Text-Based Communication Tool for Mathematical Documents for Visually Handicapped

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Abstract: *In this study, we consider the expression methods of mathematical documents, for which the receiver is visually impaired, and the supplier is his/her support. The structure of an information graph is an important factor for the accessibility of the documents in this situation. We developed a software tool to express this structure for support and a tool to access this document for the visually handicapped. Moreover, we consider the information element for which we need some extra explanations.*

1. Introduction

Target objects in elementary geometry are often expressed by figures. These must be serious barriers in communication for people with a visual disability; however, the translation to nonvisual information elements is very difficult. There are two common expression methods: text expression and tactile explanation. We are not able to express complicated documents using these methods. Moreover, they must be difficult to understand, especially for congenitally blind persons. In this report, we propose an expression method for mathematical documents. We developed two software tools to create a mathematical document according to an expression rule and to receive these documents as text-based content. In these software tools, we use the graph structures of the corresponding documents. In general, there are several edges with the same end point. These connections are simultaneously provided in graphical content. However, the order of these connections is an important factor that can aid in understanding the content quickly and easily.

We assume that sighted persons, who support the visually impaired, create mathematical documents using our software. We have developed a handwriting input system for some figures in elementary geometry [01]. As an easy application of this system, we developed our software to create documents. Moreover, we assume that the created documents are read by visually impaired persons using our other software to access these documents. This may be inconvenient for a sighted person, since there are only a few parts of the entire content, and there are no figures. Every part of the document is expressed by text and output by voice on demand. Users are able to access other parts according to the graph structure of the document.

We describe the graph structure of the content as an XML document. Because XML syntax is suitable for representing the tree structure of information, we developed an XML-based expression rule for a mathematical document. This is an improvement of the rule presented in our earlier paper [02]. Using this expression rule, we describe a graph structure of the document. In general, one node has several edges connected to other nodes. Considering a figure used to explain some mathematical problem, we are able to access all nodes and edges at once, and a well-trained brain may select the adequate nodes and edges very quickly. These tasks may be done subconciously, and for example, some teachers may not understand why some students cannot find the adequate information elements and their relations from the figure. These selections are not easy in general, and for the visually impaired, the order of the edges connected to one node may affect how easily

he/she understands the content. In this study, we have conducted a survey in the form of a questionnaire for the priorities of these connections. For our preliminary analysis of the survey results, we obtained some trends; however, finding general rules is very difficult, and this is our important future work.

2. Format of the Document

The structures of information are often expressed by directed or undirected graphs. These structures are unverbalized when we read mathematical documents. In other words, it is very difficult to understand the graph structure of the document using a standard expression method. Then, we describe mathematical documents using XML rules. In this section we explain our rules for XML expression for mathematical documents.

2.1 XML Parser

We describe the graph structures of mathematical documents, and some parts of these expressions are connected with mathematical concepts. Then, we define expression rules for our document and develop an XML parser suitable for our situations.

2.1.1 Attribute Types

In our expression rule, every attribution has its attribution type. Table 2.1 lists these types.

Table 2.1 Attribute types and their meaning

Attribute Type	Meaning
nameAttr	Short string
intAttr	Integer value
charAttr	One character
boolAttr	"true" or "false"
dbleAttr	Real number
strAttr	String
vec2Attr	Two-dimensional vector
vec3Attr	Three-dimensional data

The type "nameAttr" stands for a short string (alphanumeric and no spaces). Each element corresponds to an integer (an identifier), which is defined in a definition file. The type "dblAttr" stands for a real number; we assume that the corresponding values are treated as double-type values in the C++ language.

2.1.2 Definitions of an Element and Attribution

We describe the definitions of elements and attributions using XML rules. We define the type for each attribution; some of them are globally defined, and others are defined only for some elements. The XML file with these definitions is loaded into our XML parser as a definition file. The following expression is an example of the definition of an attribution.

```
<AttrForm name="gName" type="nameAttr">
```

```
        <NameEntry name="triangle" id="0"/>
        <NameEntry name="circle" id="1"/>
    </AttrForm>
    <AttrForm name="age" type="intAttr"/>
```

Attributions are defined using the tag "AttrForm." In a definition of the "nameAttr"-type attribution or the XML elements, we can define the names, i.e., the values of "nameAttr." These values are short strings, and each value corresponds to a unique integer value "id."

XML element definitions are described using the tag "ElmForm." The following expression is one example.

```
    <ElmForm name="Person" id="0">
        <AttrForm name="age" type="intAttr"/>
        <AttrForm name="weight" type="dblAttr"/>
        <AttrForm name="address" type="strAttr"/>
    </ElmForm>
```

Every "ElmForm" has the attributions "name" and "id." The attribution "name" stands for the tag name, and "id" is a unique integer value. This element has the subelements "AttrForm" or "NameEntry."

An attribution defined in "ElmForm" is valid only in the corresponding XML elements. In the case where an attribution is also globally defined, the new definition is valid in this element.

2.1.3 Functions of the Parser

Our XML parser is a class of the C++ language. The following are its main functions:
1. Load and save XML files.
2. Obtain subelements and attributions.
3. Seek elements by attribution values.
4. Add or remove subelements and attributions.

2.2 Expression Rules for Mathematical Documents

We use three element types for the description of mathematical documents: "InfoBody," "GrElm," and "TextElm." The attributions are "id," "ename," "etype," and "refId."

Table 2.2 Attribute types and their meaning and types

Attribute Type	Meaning	Type
InfoBody	Totality of document	------------
GrElm	An element in figure	------------
TextElm	An element in text	------------
id	Identifier	intAttr(integer)
etype	Type of element,	strAttr(string)
ename	Short name	strAttr(string)
refId	Id of link target	intAttr(integer)

For each XML element, there are some subelements in general. Standard subelements are expressed by tags. There is also a string part that is not surrounded by tags. We call such a string part a "*substring element.*" According to the format of our parser, there may be some substring elements in one XML element; however, in the rules for our mathematical document, we use only

one substring element for one XML element.

An XML element has some attributions. There must be "id," "ename," and "etype." The attribution "id" is an identifier, and we refer to an element using this number. We use "ename" and "etype" as a short explanation of the element. Using a translation table for "ename" and "etype," these string attributions are translated into a suitable short explanation. For example, attributions are expressed by English in an abridged form, and the same document can be translated into any language using the corresponding table. The attribution "refId" stands for the id number of the reference element.

In a string subelement, we can embed the "id" of a reference element in a string. The following element is an example of "TextElm" including a string subelement with reference elements.

```
<TextElm id="10" ename="ABC" etype="explanation">
        Triangle ABC @[01] is a right-angle triangle
        with the right angle A@[02].
</TextElm>
```

The expression "@[01]" represents the id number "01" of the reference element.

3. Software Tools

We assume that a visually handicapped person needs to obtain mathematical documents, and a support aids in this task. There may be some figures or tables in the documents that aid in understanding the document when a person is able to obtain visual information; however, they are barriers for the visually handicapped. Thus, we find other ways to express this content. The purpose of this study is an environmental consideration of the access to mathematical documents without visual information. We developed two software tools to create documents and to access the documents.

3.1 Software for Creating Documents

We consider two element types in our document: graphical and text elements. These elements and their connections are described with the XML rule (see Section 2). In this subsection, we explain how to create expressions using our software tools.

For graphical elements, we use a handwriting input method [01]. First, a user draws a curve or curves; then, the system recognizes the type of graphical element. After inputting the elements, we can create a new element using the developed ready-made elements or parts of them. Moreover, we can attach a string element (an explanation or a comment) and links to other elements.

For text elements, we input sentences in a text box. We can input some id number of the reference elements (see Section 2) in this string. In the text and graphical elements, we add some subelements that represent the ready-made elements. These ready-made elements are listed at the side of the input area as small icons. By selecting one icon with the right mouse button, several menu items are listed near the mouse cursor. By selecting the item "copy ID," the string "[@##]" (the part "##" is replaced by the corresponding id number) is stored. In the case where the user inputs a link in a string in a text element, he/she pastes the string "[@##]" at the point where he/she inputs the link. For the creation of a subelement, the system creates a simple element that has a connection to the XML element using the attribution "refId."

