$6 \ 3 \ 18 \ 23 \ 13 \ 8 \ 18 \ 24$$

Each pair will form a column of a matrix N.

Step 3. Compute the product  $E^{-1} N$  and reduce it modulo 29.

$$E^{-1}N = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 6 & 18 & 13 & 18 \\ 3 & 23 & 8 & 24 \end{bmatrix} = \begin{bmatrix} 36 & 80 & 75 & 78 \\ -15 & -31 & -31 & -30 \end{bmatrix} = \begin{bmatrix} 7 & 22 & 17 & 20 \\ 14 & 27 & 27 & 28 \end{bmatrix} \pmod{29}$$

When the values are written column wise they become

$$7 \ 14 \ 22 \ 27 \ 17 \ 27 \ 20 \ 28$$

which translates to **HOW\_R\_U?**

The above was illustrated as an example of a hill-2-cipher. By definition, an arbitrary *Hill n – cipher* has as its key a given invertible  $n \times n$  matrix whose entries are non – negative integers from among  $0,1,\dots, m - 1$ , where  $m$  is the number of characters used for the encoding process.

### Students’ explorations

Students found this technique very exciting. They wanted to try out examples of Hill-3 – ciphers. For many, it was a challenge to come up with 3 by 3 matrices with determinant 1. However with help from other students they managed to find some examples. Students were divided into groups of three. Out of the ten groups, six were able to work with Hill-3-ciphers. Two groups ventured into Hill-4 –ciphers and the rest perfected their work with Hill-2-ciphers. After this they designed a ‘cipherquiz’. Each group had to encode a message and post it on the whiteboard after declaring the key. The ‘receivers’ had to use the inverse key to decode the message. Students were asked to choose encoding matrices with determinant equal to 1. All students performed the computations using technology. By the end of the fifth session, all 30 students were able to work with Hill ciphers.

After this, students began to come up with queries – what if the determinant of the encoding matrix is not 1? Can anyone crack the Hill cipher? In fact one student asked whether it is possible for a person (other than receiver or sender) to crack the code, that is, figure out the encoding matrix if the plaintext and ciphertext are known. This was undoubtedly the ‘high point’ of the module for the author!

To deal with the first question, the author asked students to encode a plaintext using a matrix with determinant other than 1.  $E = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$  (with determinant is equal to -2), was one of the selected matrices. Since  $E^{-1}$  has fractional entries, the decoding process posed a problem and did not give back the original plain text. The author took this opportunity to familiarize the students

with the method of finding the inverse of a matrix in  $Z_{29}$ . This means finding a matrix  $\begin{bmatrix} x & y \\ z & w \end{bmatrix}$  so

that  $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x & y \\ z & w \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \pmod{29}$ , where  $x,y,z$  and  $w$  are integers from 0 to 28.

To answer the second query, students were introduced to the Cracking theorem.

Suppose the length  $m$  of the alphabet is a prime. Let  $P = [\bar{p}_1 \ \bar{p}_2 \ \dots \ \bar{p}_n]$  and  $C = [c_1 \ c_2 \ \dots \ c_n]$  be the plaintext matrix and ciphertext matrices respectively. Then the elementary row transformations that reduce  $C^T$  to the identity matrix  $I$  also reduce the matrix  $P^T$  to  $(A^{-1})^T$ .

A few students took up the challenge of applying the theorem. Since they were familiar with elementary row transformations, they could understand the method with some facilitation from the author. As an example, the plaintext MATH and the corresponding ciphertext MYSO were used to check if the key could be ‘cracked’. Figure 3 shows the work done by a student. It was really satisfying when some students took up the challenge of writing a C++ program for the Hill Cipher

method. During this attempt, they used their knowledge of matrix theory and logical thinking to obtain the desired output.

