Creative Thinking in Mathematics with Tangrams and The Geometer's Sketchpad

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Abstract:

The purpose of this paper is to illustrate how Tangram puzzle can be used to enhance students' creative thinking in mathematics and to use dynamic geometry software the Geometer's Sketchpad to develop the pictorial representations and geometrical shapes. In the year 2015, action research was carried out in a lower secondary school Bangkok, Thailand. The research finding revealed that the teachers used GSP to construct Tangram puzzle and Egg Tangram. The students explained that were able to use GSP to drag, rotate and translate the pieces of virtual Tangrams to form the shapes given to them. Creative thinking and problem solving skills were developed while students solved and created tangram puzzles. The students were able to express their geometric imagination, and their understanding of mathematics concept by verbal interactions. The students had fun and positive attitudes towards mathematics.

1. Introduction

The purpose of this paper is to illustrate the use of Tangram to enhance students' creative thinking and understanding in mathematics and to use dynamic geometry software The Geometer's Sketchpad (GSP) to develop their representation in mathematics. The teachers used GSP to construct Tangram puzzle and Egg tangram and to show mathematics relationships in a puzzle problems. The students used GSP to drag, rotate and translate the pieces of virtual Tangrams instead of using the pieces of Tangrams-paper to flip and turn in order to form the shapes given to them. The paper shown that this strategy will inspire teacher to go out of their way from the tradition geometry class with the problems that take students beyond rote drill. With this method, geometry class will create the learning environment of exciting, motivating, rewarding thought-provoking and challenging.

2. Teaching and Learning Mathematics in Thailand

In Thailand, the students learned mathematics five hours per week and there are 16 weeks in one semester. Geometry is one of the mathematics strands in primary and secondary levels. Learning geometry is a process of studying the conversation of graphics in space, which can enhance children's spatial ability Zhou [1]. Van Hiele [2] described a five-level model on how people learn Geometry. These levels are a product of experience and instruction, moving from visualization analysis, abstraction, deduction to rigor. Students play the tangram puzzle is one of the significant methods to enhance their geometric spatial thinking.

3. Creative Thinking in Mathematics

Geometry students should learn how geometric ideas and concepts apply to a wide range of human endeavor-in art, and in daily life outside classrooms. One of the modern themes and approaches to geometry is transformation geometry concentrates on translation, rotations and reflections. One way to provide such experience is through the infusion of creative thinking, recreation mathematics puzzle incorporate with computer software such as dynamic software the Geometer's Sketchpad. Recreation mathematics is a term for mathematics carried out for recreation rather than a strictly knowledge of advanced mathematics. Averbach, B & Chein,O [3] explained that recreation mathematics involves mathematical puzzle and games and it is used for inspiring the study of the subject. Research continues to prove that creative thinking and problem solving skills are enhanced by the challenging fun and play puzzles, games, and brainteasing activities give to the brain.

Mathematical puzzle make up an integral part of recreational mathematics. The basic idea of mathematical puzzle is to make a certain shape or design with the given pieces. Averbach, B & Chein,O [3] suggested puzzles such as Tangram, the students have to assemble the Tangram pieces put all of pieces together to form shapes of a rectangle, a triangle, a parallelogram or any figures. While the students are doing the puzzles they are finding out about the relationships of pieces of tangram to be used to construct a puzzle given. They are learning that a shape stays the same no matter how it is turned, flipped or slid. They learn that they have to try a variety of ways before they find the solution. In addition, mathematical puzzles require mathematics knowledge to solve them. The students have specific rules as do games and puzzles but they do not usually involve competition between their friends. Instead, the students have to find a solution that satisfies the given conditions.

4. Empowerment Through Tools: The Geometer's Sketchpad (GSP)

The Geometer's Sketchpad is one of the dynamic mathematics software that provides opportunities for students to investigate and discover mathematics concepts in particular geometric patterns. GSP empowers students to use their abilities to create graphical representation, to enable them in developing their mathematical thinking skills, concepts, and understanding. Khairiree [4] described that while using GSP students learned by exploring, investigating and discovering. GSP enhance students' ability in helping them visualize abstract mathematical relationships and various problem structures through pictorial representations.