3.2 Software Tool for Expressing Documents

Our document is stored as an XML file; our system loads this XML file and allocates some strings to seven text boxes. First, the system sets the target element of the root element of the XML document. The attributions "name" and "type" are output in the text box "Target," as they are strings (Figure 3.1).
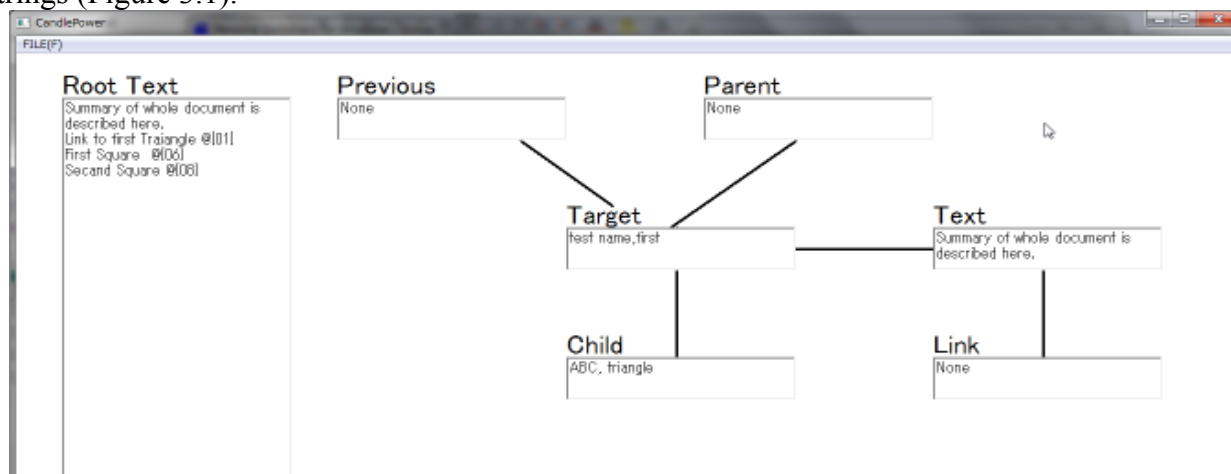

Figure 3.1 Software Tool for Expression

First, the name and the type of the first subelement are listed in the text box "Child." After depressing the right arrow key, the name and type (in short name-type) are replaced by those of the second subelement. After that, they are replaced again by the previous pair by depressing the left arrow key. Using the left and right arrow keys, we change the selection of a subelement. By depressing the down arrow key, the target element is replaced by the selected subelement. Then, the contents of the text box "Target" are replaced by the name-type of the new target element, and the name and type of the first subelement of the new target element is listed in the "Child" text box.

The upper-left text box is "Previous." The name-type of the previous target is listed here. The upper-right text box is "Parent." If the target element is a subelement of some element, the name-type of this parent element is listed here. These elements do not always exist; in this case, the message "None" appears.

The left large text box is "Root Text." A brief explanation for the entire document or a list of the contents is described in the text subelement of the root element. One disadvantage of our system or of nonvisual communication, is its nonbrowsability. Thus, this part may be very important for almost all cases, and we prepare this text box to access this subelement.

The text box to the right of "Target" is "Text." The first line of the text subelement of the target element is listed here. The output line of the text subelement is replaced by the other line using the up and down arrow keys with "SHIFT." The text box under the this text box is "Link." The name-type of the linked element is listed here. When there are several linked elements in a line, the first element is listed, and this can be replaced by using the left and right arrow keys with "SHIFT." By using the up arrow key, the target element is replaced by the parent element, and when the up arrow key is pushed with "SHIFT," the target element is replaced by the previous element. For the jump to a linked element in a text subelement, we use the down arrow key with "CTRL." In the case where the linked subelement has the attribution "refId," the target element is replaced by the element corresponding to this number ("id" of the element equals "refId").

As this software tool is developed for the visually impaired, the main operations are controlled by

keystrokes, and the contents are given as voice output. By using a screen reader, some basic tasks can be done without visual output (for example, load a file, exiting the system, and so on). It may not be access-friendly if we only use the screen reader to access a text document in our system. We assign our main functions to keystrokes for the quick and convenient use of our system. Table 3.1 lists these keystrokes.

Table 3.1 Keystrokes assigned to main functions

Keystroke	Action	Keystroke	Action
→	Next Child Element	L	Voice Output of Target Name-type
←	Previous Child Element	P	Voice Output of Parent Name-type
→+SHIFT	Next Link in Text	P+SHIFT	Voice Output of Previous Name-type
←+SHIFT	Previous Link in Text	T	Voice Output of Text (One Line)
↓	Change to Selected Child	N	Voice Output of Selected Child
↓+SHIFT	Next Line in Text	N+SHIFT	Voice Output of Selected Link
↓+CTRL	Change to Selected Link	R	Voice Output of Root Text
↑	Change to Parent Element		
↑+SHIFT	Change to Previous Element		

4. Elements Requiring Additional Explanation

We can create and express a mathematical document, and the most difficult problem remaining is to create a good document. In this section, we try to find some clues to solve this problem. We conducted a survey in the form of a questionnaire and found the graphical elements for which we need some additional explanations using the results. We analyze these data and provide some possible reasons for the results.

4.1 Questionnaire

We create three documents for the questionnaire. These are proofs of "the theorem of the circumferential angle," "the pythagorean theorem," and "the power of point theorem." There are some figures in the document (Figure 4.2), and without them, it may be difficult to understand the proofs. This document is written in Japanese, and after each graphical element, an element number is listed in parentheses. Figure 4.1 shows part of the document (English translation).

Proof
The angle ∠A(2) is right angle in the triangle △ABC(1).
Then we show the equation
$$|AB|^2 + |AC|^2 = |BC|^2$$
Consider three squares □ACPQ(6), □ABRS(7), □BCTU(8).
Each square includes a side AC(3), AB(4), or BC(5) respectively.
Let L(11) be a straight line that includes the point A(9) and is orthogonal to the side BC(10).
……………………………………………

Figure 4.1 Part of the Document (English Translation)

Originally, the element numbers were listed in ascending order. In general, the same element has several numbers. Owing to the differences in the word order in English and Japanese, the element

numbers are not listed in ascending order.

The participants of this questionnaire are university students from the faculty of engineering. All of them are sighted persons. They can understand these proofs in a few minutes after reading the documents, and they probably did not know the story of the proofs in advance. Thus, the graphical elements are important features that enable their understanding of the documents.
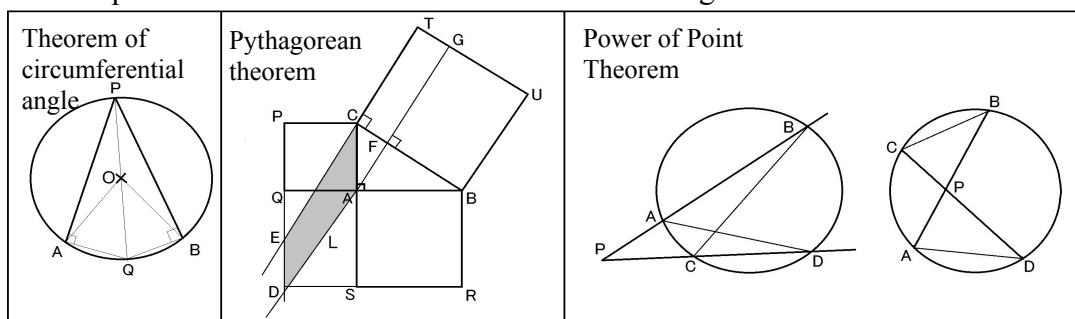

Figure 4.2 Figures in the Documents

After explaining the purpose, the questionnaire is carried out according to the following procedure:

1. Prepare the documents with and without figures.
2. The participant reads the document with figures until he/she understands the document.
3. Explain the next procedure for reading the document without figures and finding the elements that need some extra explanations.
4. According to this procedure, the participant reads the document, and the interviewer writes down the corresponding number immediately after finding the element.
5. Check all numbers after reading the document.

4.2 Analysis of the Responses

Several persons gave responses to these questionnaires: 33 persons for the circumferential angle

Table 4.1 Top five elements requiring explanations

Circumferential (33)		Pythagorean (36)		Power of Point (27)	
Element	Ratio (%)	Element	Ratio (%)	Element	Ratio (%)
$\angle$AOQ	78	$\square$DACE	89	$\triangle$PBC	70
AB (arc)	67	$\square$FGTC	69	$\triangle$PAD	67
Q (cross pt)	67	$\square$ACPQ	61	$\angle$BPC	67
$\triangle$APB	63	TC (side)	53	P (point)	63
$\angle$APB	59	$\square$BCTU	53	First Line	5

theorem, 36 persons for the pythagorean theorem, and 27 persons for the power of point theorem. The top five elements, amongst many others, which require further explanations are listed in Table

4.1.