Cracking Theorem  
 $[C^T | P^T] \rightarrow [I | (A^{-1})^T]$   
 Plain text  $\rightarrow$  MATH  $\rightarrow$  12 0 19 7  
 Cipher text  $\rightarrow$  MYSO  $\rightarrow$  12 24 18 14  
 $P = \begin{bmatrix} 12 & 19 \\ 0 & 7 \end{bmatrix}$      $C = \begin{bmatrix} 12 & 18 \\ 24 & 14 \end{bmatrix}$   
 $[C^T | P^T] = \left[ \begin{array}{cc|cc} 12 & 24 & 12 & 0 \\ 18 & 14 & 19 & 7 \end{array} \right] \xrightarrow{R_1 \rightarrow 17R_1} \left[ \begin{array}{cc|cc} 204 & 408 & 204 & 0 \\ 18 & 14 & 19 & 7 \end{array} \right]$   
 $\equiv \left[ \begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 18 & 14 & 19 & 7 \end{array} \right] \xrightarrow{R_2 \rightarrow R_2 - 18R_1} \left[ \begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -22 & 17 & 7 \end{array} \right]$   
 $\equiv \left[ \begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 7 & 1 & 7 \end{array} \right] \xrightarrow{R_2 \rightarrow 25R_2} \left[ \begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 175 & 25 & 175 \end{array} \right]$   
 $\equiv \left[ \begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 25 & 1 \end{array} \right] \xrightarrow{R_1 \rightarrow R_1 - 2R_2} \left[ \begin{array}{cc|cc} 1 & 0 & -49 & -2 \\ 0 & 1 & 25 & 1 \end{array} \right]$   
 $\equiv \left[ \begin{array}{cc|cc} 1 & 0 & 9 & 27 \\ 0 & 1 & 25 & 1 \end{array} \right] = [I | (A^{-1})^T]$   
 $\therefore A^{-1} = \begin{bmatrix} 9 & 27 \\ 27 & 1 \end{bmatrix}$  in  $Z_{29}$   
 To find  $A^{-1} \left[ \begin{array}{cc|cc} 9 & 27 & 1 & 0 \\ 27 & 1 & 0 & 1 \end{array} \right] \leftrightarrow \left[ \begin{array}{cc|cc} 1 & 0 & 1 & 4 \\ 0 & 1 & 2 & 9 \end{array} \right]$   
 $\downarrow$   
 $A^{-1}$

**Figure 3** A student’s work on applying the Cracking Theorem

**Discussion**

Throughout the module student’s explorations were facilitated by the author. Graphics calculators and Excel helped to trivialize lengthy matrix calculations thus enabling students to focus on exploring the models and interpreting solutions. Students were able to understand the need for matrices and matrix operations. Paper and pencil computations assumed greater importance as in many cases students preferred to do the calculations by hand and then verify their answers using technology. In the Hill Cipher method, the lengthy calculations such as multiplying matrices of large orders and reducing matrices modulo 29, was ‘outsourced’ to the calculator. Throughout the module the graphics calculator or Excel served as a ‘mathematical investigation assistant’, as proposed by Arnold [1], giving students control over what they were learning. Finally, the most important outcome of the module was that technology enabled explorations of the mathematical applications (such as the Hill Cipher) gave students access to higher level mathematical concepts. According to Heid [3], the technology acted as an ‘amplifier’ as students learnt new mathematical content, some of which was far beyond the level of grade 12. For example, the Hill cipher method led to the discussion of the some ideas in number theory and the Cracking Theorem which was beyond the curriculum. To summarise, it will be apt to say that the integration of applications along with technology, along with facilitation from the teacher, provided students with a rich and motivating environment for learning mathematics.

**4. Research Study II. Integration of graphics calculators in the senior secondary mathematics classroom**

Among the various technology tools available for mathematics instruction, handheld technology in the form of graphics calculators, have assumed great importance in many countries as far as the teaching and learning of mathematics is concerned. The computer-like features and easy portability of these devices help to integrate mathematical modelling activities in the classroom and bring the power of visualization right onto the palm of the hand. Mathematics teachers and educators have been investigating various ways of integrating these devices with classroom teaching in order to make mathematics accessible to students of varying levels of ability. Graphics calculators have significant mathematical capabilities allowing the user to deal with matrices, complex numbers,



symbolic integration and differentiation as well as data analysis. In countries like India, where, having a large number of students in a class is almost a norm, graphics calculators can play a significant role in making technology accessible to students for mathematical exploration. However, hardly any research studies have been conducted to study the impact of integration of handheld technology in the mathematics classroom in India. In this section we shall describe a pilot study (initiated by Casio India company) which was conducted in 15 schools of Delhi to study the impact of integrating graphics calculators (Casio fx-CG20) in mathematics teaching and learning in senior secondary school. The primary objective was to observe the impact of graphics calculators on students' problem solving ability, conceptual understanding and also to assess students' perception of learning mathematics after using graphics calculators. The research questions of the study were identified as follows