5. Tangram

From the educational point of view, Tangram assists in teaching geometry via developing: geometrical knowledge, reasoning, geometrical imagination. Geometrical imagination is ability to sense, geometrical shapes, their size and position in space, a given shape in different space positions, changes of shapes in their size, structure. Two types of Tangram are employed in this studied.

1. Ancient Chinese Tangram Puzzle

Tangram puzzle is an ancient Chinese art and Tangram is a popular mathematical problem solving activity. Slocum, J.,et al., [5] described that the Tangram puzzle is in the shape of a square. It

consists of 7 geometric pieces. The pieces called “tans” are used to create different patterns. Tangram rules of the puzzle stated that all seven pieces must be used, all pieces must lie flat, all pieces must touch and no pieces may overlap. Pieces may be rotated and or flipped to form the desired shape. The Tangram can be arranged in more than 3,000 patterns. The invention of the tangram puzzle is unrecorded in history. The earliest known Chinese book is dated 1813 but the puzzle was very old by then. One reason for this could be that in China the country of origin, at that time it was consider that Tangram was a game for woman and children [5].

2) *Egg Tangram Puzzle*

Tat, et. Al. [6] explained that Egg Tangram puzzle or the Egg of Columbus or Magic Egg consists of nine pieces of geometric shape. The nine pieces can be arranged into various patterns, the most popular are birds. The goal of Egg Tangram is to rearrange the pieces to form other specific shapes. It is called flip, slide and turn, it is a very fun puzzle to do in the classroom. The process of arranging pieces together enhance students’ thinking skills, reasoning and visual problem solving skills.

6. Tangram, GSP and Action Research in Thailand

In Fiscal Year 2015, Suan Sunandha Rajabhat University allocated budget to conduct community academic services to mathematics teachers. Under the community services, the author received budget to conduct the training workshops on the use of GSP as a tool in mathematics classes. In order to have direct experiences the author conducted the action research in mathematics class in order to explore the classroom environment of creative thinking in mathematics with Tangram puzzle and GSP approaches. The Tangram puzzle and GSP are introduced in Secondary Mathematics Textbooks in Thailand.

Russell & Bologna [7] explained that in geometry teaching, tangram puzzle could be used as an aid in presenting specific mathematical concepts, inspiring children’s observation, imagination, shape analysis, creativity and logical thinking. The use of creative mathematical virtual Tangram puzzles and GSP are instructional tools in geometry classroom that provide an environment for the students to work on something familiar but from a different perspective. One significant of this occurs in the stage of using GSP to create virtual Tangram puzzles. For example, when students want to draw a rectangle with ruler they requires a knowledge of the side lengths and angles measurement. In order to construct a rectangular shape from pieces of virtual tangram puzzles with GSP, students need more than basic knowledge of a rectangle. They have to employ knowledge on symmetry and transformation such as rotation, reflect and translation and use GSP to drag, rotate, and flip the Tangram pieces to construct a rectangular shape. This action research focused on how teacher implemented creative mathematical puzzle virtual Tangram and GSP in mathematics classes.

8. Action Research Questions

The action research questions were:

1. How to implement creative thinking in mathematics with Tangram and GSP effectively?
2. What are the effects of using creative thinking in mathematics with Tangram and GSP towards students’ attitudes in mathematics?

Research Question 1: How to implement creative thinking in mathematics with Tangram and GSP effectively?

The following examples show activities in constructing Tangram with GSP and implementing creative thinking in mathematics with Tangram and GSP in the action research in Thailand.

1. Constructing Tangram with GSP.

The teacher constructed Tangram puzzles with GSP. The steps in constructing ancient Chinese Tangram and Egg Tangram are as follows.

(1) *Ancient Chinese Tangram*

Tangram puzzle is an ancient Chinese art and Tangram is a popular mathematical problem solving activity. The Tangram puzzle is in the shape of a square. It consists of seven geometric pieces. Tangram puzzle was constructed using GSP as follows.