There are some trends in the selections of these elements. In these cases, there is one base graphical element, and the participants grasp a subelement easily. It is very difficult to understand the case in which an element cannot be defined uniquely only by the explanation. The quadrangles ACPQ and BCTU in the proof of the pythagorean theorem and the point P in the proof of the power of point theorem are examples of this. For all of them, more than half of the participants required additional explanation. Moreover, there are some elements that need several steps to define them. The cross points E, G, and F are typical elements. For point E, we need four steps to define this element, and the points G and F require three steps. 50% of the survey participants indicated the need for extra explanations for point E, and 25% of the survey participants indicated the need for extra explanations for points G and F.

4.3 Feature Values for Confusion

We created three explanations using figures in our questionnaires. In these explanations, some graphical elements were difficult to understand without figures. In this subsection, we try to evaluate this confusion for the graphical elements.

For this evaluation, we consider three feature values: Complexity, Dimension, and Generation. For Complexity and Generation, we define the concept of a "Base Element." In our situation, there is one fundamental element for each explanation, and all elements are derived from this element directly or indirectly. For the circumferential and power of point theorems, the base elements are circles, and a right-angle triangle is the base element for the pythagorean theorem. Complexity describes how difficult ti is to explain an element. This is rated by three phases of evaluation. Dimension (+1) is also rated by three phases; it takes the value 1 when the element is a point, 2 for a curve or line, and 3 for a polygon. When all point subelements are in the base element, the value of Generation is 1. In the case where an element consists of some new elements and some other elements for which values of Generation are given, and set the the maximum number of Generation n, the value of Generation of this element is $n+1$.

Table 4.2 Features and their values

Complexity			Dimension(+1)			Generation	
1	Only Base Elements		1	Point		1	Base Elements
2	With Other Elements		2	Line Segments, Circle		2	Base Element and Other
3	With Ambiguity		3	Polygon		N	Under N-1-th Generation Element and Other.

4.4 Approximation of Confusion

In our questionnaires, we obtained elements that need some explanations. Then, we defined the degree of confusion as the proportion of people who needed additional explanations. Using the data obtained in the questionnaires for the circumferential and power of point theorems, we approximated the degree of confusion by a linear regression. We used normalized values (in [0,1]) for each feature value, and we obtained the regression

$$\text{Confusion Ratio} = 0.341 \times \text{Generation} + 0.253 \times \text{Dimension} + 0.361 \times \text{Dimension}$$

Table 4.3 lists the results for the elements of the pythagorean theorem.

Table 4.3 Linear regression for the confusion ratio

No.	Graphical Element	Group	Confusion Ratio	Regression Value	Error	Gene-ration	Di-mension	Com-plexity
1	△ABC	1	8%	25%	17%	0.0	1.0	0.0
2	∠A	1	6%	13%	7%	0.0	0.5	0.0
3	AC	1	6%	13%	7%	0.0	0.5	0.0
9	A	1	3%	0%	3%	0.0	0.0	0.0
6	□ACPQ	2,3	61%	73%	12%	0.3	1.0	1.0
17	E	2	50%	52%	2%	1.0	0.0	0.5
18	□DACE	2	89%	77%	11%	1.0	1.0	0.5
24	F	2	25%	41%	16%	0.7	0.0	0.5
25	G	2	36%	41%	5%	0.7	0.0	0.5
27	□FGTC	2	69%	66%	3%	0.7	1.0	0.5
28	△PEC	2	44%	77%	33%	1.0	1.0	0.5
33	CF	2	39%	53%	15%	0.7	0.5	0.5

4.5 An Example of Graph Improvement

Using our evaluation, we obtained an approximate confusion ratio. In the explanation of the pythagorean theorem, the approximate confusion ratio for the quadrangle DACE is 77%; therefore, we can understand the requirements for the additional explanation of this element. If we create a graph structure for this document (using our software, for example), the explanation for the quadrangle DAEC may be complicated (Figure 4.3) and difficult to understand without a figure. Thus, we need some additional explanation for correct understanding of the document.
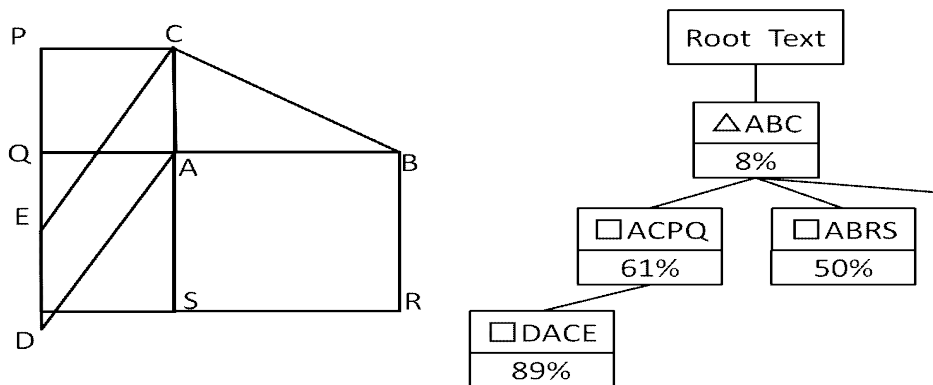
Figure 4.3 Improvement of Graph Structure

5. Conclusions

We developed software tools for the creation and expression of mathematical documents to

support visually impaired persons. Our documents describe the graph structure of a mathematical document using XML syntax. The expression tool represents every part of the document using a keyboard and voice output.

For improving documents, we analyzed the graphical elements for which many people required some additional explanations. We found some tendencies for these elements, and we will be able to create tools for automatic improvement of the documents if we find some adequate evaluation method for these tendencies. This is planned for future work.

Acknowledgement

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Comparison of Chinese and Australian attitudes to CAS in education

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Abstract

Various articles in both academic journals and the popular press over the past decade have attempted to uncover the reasons for Chinese success in school mathematics. As ranked by PISA, Chinese students consistently do better than their Western counterparts. Reasons which have been proposed (in the West) include more time for mathematics in schools, greater knowledge of mathematics (as well as ability) by the teachers, greater involvement by parents in their child's education, and an over-riding ethos that every child can succeed. It is notable that Chinese mathematics (prior to university) puts a high emphasis on technical skills in algebra and geometry (and in some provinces, calculus), and is very focused on success in examinations. The Chinese approach has its critics, but there is no doubt that whereas mathematical standards, as measured by international tests, seem to be slipping in the West, those standards remain very high in China. The Chinese approach however seems to provide little space for experimentation in mathematics, and in particular for the use of technology. Educators in the West are slowly realizing that using computer algebra systems can greatly enhance learning of, and interest in, mathematics, and that the use of such systems seems not to impair technical skills. This article is an initial investigation into the use of CAS in the context of the Chinese education system, and how attitudes differ from those in the West.

1 Introduction

Every few years the Programme for International Student Assessment (PISA) of the OECD tests 15 year old school students across a range of disciplines, including of course mathematics. Table 1 shows the results from China and a handful of predominantly English speaking countries, for the two most recent years of PISA assessment [6, 7].

It is immediately seen that Chinese students consistently outperform students in other countries. The 2012 results unleashed an unedifying storm of comments [1, 10]; some (American) writers insisting that the Shanghai scores were due to only the best students being tested, rather than all students; these accusations were refuted by other American as well as Chinese writers and academics.

Place	2009	2012
China Shanghai	600	613
Singapore	562	573
China Hong Kong	555	561
China Macao	525	538
Canada	527	518
New Zealand	519	500
Australia	514	504
OECD Average	**494**	**498**
United Kingdom	492	494
USA	487	481
(Lowest)	333	368

Table 1: PISA mathematics results from selected countries and regions

However, a study in Australian schools [11] has found that Chinese international students tend to perform better than their local peers, which would seem to indicate some cultural element.

The following have been proposed for Chinese success in school mathematics [8] :

1. There is more school time given to mathematics.

2. Teachers know more, and are themselves better at, mathematics than their Western counterparts.

3. Parents are heavily invested in their child's education, and put time and effort into ensuring their success: extra tutoring for example.

4. Children are expected to succeed.

5. The Confucian ethos of hard work and persistence does not exist to the same extent in the West.

However, the Chinese themselves are aware that their mathematics instruction methods are not necessarily perfect: that the "traditional deference to authority, breeds a conformity which could be detrimental to creativity" [10] , and also that there is nowhere to go in a success-driven system for students who, for whatever reason, don't succeed.