- (i) How does the integration of graphics calculators impact students' understanding of mathematical concepts?
- (ii) How does the integration of graphics calculators impact students' problem solving skills?
- (iii) How does use of graphics calculators affect student's perception of mathematics?

### Definition of the constructs and variables

Two research constructs and their respective variables were identified and defined as follows

- (i) The impact of graphics calculators on students' perception of mathematics learning (Variables: students' rating of 10 items through a questionnaire, based on their perception of the graphics calculator technology and on perception of mathematics learning through the technology)
- (ii) The impact of graphics calculators on students' conceptual understanding and problem solving ability (variables: pre-test and post-test scores conducted before and after a module)

### Hypotheses

The hypothesis of the study is that the graphics calculator has a positive impact on students' conceptual understanding and problem solving skills. The null hypothesis,  $H_0$ , is that there is no difference between the performance of students in the pre-test and post-test. The alternate hypothesis,  $H_A$ , is that the performance of students in the post-test is significantly better than the pre-test. The level of significance will be taken as 5% or  $\alpha = 0.05$ .

### Sampling

To measure the impact of integrating graphics calculators on the performance of students' problem solving ability and conceptual understanding, a sample of 497 students from classes 11 and 12 were selected from 15 Kendriya Vidyalaya schools in Delhi through convenience sampling. All 497 students went through a module in which they were taught topics from the syllabus using graphics calculators over a period of 6 months. These students will be referred to as the GDC group. From this group, 75 selected students of class 11 (five from each school) attempted a pre-test prior to the module and a post-test after the module. These 75 students will be referred to as the *experimental group*. Another 75 students (five from each school) were selected from class 11 sections of these 15 schools, who were taught the same topics of the module *without* the use of graphics calculators. These 75 students will be referred to as the *control group*. Both pre-test and post-test aimed to test students' conceptual understanding and problem solving skills related to the

topics taught to them using graphics calculators. At the end of the module, all 497 students responded to a questionnaire comprising 10 items which tested their perception of using GDCs in mathematics learning.

## Method

The study was conducted in four phases. In the first phase, mathematical content related to some topics of classes 11 and 12 was developed in the form of a manual. Specifically, the content was based on the topics linear inequations, functions and conics from the syllabus of class 11 and calculus, linear programming and matrices from the syllabus of class 12. The manual comprised chapters which explained the concepts of the topics through the graphics calculator, supported by steps of exploration. The manual was meant to provide the student with opportunity for visualization and exploration and also provide the teacher with a guideline for integrating graphics calculators in the teaching learning process. In the second phase, 15 selected teachers (one teacher from each school) underwent a series of workshops which familiarised them with the features of the graphics calculator and provided concrete suggestions to enable them to integrate the technology in their regular teaching. In the third phase, these 15 teachers conducted the modules in their respective classes. In one class the topic was taught by each teacher through the use of graphics calculators and students were allowed to use the calculators in all the class sessions of a specific topic. In the other class the same topic was taught, by the same teacher, in a traditional manner, without the use of graphics calculators. In the fourth phase, the pilot study was evaluated based on the following criteria.

- A. Comprehensive feedback of students regarding the use of GDCs in the teaching learning process through a questionnaire.
- B. Impact of GDCs on the performance of students' problem solving ability and conceptual understanding through a pre-test and a post-test.