- Open the Geometer's Sketchpad, in the File Menu, choose New Sketch and follows the instruction step-by-step.
  - Construct a square ABCD
  - Construct the midpoints *E F* and of the line segments *AB* and *BD*
  - Construct line segments *AD* and *CB*
  - Construct point *G* and point *H*, the point of intersection of segments *AD* and *CB*.
  - Construct a line segment *CH*.
  - Construct a line through point *E* perpendicular to a line segment *EF*
  - Construct point **I**, the point of intersection of the perpendicular line and a line segment *AD*.
  - Construct a line through point *H* parallel to a line segment *BD*
  - Construct point **J**, the point of intersection of the parallel line and a line segment *AD*.
  - Construct the lines segments EI and HJ and hide the perpendicular and the parallel line.
  - Construct the polygon's interior as shown in the Figure 1

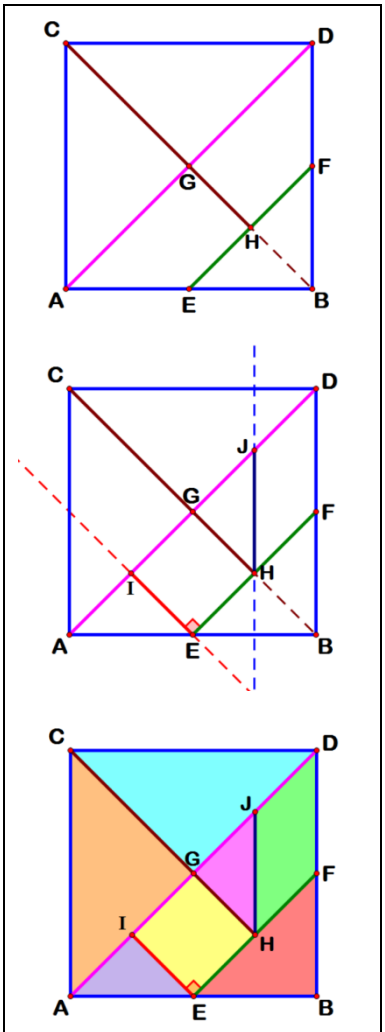

Figure 1: Constructing an Ancient Chinese Tangram

(2) *Egg Tangram Puzzle*

Egg Tangram puzzle or Magic Egg consists of 9 pieces of geometric shape. Egg Tangram puzzle was constructed using GSP as follows.

- Open the Geometer's Sketchpad, in the File Menu, choose New Sketch and follows the instruction step-by-step.
- Using Segment tool to draw a line segment *AB*, let *AB* = 5 cm and construct a line segment *CD* through point *E* perpendicular to line segment *AB*;
- Construct a circle with center *E* and set radius 5 cm (can set as parameter t[1] = 5 cm);
- Construct point ***F***, and ***G*** the point of intersection of the circle and the perpendicular line;
- Construct rays *AF* and *BF* ;
- Construct a circle with center *A* and radius *AB*;
- Construct a circle with center *B* and radius *BA*;
- Construct a circle with center *F* and radius 3 cm as shown in the Figure 2.

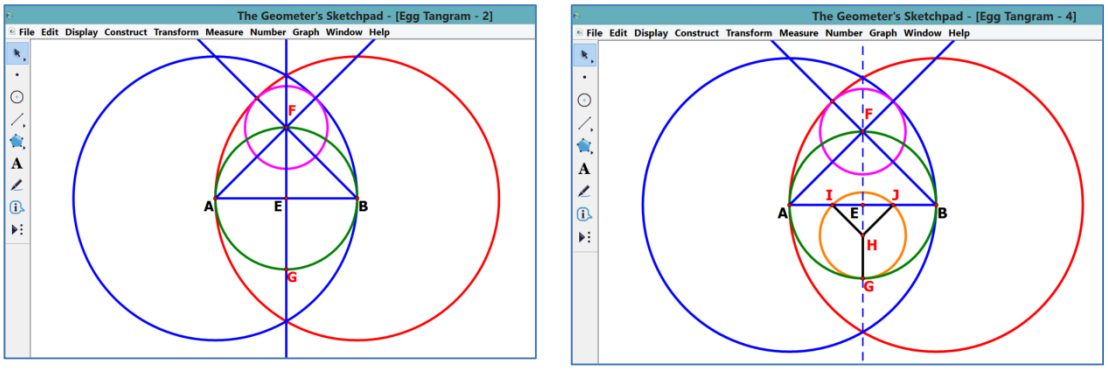

Figure 2: Constructing an EggTangram

- Construct point *H* on a line segment *CD*, let *EH* = 2 cm;
- Construct a circle with center *H* and radius *HE*;
- Construct line segments *HG*, *HI*, and *HJ*; and
- Construct line segments and arcs on circles to complete Egg Tangram as shown in Figure 3.