There has been a considerable sharing of ideas between China and the West in regards to mathematics teaching, with numerous teacher exchanges and an open-mindedness to learn from each approach. Chinese mathematics teaching tends to be highly prescriptive, textbook-based, and measured–every child in a class will be on the same page of the same textbook at the same time. Western educators favour a more flexible approach where students might be streamed into different groups working on different problems, or moving through their texts at different speeds. Chinese educators will have spent much of their university degree learning about mathematics and mathematics education–even in primary schools teachers are discipline specific –generalist Western primary teachers will not have had nearly as much exposure to mathematics in their teaching degrees.

The Chinese approach to learning and teaching means it is very difficult for a teacher to develop a new curriculum or teaching style: what is to be taught, and how it is to be taught, are mandated

by Government decisions; it is the job of the teacher to ensure that the Government's regulations are precisely followed. Teachers in the West have far more freedom in how they decide to teach the curriculum.

The introduction of technology in mathematics teaching and learning brings much of the differences between China and the West into sharp relief. Several decades of intense research has shown that the use of such technology as CAS Calculators can re-engage students, and revitalize tired curricula. See, for example, Herron et al. [2] for the results of a recent study. Moreover, students who have learned their mathematics experimentally and experientially through CAS show no lacking in technical skills from students who have learned a more traditional skills-based curriculum. However, old habits die hard, and certainly many educators are doubtful about the use of computational tools as part of a mathematics curriculum.

There has been hardly any research or discussion into mathematics education at a tertiary level, and the differences inside and outside China, although this state of affairs is changing [3]. However, we may take as axiomatic that differences in schools would lead to similar differences in universities and colleges: students and teachers in both places have gone through the same educational background, and would be expected to teach and learn in a similar manner to that at school.

2 The Survey

A survey on the use of CAS in universities was distributed to academics in both Australia and China–the survey was prepared and distributed according to the regulations of the Human Ethics Research Committee at Victoria University; this committee oversaw the survey, and ensured that it satisfied all of the ethics requirements.

The survey consisted of a number of questions, Likert scale, Yes/No, free text, to solicit respondents' knowledge of CAS, uses of CAS, and opinions of CAS.

The survey consisted of several different sections: demographic (age, position, students taught, experience of using CAS), and then branches depending on whether the participants was currently using CAS, had used CAS in the past but not currently, or have never used CAS. The survey ended with some general review questions about attitudes towards CAS in teaching, as the flow chart in figure 1 shows.

This means that not everybody in fact completed the same set of questions; the branching ensured that the questions were geared to the participants knowledge of, and use of, CAS.

The survey was distributed to tertiary mathematics educators in Australia, and in Shanghai, and there were about 40 Australian responses and 60 Chinese responses. There were some immediate differences in the demographics:

- Far more Chinese educators were in the 45–54 age bracket than Australians (25% from China; 10% from Australia)

- Far more Australian educators were in the 55-65 age bracket (32% of Australians; 5% of Chinese)

- Far more Australians had student cohorts primarily of science and engineering (82% of Australians, 21% of Chinese) and also of Education (58% of Australians, 7% of Chinese). However, these figures are misleading, as fewer Chinese educators named different cohorts.
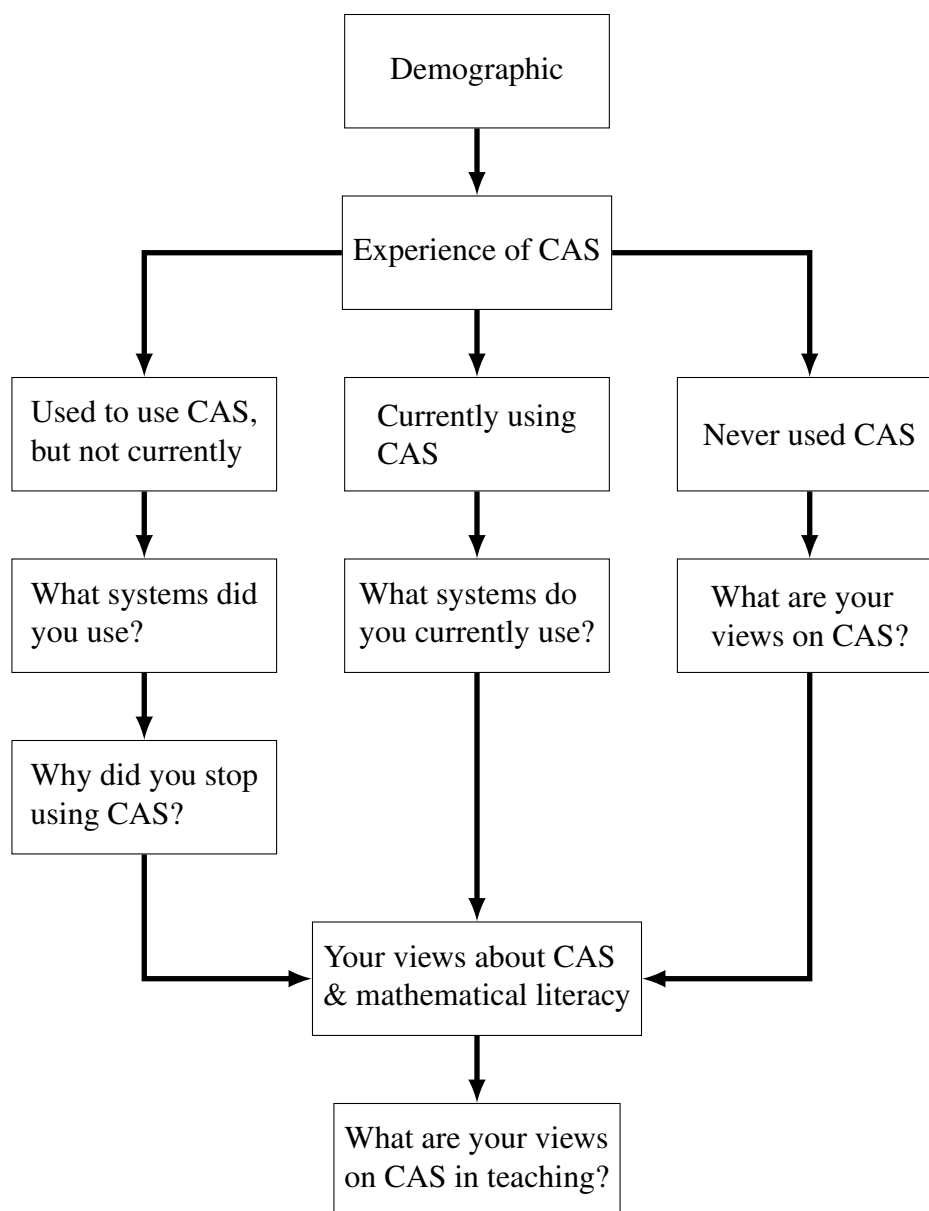
Figure 1: The schematic of the survey

With experience of using CAS in teaching, far more Australian had, or were currently using CAS, than Chinese:

- 79% of Chinese respondents had never used CAS for teaching, as opposed to only 26% of Australians

- 3% of Chinese had once used CAS but not currently, as opposed to 18% of Australians

- 10% of Chinese educators are currently using CAS, and 21% of Australians.

These numbers alone, irrespective of further analysis, indicate what we would expect from the didactic and prescriptive model of Chinese mathematics education. In the current Chinese system, it seems

difficult for an individual academic to experiment with new methods of teaching, or to replace a drill-and-practice pedagogy with a more experimental one.

Of the educators currently using CAS, the Chinese educators were evenly spread between using it in the classroom and using it to prepare material (for example, calculus questions which have simple, integral answers), with one lone respondent claiming to use CAS for assessment. In comparison, Australian educators were evenly spread amongst those three uses.

There were several questions asking about exposure to different systems, including commercial (Maple, Mathematica, Matlab), open source (Octave, Sage, Maxima), handheld (TI-nspire CAS, Casio ClassPad, HP Prime calculators), assessment software (Pearson MyMathLab, Wiley Assist, MAA WeBWorK, STACK), dynamic geometry software (GeoGebra, Cabri Geometry, Geometers Sketchpad). Respondents were asked to note whether they had heard of this software, and whether they had ever, or were now, using it. As might be expected, the commercial entities, in particular Mathematica, Maple, Matlab and some of the CAS Calculators, had the greatest exposure among both cohorts. However, there was a diversity of exposure to the open-source systems and to the assessment systems. Far fewer Chinese than Australians had exposure to open-source systems such as Maxima, Sage, Octave, GeoGebra and to assessment software. And of the Chinese respondents who had used CAS for teaching in the past, none of them had used open-source systems.