### **A. Comprehensive feedback of students regarding the use of GDCs in the teaching learning process through a questionnaire.**

All 497 students who participated in the study were required to respond to a questionnaire of 10 items (see Table 2) by entering a number from 1 to 5, where the numbers indicate the following 1-Strongly disagree, 2-Disagree, 3- Not sure, 4- Agree, 5- Strongly agree. These items focused on the student's perception regarding the use of the graphics calculator in their mathematics class.

### **B. Impact of GDCs on the performance of students' problem solving ability and conceptual understanding.**

To measure the impact of integrating graphics calculators on the performance of students' problem solving ability and conceptual understanding, two written tests were administered to the experimental group (75 students who had used GDCs) and the control group (75 students who had not used GDCs). These tests were designed with the aim to compare the performance of the two groups in problem solving skills and conceptual understanding. The first test, the pre-test, was administered to students of both groups prior to the integration of graphics calculators in the teaching - learning process. The second test, the post- test, was administered to both groups of students after the completion of the module. Both tests comprised 20 multiple choice questions based on the topics, linear inequations, functions, conic sections, limits and derivatives. Each

question had four options out of which only one answer was correct. The questions were based on the following attributes

(i) Application of formulae or results - These questions required the student to apply a formula or a result to arrive at the solution. Typically the answer could be obtained in one or two steps. An example from the pre-test,

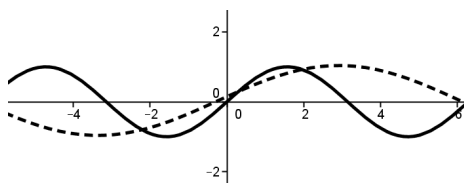
If  $x^2 + y^2 - 4x + 6y + k = 0$  represents a circle with radius 5 units, then  $k$  equals

- A. 12      B. -12      C. -13      D. 13

(ii) Graphical reasoning – In these questions a graph was provided and the student was required to apply her conceptual understanding and visualization skills to select the correct answer. An example from the pre-test,

The thick line represents the graph of  $y = \sin x$ . The dotted line represents the graph of

- A.  $y = 2 \sin x/2$     B.  $y = \sin x/2$     C.  $y = \sin(x + 1/2)$     D.  $y = \frac{1}{2} \sin x$



(iii) Application of procedural knowledge – Questions based on this attribute required the student to use her conceptual understanding as well as an appropriate method to arrive at the answer. The solution required more than two steps. An example from the post-test,

The circles  $(x - 1)^2 + (y - 4)^2 = 36$  and  $(x - 6)^2 + (y + 1)^2 = 26$  meet at

- A. (1, -2) & (7,4)    B. (1, 2) & (7,-4)    C. (1, 2) and (-7,4)    D. (-1, 2) and (7,4)

(iv) Application to a realistic context – This category of questions presented a real life situation and the student was required to use conceptual knowledge and procedural skills to arrive at the answer. An example from the post-test,

An experimental breeding colony of ants is set up and the size of the colony at time  $t$  months ( $0 \leq t \leq 8$ ) is given by  $N = 20 - \frac{400}{t-10}$ . The original size of the colony is

- A. 20 ants      B. 30 ants      C. 50 ants      D. 60 ants

Both the pre-test and post-test comprised questions based on all four attributes. One mark was awarded for each correct answer. If a student marked the correct option without supporting it with at least a partial solution (space was provided for solutions), it was not given a score. This was done to avoid students selecting options blindly or just making guesses. However, there was no negative marking for incorrect answers. By the end of the study, all 150 students had attempted the pre-test as well as the post-test. Thus there were two scores (out of 20) available for each student.

### Data Analysis

This section will present the analysis of the data which will be discussed under the heads A and B as described in the previous sections.