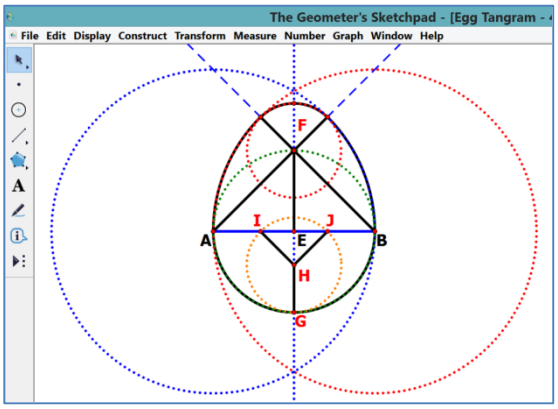

Figure 3: Constructing an EggTangram (cont.)

- Hide circles, segments, and coloured Egg Tangram as shown in Figure 4 and 5.

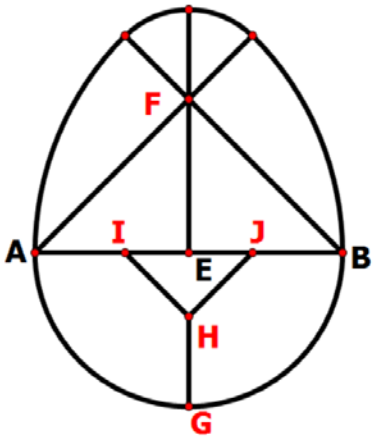

Figure 4 Egg Tangram

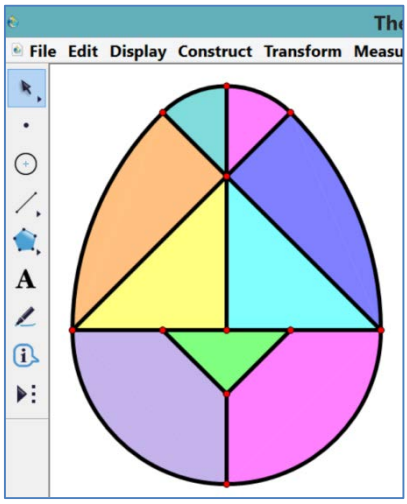

Figure 5 Coloured Egg Tangram

2. Implementing creative thinking in mathematics with Tangram and GSP

The examples of the implementation of creative thinking in mathematics incorporate Tangram puzzles and GSP in geometry in mathematics classes which used in the action research are as follows.