The lack of open-source software usage in China may simply reflect a language issue. Mathematica, Maple, and Matlab all exist in Chinese versions; however open source systems require volunteers to translate both the system and the documentation into other languages. For example, although there exist a number of documents describing Maxima and its use in Chinese, all the examples and screenshots are in English. Figure 2 shows such a screenshot describing Maxima.

Figure 2: Maxima in Chinese

The explanation is in Chinese, but all the commands are in English. This fact alone is a serious impediment to the take-up of open-source software in China. Maple has long provided language packs in a number of different languages, and currently both the user interface and context-sensitive menus are available in Simplified Chinese, and most of that functionality is available also in Traditional Chinese. Maple TA (the teaching and assessment package for Maple) has had a Chinese language pack since 2010, which makes it all the more remarkable that its take-up, at least as indicated by our

survey, appears to be not wide.

There are some open-source software systems with Chinese language packs; for example GeoGebra, shown in figure 3
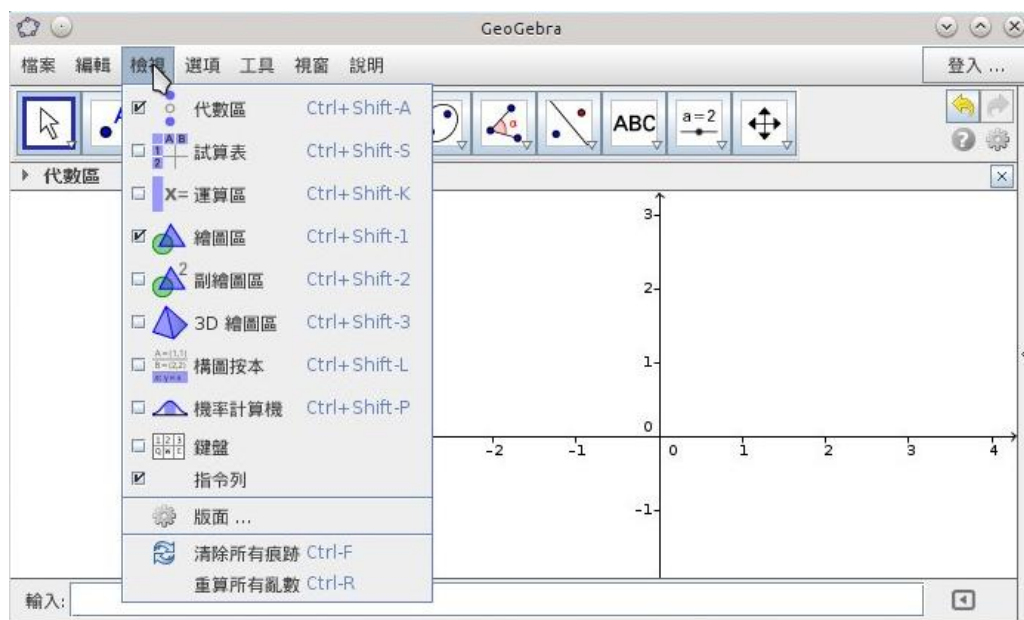

Figure 3: GeoGebra in Chinese

However, even with systems that supposedly have Chinese language packs, information in Chinese is interspersed with the Latin text of the CAS commands, as figure 4, which is taken from a published article, shows.

Figure 4: Discussion of Mathematica in Chinese

There were considerable differences in attitude towards CAS by academics who never used CAS, between the Chinese and Australian respondents. Thirty of the Chinese (50% of the total) and 17 Australians (slightly over 50%) claimed never to have used CAS in teaching. Of those respondents:

- 33% of the Chinese, but only 11% of the Australians, claimed to not have time to learn about CAS.

- 27% of Chinese, but only 11% of Australians, agreed that the use of CAS doesn't prepare students for future mathematical work. This is a belief that flies in the face of much recent research, so we would expect that these people were primarily research mathematicians rather than educators.

- 40% of Chinese, but only 17% of the Australians, claimed not to have time to change the syllabus to include CAS.

- More telling still, over 50% of the Chinese, but only about 11% of the Australians, claimed not to have sufficient support to make CAS available, or to make online assessment available.

- Finally, nearly a quarter of the Chinese, but none of the Australians, chose "Agree" to the statement that "CAS can't be used in assessment, so I don't use it in teaching".

In the final sections, first about mathematical literacy there was one major difference:

- only about 10% of the Chinese respondents felt that CAS enables mathematicians to work more efficiently; whereas over 50% of the Australian thought so.

In the section about attitudes to CAS in teaching:

- over 61% of the Chinese respondents but less than 30% of the Australians believed that CAS encouraged students to examine their solutions more carefully

- about 30% of the Chinese respondents but only about 12% of the Australians believed that CAS usage does not make classes more interesting for students

- 50% of the Chinese respondents but less than 20% of the Australians believed that CAS helps students obtain a deeper understanding of concepts

- nearly 60% of the Chinese, but only 30% of Australians, thought that CAS encouraged collaboration amongst students

- over 35% of the Chinese, and only 16% of the Australians, also felt that CAS does not help students understand mathematical concepts

- over 64% of the Chinese, and only 22% of the Australians, believed that CAS examples improve student attention in class

- over 30% of the Chinese, and only 16% of the Australians, believed that CAS use distracts from understanding mathematical concepts.

These are shown in figures 5 and 6.

These numbers seem at first to be contradictory, but in fact there are several strands here:

1. We have seen that Chinese educators, in general, have had less exposure to CAS than Australians; attitudes from the Chinese would seem to be more idealistic than those from the Australians.

2. In both countries there is a strong opposition to the use of CAS in teaching; based on a notion that the use of CAS is not really "doing mathematics", and that pencil-and-paper drill-and-practice is the only way of properly learning mathematics. As noted previously, the now extensive literature strongly points to the opposite conclusion.

3. Mathematics educators in both countries seem to be coming aware of the important role that technology now plays in mathematical research and practice, however in both countries tertiary mathematics education consists mainly of "covering" a syllabus, and as such is heavily teacher-centred.
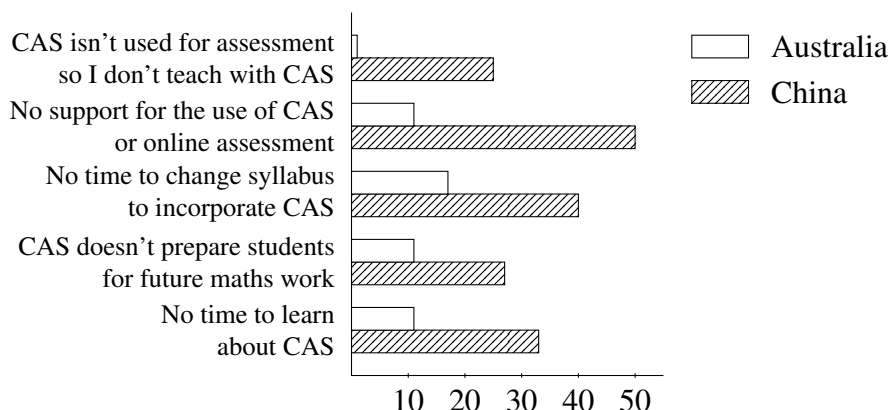
Figure 5: Attitudes to CAS

3 Discussion of the survey

We have seen that there are both similarities and differences between Chinese and Australian attitudes to the use of CAS and other computer systems in educations. Much of the differences seem to come from Chinese lack of exposure to CAS, and to the more prescriptive nature of mathematics education. At a university, a syllabus will be prescribed, and there is little option for a single staff member to amend either the syllabus, or the modes of teaching, or the methods of assessment.