A. The data of 497 student’s responses to the questionnaire are shown in Table 2.

Table 2. Responses of students (N = 497) to the questionnaire

Item no.	Item	SA	A	NS	D	SD
1.	I found the classes taught using graphics calculators more interesting than the traditional (regular) classroom teaching which do not use graphics calculators.	118	325	28	20	6
2.	The use of graphics calculators should be included in most topics of the curriculum in classes 11 and 12.	133	248	83	27	6
3.	Mathematics as a subject will seem less abstract if topics are taught with the help of graphics calculators.	54	171	145	107	20
4.	My confidence level in solving problems has decreased after using graphics calculators.	20	49	56	292	80
5.	I feel that graphics calculators helped me to visualize and explore concepts and thus gain a deeper insight into the subject.	131	277	53	27	9
6.	I feel that graphics calculators can help to perform calculations very easily. This can lead to saving of time which can be spent on exploring mathematical concepts.	165	244	46	21	11
7.	The use of graphics calculators will have a harmful effect on my paper and pencil skills.	50	85	72	207	86
8.	I feel math classes should use a mix of graphics calculators and paper pencil methods.	179	262	32	15	9
9.	Problem solving without using graphics calculators (as in our regular classes) is more challenging and fun than using graphics calculators.	52	176	130	122	17
10.	Graphics calculators are easy to use and anyone can learn how to use them.	174	231	60	16	16

Table 3 shows the sample means and standard deviations for agreement scores (1 to 5) for all items of the questionnaire. These have been used to estimate the population mean scores by calculating 95% confidence intervals. A mean score of 3.75 and above indicates agreement with the statement given for all items except 4 and 7. For all 8 out of the 10 items (excluding item 3 and item 9) the interval estimates indicate strong agreement with the statement. This is evident from the observation that the sample mean for these item lies well within the 95% confidence interval. The estimates indicate that a majority of students felt that the classes taught using graphics calculators were more interesting than traditional classes (item 1) and a majority agreed that graphics calculators should be integrated in classroom teaching in grades 11 and 12 (item 2). A large

proportion of students were unsure if the use of graphics calculators would make Mathematics “less abstract” (item 3). Most students agreed that graphics calculators helped them to obtain a deeper insight into the subject (item 5) and also helped in performing difficult calculations leading to saving of time which could be utilised on exploration of concepts (item 6). It is interesting to note that a majority of students felt that the use of calculators did not hamper their paper – pencil skills (item 7) nor did it reduce their confidence in solving problems (item 4). In fact they endorsed the use of a combination of paper – pencil methods and calculator use (item 8). Most students also agreed that graphics calculators are easy to use (item 10).

Table 3. 95% confidence intervals of population mean for agreement scores

Item no.	Sample mean (N = 497)	Sample Standard deviation	Population Mean (95% confidence interval)	
			Upper Bound	Lower Bound
1	4.06	0.75	4.13	4.00
2	3.96	0.87	4.03	3.88
3	3.27	1.04	3.36	3.17
4	2.27	0.98	2.36	2.18
5	3.99	0.87	4.07	3.92
6	4.09	0.89	4.17	4.01
7	2.63	1.24	2.74	2.52
8	4.18	0.82	4.25	4.12
9	3.25	1.05	3.34	3.16
10	4.07	0.94	4.15	3.19

Note: A 95 % confidence interval is highly indicative of the population mean. This implies that if various samples of this size (N = 497) are taken from the population and their confidence intervals are calculated, then in 95% of the cases the population mean is likely to lie within the estimated interval.

### Comparison of means

To analyse the pre-test and post-test scores of all 150 students (experimental and control groups) a comparison of mean scores of both tests was done for all 20 questions. Similarly comparison of mean scores was also done for the scores obtained by students on 8 graphical reasoning questions in both the tests. The scores are shown in table 4. The mean score of the experimental group improved marginally in the post-test. However on the questions based on graphical reasoning the improvement in the post-test is more evident.

Table 4. Pre-test and post-test mean scores of control and experimental groups

	Pretest	posttest
Comparison of Mean scores of control group (scores out of 20)	8.89	8.72
Comparison of Mean scores of experimental group (scores out of 20)	10.04	10.36
Comparison of Mean scores of control group in graphical reasoning questions (scores out of 8)	3.4	3.29
Comparison of Mean scores of experimental group in graphical reasoning questions (scores out of 8)	3.74	4.01

### Paired t test

A paired t test was conducted on the pre-test and post-test scores of the experimental group on the entire test (out of 20) and on the scores of graphical reasoning questions (out of 8). Also after carefully examining the school wise mean scores, paired t test were conducted on the pre-test and post-test scores of students of selected 6 schools. In a paired t test, a p – value of 0.05 or less indicates that the null hypothesis needs to be rejected. Since, in the study, the null hypothesis states that there is no difference in the performance of students, before and after use of graphics calculators, rejecting the null hypothesis would imply that the difference between the pre-test and post-test scores is significant at 5% level of significance ( $\alpha = 0.05$ ).