2.1 The Figures below show the examples of ancient Tangram in geometry and puzzle patterns.

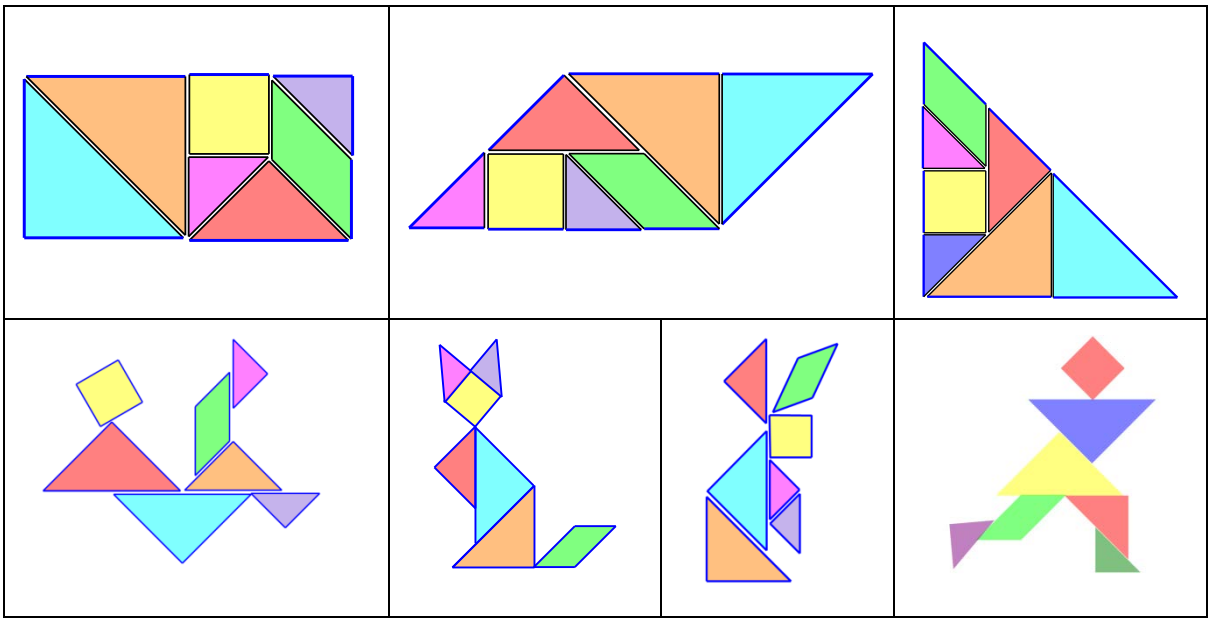

Figure 6 Tangram Puzzles

2.2 The Figures below show the examples of Egg Tangram puzzle patterns.

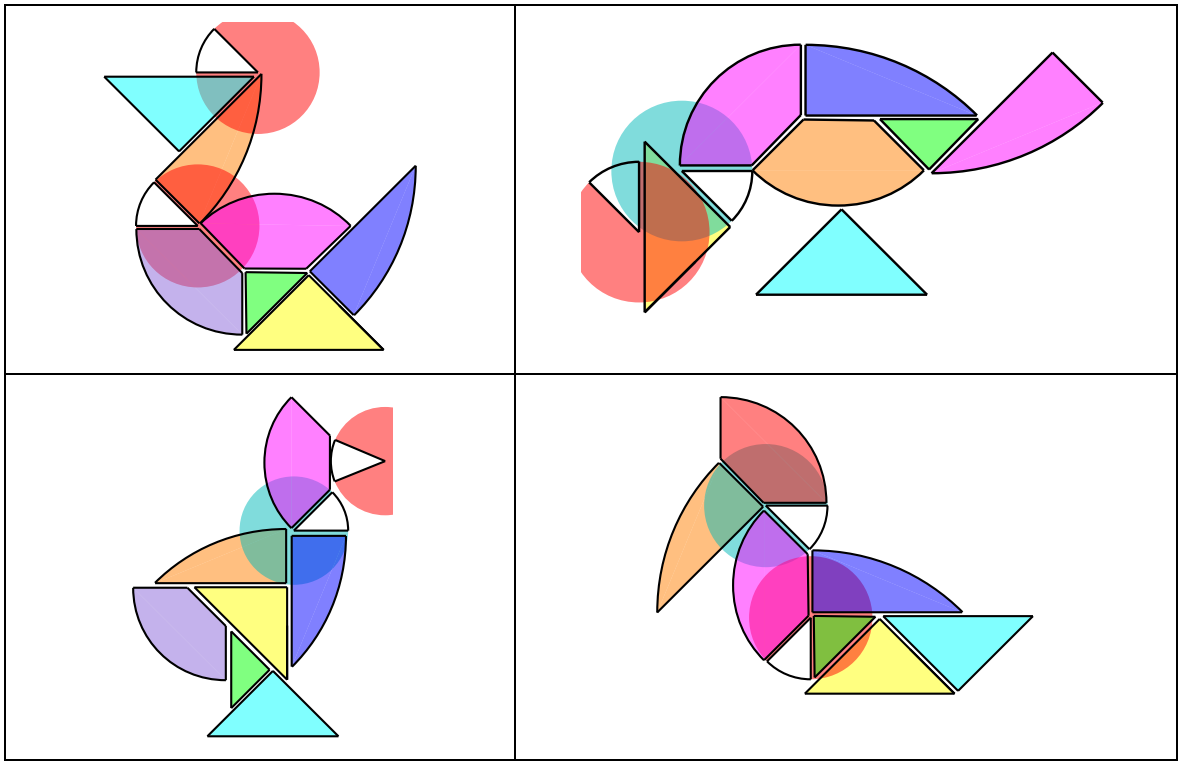

Figure 7 Egg Tangram Puzzles

2.3 How student learned geometry by using Tangram and GSP

Students used GSP tools to drag, rotate the virtual Tangram pieces to form the puzzle given to them. These activities enabled students to construct geometric Egg Tangram Puzzles using GSP to create patterns by rotate and flip a piece of virtual Egg tangram. The students discover the relationships of graphics in geometry by playing virtual Tangram puzzles, recomposing and representing the visuals. This would help students concentrate more on thinking and finish the composition more precisely.

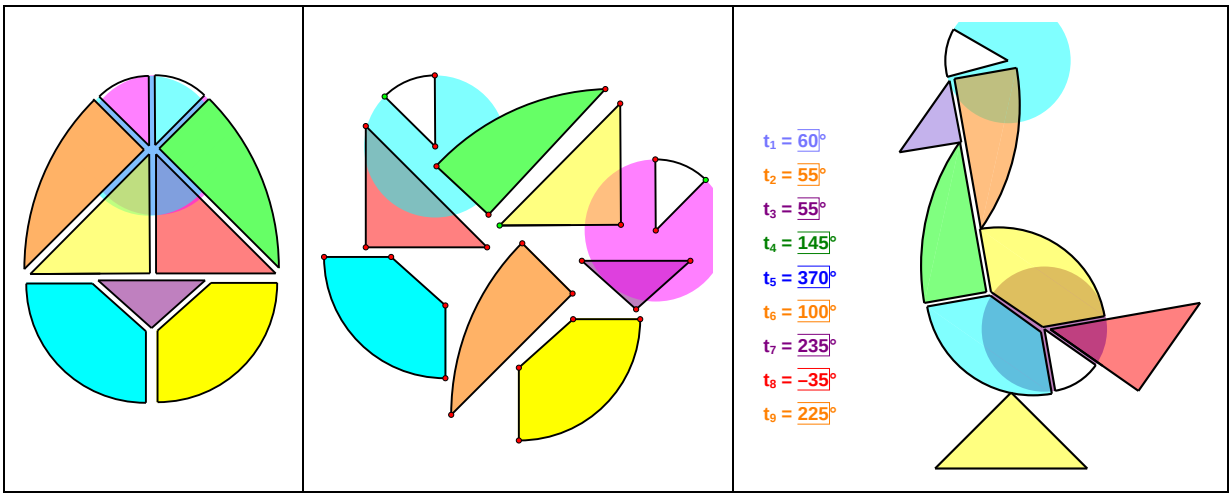

Figure 8 Students created Egg Tangram Puzzle using GSP

Question 2: What are the effects of using creative thinking in mathematics with Tangram and GSP towards students' attitudes in mathematics?

Based on the interview, the researcher found out that the students in the sample schools now liked to learn mathematics and they had more understanding on translation rotation and reflection. The students used GSP to drag, rotate, translate and reflect the pieces of virtual tangram to form the tangram puzzles given to them. The students were able to explain, knew what to do and knew why they had to do. In addition, the students revealed that with virtual tangram puzzles and GSP strategy they were able to visualize and create graphical representations, which will enable them to develop their creative thinking in mathematics, concepts and understanding. The students explained that it was fun in learning mathematics by this method. Based on these evidences the students have acquired positive attitude toward mathematics.

9. Conclusions

I had shown that with the use of virtual Tangram puzzles and GSP approach was very useful and effective method for both teacher and students. This method inspired teachers to go out of their way from the tradition geometry class with the problems that take students beyond rote drill. With this method, geometry class will create the learning environment of exciting, motivating, rewarding thought-provoking and challenging.

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[i] The author gratefully acknowledges to the Principal of Secondary School, Bangkok, Thailand, students and the teachers who were involved in this research. The author also acknowledges her gratitude to International College, Suan Sunandha Rajabhat University Bangkok, Thailand for supporting and funding this research project