It is worth nothing that Chinese syllabi are in general more prescribed that those in Australian or American universities, where there is greater autonomy. The Chinese Government, through a number of influential and powerful committees, produce guidelines as to the content of mathematics syllabi [5]. Even though they are officially only guidelines, and individual universities or academics may wish to (and occasionally do) teach different material, in practice most teaching at universities follows the guidelines as though they were prescriptive. This environment would appear to make it more difficult for a single academic to radically reform a syllabus, or to change methods of teaching—Chinese
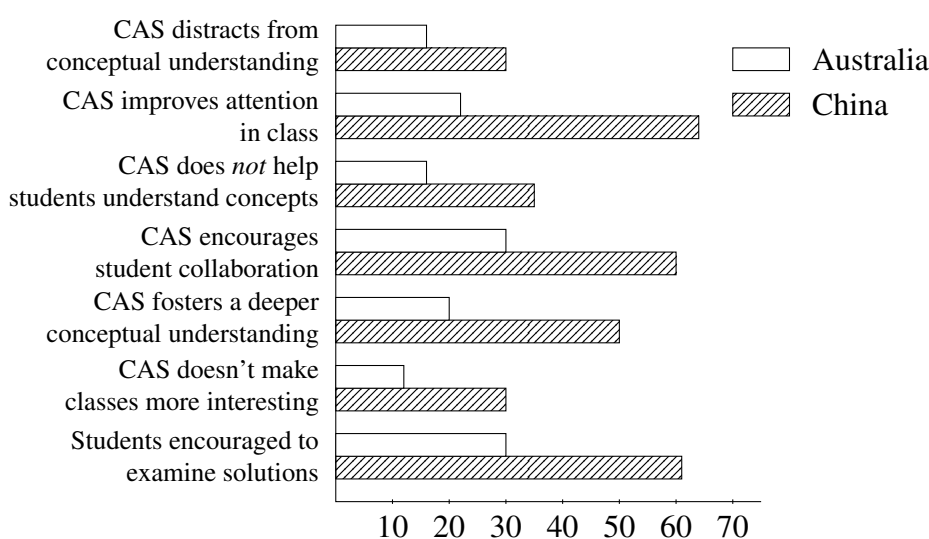
Figure 6: Attitudes to CAS in education

university teaching is predominantly lecture-based, and sometimes students find the transition from secondary school to university difficult.

Curiously, a study some years ago [9] found that Chinese teachers in "lower grades" had more confidence in employing technology than did their US counterparts; the author's interpretation was that Chinese mathematics teachers in lower grades are in general better trained and prepared. Note that technology in the context of this article meant generic technology: AuthorWare, Flash, PowerPoint. The author claimed that Chinese teachers were more keen to use technology for "instruction". There was no indication that teachers would put such technology in the hands of learners.

4   Conclusions

Although there appear differences between the Chinese and Australian attitudes, as evidenced by our survey, the similarities in fact outweigh them. In both countries tertiary mathematics is teacher-centred and syllabus-driven; in both countries the driving force behind most teaching is that of "covering the syllabus"; of "getting through" all the material. This leaves little time for experimentation, and for the use of more student-centred learning models such as problem-based learning, in which one may expect technology to play a major part.

The statistics presented in this paper have been descriptive only; further work will uncover statistical significances, and the degrees to which attitudes in both countries differ.

However, based on the results so far, indications are that academics in both countries have much of the same attitudes: that the use of CAS may well deepen student understanding of mathematical concepts, but there is no room in the syllabi, and little or no local support, for its current use.

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Figure Drawing using K_ETCindy and its Application to Mathematics Education
– Practical example of application of mathematics to mathematics –

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Abstract

Geometric construction normally means generation of a gure suited for given conditions using rulers and a pair of compasses only for a nite number of times. Hereinafter, this is referred to simply as geometric construction. This paper presents a discussion of, in addition to geometric construction, the practice of drawing gures by adding mathematical contents.

When mathematical material is added to geometric construction using rulers and compasses, the use of dynamic geometry (DG) software is one option, whereas K_ETCindy is used for this study because K_ETCindy is equipped with DG′s Cinderella as GUI and can be used for drawing gures by Script as CUI. Therefore, mathematically precise gures can be drawn with ease, producing beautiful results. This paper explains gure drawing while the quadratic curve concept is added to geometric construction. The author considers that gure drawing by Script is extremely useful for mathematics education from the viewpoints of application of mathematics to mathematics. This point will be discussed hereinafter.

1 Introduction

Quite a few teachers use LATEX for the production of handouts to be used for math classes and test questions. The author encountered teachers who were struggling to nd and use a system that can output precise and good-looking gures and graphs. The author tested several software packages and now uses K_ETCindy because it can output precise and good-looking gures and graphs by simple manipulations.

One bene t of K_ETCindy is that it uses dynamic geometry software Cinderella as the GUI, thereby allowing visual operation. Regarding GUI operation, although operating environments are almost identical to those of other software, an important bene t of Cinderella is that the generation of gures and graphs is made possible by a Character User Interface (CUI).

An additional bene t is that necessary commands and function formulae can be added by programming[1]. Figures in Japanese mathematics[2], which might be drawn only slightly, can be drawn now to a greater degree by the addition of mathematical contents such as quadratic curves.

In addition, with KETCindy, the quality of gures and graphs is equal to or greater than those depicted in the textbook. Furthermore, gures are generated precisely by simple manipulation. Results thus obtained can be output beautifully by LaTeX. Thanks to this system, the quality of gures and graphs used in the data for author′s class and academic papers were improved remarkably. The prime reason why the author uses KETCindy in LaTeX lies here.

All gures shown in Chapter 2 are drawn by KETCindy. The high quality of these gures is readily apparent: they are equal to or greater in terms of quality than those depicted in the textbook. Figure drawing in which Script is used concomitantly is explained taking Japanese mathematics problem as an example. At the same time, the bene ts of rendering gures using LaTeX and KETCindy are reported. In Chapter 3, future tasks of mathematics education [1] and fostering of mathematics teacher will be discussed based on the contents of Chapter 2 [2]. In the Appendix, the contents of the Encyclopedia of Geometric Solution [3], which presented the problem consciousness for this paper, are cited.

2 Quality gures drawn using Script and KETCindy

Wasan dealt with numerous problems related to gures [4]. If a drawing meeting with the conditions is given, then the solution itself leading to an answer is not so di cult.

However, in some cases, the generation of a gure is di cult even if the conditions are being given. Here, example problems 1, 2 and 3 shown below are addressed [5]. In fact, the center of circle P can be drawn by adding a quadratic curve concept to gure drawing using rulers and compasses. This illustration presents a quadratic curve drawn using Script. Beautiful drawings can be output by KETCindy.

Ex 1: Find the length of one side of a square when the diameter of circle P is 1.

Ex 2: Find the length of one side of a square when the diameter of circle P is 3.

Ex 3: Two large circles Q and two small circles P are shown in outer circle O.
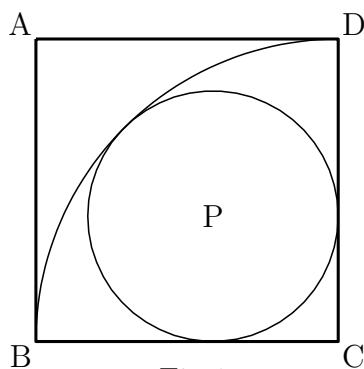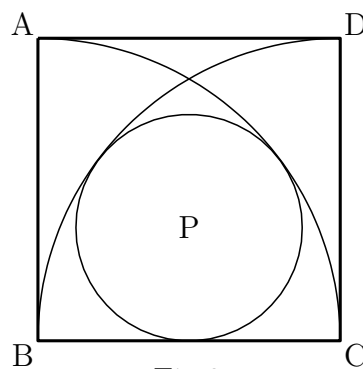When the diameter of the small circle P is 2, nd the diameter of the large circle.
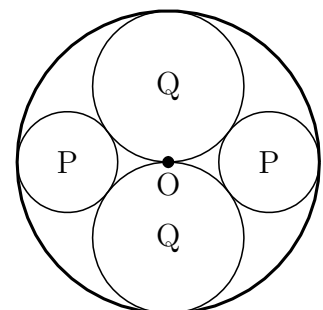

Fig.1.

Fig.2.

Fig.3.

[1] Function formula capable of creating parabola, ellipsoid, hyperbolic curve, and symbols showing that the lengths of line segments are equal are added by the program.

[2] Japanese mathematics: Wasan which was developed during the Edo Period (1603 – 1867).

2.1 Figure drawing of Example 1: Utilization of a parabola

Example 1 : As illustrated at right, in square ABCD, circle P is inscribed in quadrant C having its center at C, and inscribed on side BC and side CD.
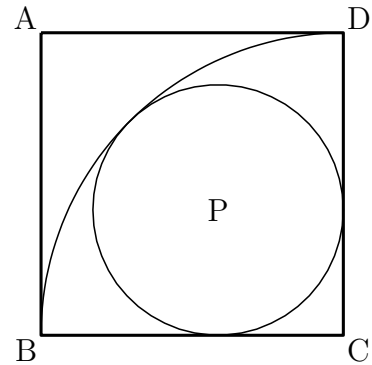Find the length of one side of the square when the circle diameter is 1.