### Results of the study

A quantitative analysis of the responses of the 497 students to the questionnaire revealed that a majority of the students developed a very positive attitude regarding the use of graphics calculators in their mathematics class. Most of them agreed that the calculator helped them to develop conceptual understanding by providing opportunities for visualization and exploration and that the use of calculators did not hamper their paper – pencil skills nor did it reduce their confidence in solving problems. They also expressed that the classes taught using graphics calculators were more interesting than traditional classes and agreed that graphics calculators should be integrated in classroom teaching in grades 11 and 12. The responses of the 497 students is highly representative of the population and is very encouraging as it suggests that the integration of graphics calculators in the senior secondary level can have a positive impact on the teaching learning process.

The paired t tests (conducted at a significance level of 5% ,i.e,  $\alpha = 0.05$  on the data of all 15 schools taken together) reveal that the mean score improved in the post-test after the implementation of the module. Similarly the mean score (out of 8) on the graphical reasoning questions also improved in the post-test. However, this improvement is only marginal and not statistically significant at 5% level of significance. A school wise analysis of data, however, shows that in 6 out of 15 schools, the difference in overall pre-test and post-test scores of the experimental group (out of 20) and in the graphical reasoning questions (out of 8) is highly significant at 5% level of significance. This suggests the possibility that in these 6 schools the graphics calculators were used more effectively during the module than in the remaining 9 schools. The overall results indicate that the module has had a positive impact on students' problem solving skills and conceptual as well as graphical understanding.

### Conclusion

In this paper we have discussed two research studies conducted with senior secondary school students where technology played a key role in helping them visualize and explore various concepts. The mathematics curriculum in India does not traditionally prescribe the use of technology. The emphasis is primarily on the development of manipulative skills and procedural knowledge. Thus studies of this kind have an important role to play in convincing curriculum makers of the benefits and affordances of technology. In the first study, students underwent a module on the topic matrices and determinants, in which they explored various operations and properties of matrices through real life applications. Graphics calculators and Excel took over lengthy calculation and enabled them to explore applications of matrices such as the Hill Cipher method. In the process they learnt of the relevance of mathematics to real life and as proposed by Heid [3] technology played the role of an 'amplifier' giving them access to higher level mathematical concepts. One of the drawbacks of the study was that the module was time

consuming and required students to work beyond their school hours. In the second study, students of grades 11 and 12 were taught several topics of their regular curriculum using graphics calculators. They had access to calculators in their regular classes for about six months. Statistical tests reveal that the students had improved in their ability to reason graphically and also in their problem solving skills. A comprehensive feedback through a questionnaire shows that students had developed a very positive attitude towards the use of graphics calculators in their mathematics class. The calculator helped them to explore concepts graphically, symbolically and numerically and this supports Heid's [3] theory that technology facilitates a multi-representational approach to learning mathematical concepts. Teachers felt that graphics calculators can be easily integrated in the classroom and using them do not hamper students' paper – pencil skills. According to them an effective pedagogy for the mathematics classroom would be to emphasize the exploration of concepts through graphics calculators and later focus on developing procedural skills. This supports the 'concepts before skills' approach and as proposed by Heid [2], technology acts as a 'reorganiser' by changing the order and sequence in which the mathematical content can be taught. To summarise, it may be concluded that integrating technology in mathematics teaching and learning at the senior secondary level led to a very satisfying combination of technology use and 'by-hand' skills. The studies indicate that the appropriate use of technology can augment learning and provide a rich and motivating environment for exploring mathematics.

**Acknowledgements:** The author would like to acknowledge the support provided by Casio India Company in facilitating the second research study described in this paper. The technical support provided to the schools during the implementation of the modules and during the process of data collection led to the smooth execution of the study.

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