Answer : The length of one side of the square is $\dfrac{1+\sqrt{2}}{2}$.

Fig.1.

2.1.1 Procedures for drawing circle P

Procedures for drawing circle P shown in Example 1 are the following.

1. Center P of the circle passes through intersection E of quadrant C and diagonal line AC.
2. Center P of the circle draws a parabola while E is the focal point and side BC is the directrix.
3. Intersection of a parabola and diagonal line AC is the center P of the circle to be obtained.
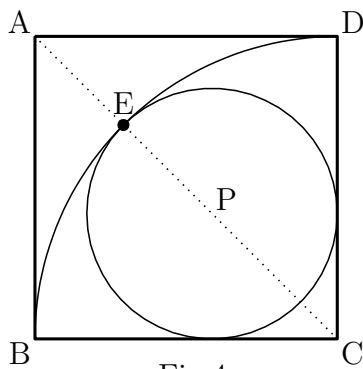
Fig.4.
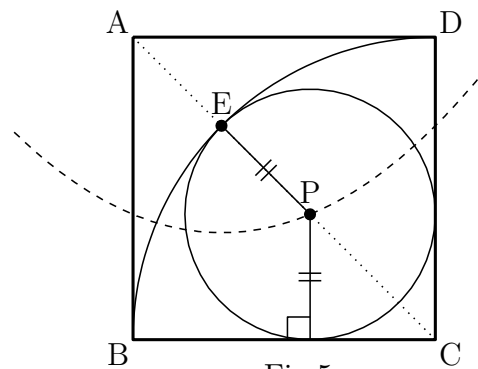
Fig.5.

2.1.2 Example of Script of Example 1

Here, the example of Script of Fig.5 is presented.

1:Fhead="Parabola.tex";	File name: Parabola.tex
2:Ketinit();	Initialization of K$_E$TCindy
3:Addax(0);	Coordinate axes are not drawn
4:Listplot([A,B,C,D,A],["dr,2"]);	Square ABCD is drawn by line 2
5:Listplot([A,C],["do,1"]);	Diagonal AC is drawn by dotted line
6:Circledata([C,B],["dr,1","Rng=[pi/2,pi]"]);	Quadrant C, the part of circle C from $\pi/2$ to π
7:Parabolaplot("1",[E,B,C],["da,1"]);	Parabola with focal point E and directrix BC
8:Putintersect("P","rt1para","sgAC");	Intersection P of parabola and diagonal line
9:Circledata([P,E],["dr,1"]);	Circle passing through center P and point E
10:Pointdata("1",E,["size=7"]);	Point E is shown by size 7
11:Pointdata("2",P,["size=7"]);	Point P is shown by size 7
12:Letter([A,"nw","A",B,"sw","B",C,"se","C",D,"ne","D",E,"n2","E",P,"n2","P"]);	Points A, B, C, D, E, P are shown
13:Windispg();	Displayed on the display

Notes : The le name written on the rst line and the script shown above are written between the 2nd line and the 13th line. Then the gure can be output by KETCindy on Cinderella, which is the GUI. Furthermore, actual gures can be con rmed by PDF. For the alteration of symbols in the gure and the vertex, the gure might be changed directly returning to Cinderella or changed by Script. At the same time, a tpic le which can be inserted by LATEX can be generated by the name of the rst line. If the le name in the rst line is inserted under the layer environment, then a gure can be inserted at the desired point[3]. KETCindy includes expressions of various kinds used in the textbook from mathematics education viewpoints. For example, for the line type, line, dashed line, and dotted line available, each is designated by "dr,n", "da,n" and "do,n" and the thickness can be designated too. The script in the fourth line means a line and thickness 2. The default of the line type and thickness is "dr,1". Function formula Parabolaplot in the seventh line for drawing parabola is added by the program. With KETCindy, function formula and symbol can add functions to be generated as necessary. Those added as necessary are explained hereinafter in each case.

2.1.3   Bene ts of using KETCindy

As described previously, the author uses KETCindy to employ mathematically precise gures which are also quality printing materials for mathematics education. Quality printing materials can then be inserted into LATEX. Comparison of quality of the drawing between DG (a gure of Cinderella is used here) and a gure by KETCindy reveals the di erence between the two at a glance.

Furthermore, it might be cited that with KETCindy, the description of Script is mathematical and brief. For example, as represented by the seventh line of parabola Script in the statement above, the focal point and directrix are simply designated. At designation, the focal point and directrix are simply designated by symbols referring to the gure of Cinderella.
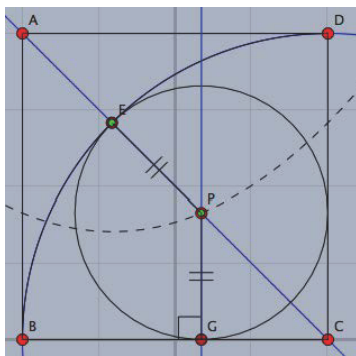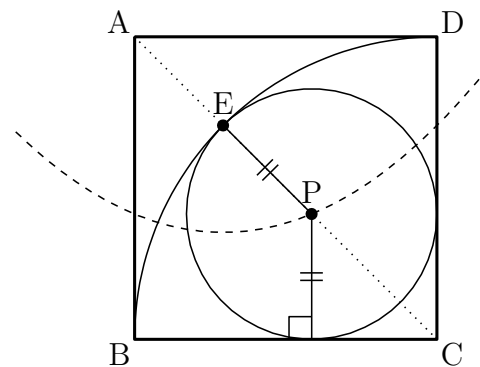

Fig.6.

Fig.5.

The following points can be cited as bene ts of KETCindy:

1. Cinderella is useful as DG.
2. Precise and quality gures are drawn by DG and might be presented to students as printing materials.
3. Alteration of drawn gures is simple.
4. Program of KETCindy is mathematical and easy to understand.
5. It is freeware and can be introduced easily into school education.

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[3] Details are shown in the Appendix provided at the end of this paper.

2.2 Figure drawing of Example 2: Utilization of ellipsoid

Example 2 : As illustrated at right, circle P is in square ABCD and is tangent to two quadrants B and C having its center at point B and point C and to side BC. When the length of the diameter of the circle is 3, nd the length of one side of the square.
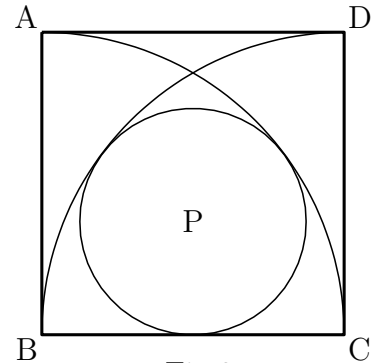
Fig.2.

Answer : The length of one side of the square is 4.

2.2.1 Procedures for drawing circle P

Procedures for drawing circle P shown in Example 2 are the following.

1. Circle P passes through midpoint E of side BC.
2. Circle P is inscribed to quadrant B at point Y.
3. BP + EP = BP + PY = BY = BC (constant).
4. Circle P passes through point E inside of circle B (quadrant B). Therefore, point P is on the ellipsoid having focal point on two points B and E.
5. Center P of the circle is also on the vertical bisector of side BC.
6. Therefore, center P of the circle to be obtained is the intersection of the ellipsoid and vertical bisector.
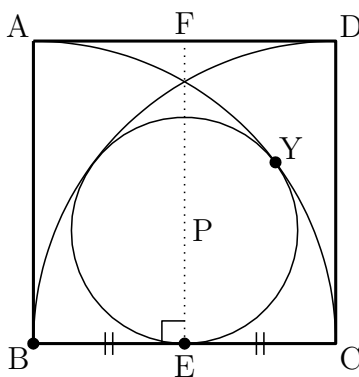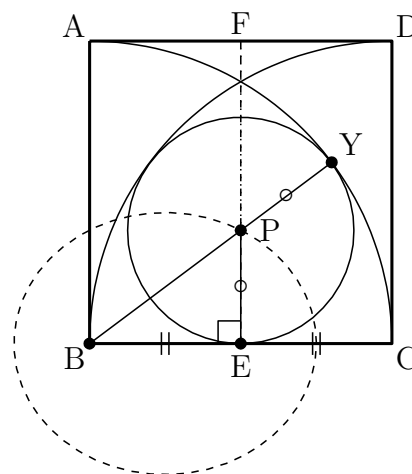
Fig.7.

Fig.8.

2.2.2 Example of Script of Example 2

Here, the example of Script of Fig.8 is presented.

4:Listplot([A, B, C, D, A],["dr,2"]);	Square ABCD is drawn by line 2
5:Circledata([B,C],["dr,1","Rng=[0,pi/2]"]);	Quadrant B, the part of circle B from 0 to $\pi/2$
6:Circledata([C,B],["dr,1","Rng=[pi/2,pi]"]);	Quadrant C, the part of circle C from $\pi/2$ to π
7:Listplot([E,F],["do,1"]);	Line segment EF: Vertical bisector EF
8:Ellipseplot("1",[B,E,|B C|],["da,1"]);	Ellipsoid having two focal points B and E
9:Putintersect("P","rt1elp","sgEF");	Intersection P of the parabola and segment EF
10:Circledata([P,E],["dr,1"]);	Circle passing through center P and point E
13:Pointdata("1",P,["size=7"]);	Point P is shown by size 7
15:Drawsegmark("1",[B,E],["Type=2"]);	Mark showing equivalent line segment
16:Drawsegmark("2",[C,E],["Type=2"]);	Mark showing equivalent line segment

Notes : In Example 1, all statements of Script are shown. In Example 2, only major Scripts are shown. Therefore, some line numbers are missing. Drawsegmark shown in the 15th line and 16th line means that two line segments are equal as shown in Fig.7 and Fig.8. Namely, BE = CE. It is visually understood by this symbol that point E is the midpoint of side BC. This expression is used frequently in Japanese mathematics textbooks. In Fig.8, if results show that two points B and E are of focal points, then point P to be obtained should satisfy BP + EP = 2a (2a is the sum of distance) as characteristics of the ellipsoid. It is important to recognize that it is the radius of quadrant B. If this is found, |B   C| is simply designated in K$_E$TCindy. This is similar to vector notation and is mathematically simple.

2.3   Figure drawing of Example 3: Utilization of hyperbola

Example 3 : As illustrated at right, two large circles Q
and two small circles P are shown in outer
circle O.
When the diameter of small circle P is 2,
nd the diameter of the large circle.

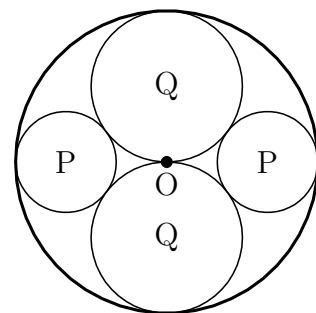**Answer** : The diameter of the large circle is 3.

Fig.3.

2.3.1   Procedures for drawing circle P

Procedures for drawing circle P in Example 3 are the following. Two large circles Q are designated as Q1 and Q2. Two small circles P are designated as P1 and P2. Locations of the centers of two large circles Q1 and Q2 might be readily apparent. Procedures drawing center P1 and P2 of two small circles are explained hereunder.

1. Points of tangency of the circle P and circle O are designated respectively as A and B. The line segment AB is a diameter of circle O.
2. Circle P1 and circle Q1 are circumscribed at point Y.
3. Point P1 is outside of circle Q1.
4. |Q1P1   AP1| = |Q1P1   P1Y| = |Q1Y| = |Q1O| (constant).
5. Therefore, P1 is a hyperbola having two focal points of A and Q1.
6. The intersection of hyperbola and diameter AB is center P1 to be obtained.

Similarly, drawing of the center of small circle P2 is obtainable by the hyperbola.
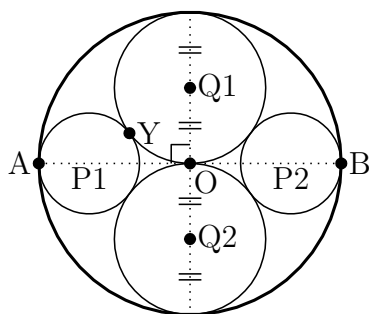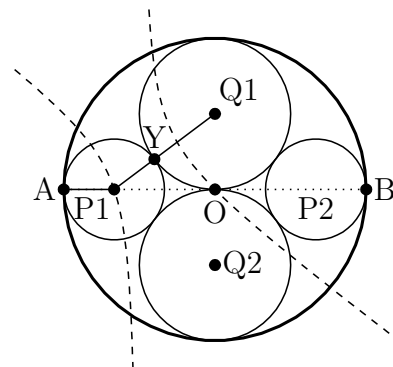

Fig.9.

Fig.10.

2.3.2 Example of Script for Example 3

Here, the example of the Script for Fig.10 is presented.

4:Circledata([O,A],["dr,1"]);	Line type 1 passing through A at center O
5:Circledata([Q1,O]);	Circle having its center at Q1 and passing through O
6:Circledata([Q2,O]);	Circle having its center at Q2 and passing through O
7:Listplot([A,B],["do,1"]);	Line segment AB is shown by dotted line
8:Hyperbolaplot("1",[A, Q1, |Q1 O|]);	Hyperbola having focal points A and Q1
9:Putintersect("P1","rt1hyp","sgAB");	Intersection P1 of hyperbola and line segment AB
10:Hyperbolaplot("2",[B, Q1, |Q1 O|] ,["notex"]);	Hyperbola of focal points B and Q1 which is not shown in T$_E$X
11:Putintersect("P2","rt2hyp","sgAB");	Intersection P2 of hyperbola and line segment AB
12:Circledata([P1,A]);	Circle having its center at P1 and passing through A
13:Circledata([P2,B]);	Circle having its center at P2 and passing through B
17:Pointdata("1",P1,["size=7"]);	Point P1 is shown by size 7

Notes : Only the major Script relating to Example 3 are shown. Therefore, some line numbers are missing. Regarding the drawing of the hyperbola shown in the eighth line, if results show that in Fig.10 two points of A and Q1 are focal points similarly to Example 2, then the point to be obtained should satisfy $|AP1   Q1P1| = 2a$ (where $2a$ is the difference of distance) as characteristics of the hyperbola. An important matter is recognition that it is the radius of circle Q1. If this is found, then it is designated simply as $|Q1   O|$ in K$_E$TCindy.

For drawing of P2, a hyperbola having two focal points of B and Q1 shown in the tenth line should be generated. However, this will complicate Fig.10. Then, "notex" is included, which means that no drawing is provided at the end of the tenth line by the designated option.

3   Summary and Future Tasks

The problems of figure drawing shown in Examples 1, 2 and 3 taken up here are resolved by K$_E$TCindy, in which contents of the quadratic curve are added to geometric construction using rulers and a pair of compasses.

Generating a drawing using rulers and a pair of compasses only for a finite number of times is normally designated as geometric construction in the mathematical field. According to the author, generation of a figure by adding mathematical contents to said figure drawing is extended geometric construction. When extended geometric construction is used, a regular heptagon that is impossible to draw accurately can be drawn with extended geometric construction manner by solving high-degree equations. Such educational material is regarded as effective from the perspective of application of mathematics to mathematics. Extended geometric construction is to apply mathematics itself to mathematics and the author considers that from viewpoints of utilization of the learned contents; it is appropriate content for challenge learning in high school and for application of mathematics and mathematical exploration currently proposed.

Next, as represented by K$_E$TCindy, a system that can output generated results beautifully as the printed educational material is conducive to mathematics education for pupils and students. Simultaneously, it might be used as a means for research announcements by pupils engaged in

SSH research[4]. The author considers that the application of mathematics to problem-solving is urgent to meet the demands of society as well as curriculum guidelines. Therefore, the author intends to report research outcomes at international conferences in the form of an academic paper under the title of "Application of Mathematics to Mathematics" [1].

With the program used here for figure drawing, parabolas, ellipsoids, and hyperbolas can be generated by designating a focal point and a directrix in a similar manner as the definition of quadratic curves. As a future task, such a program will need Script. To do this, some programming knowledge is necessary for mathematics teachers in secondary school so that they can solve their own problems as well as ICT applications. To that end, the ICT training system will become increasingly important.

Finally, to draw figures meeting the conditions shown in this paper mathematically precisely and beautifully, problems listed in the Encyclopedia of Geometric Solution (1959) [3] should be studied. Then, figures can be drawn easily and mathematically precisely. The author wonders why, although tips for extended geometric construction were listed in the literature published a half century ago, no extended geometric construction idea as introduced by the author has been devised. The reasons for this might be that teaching of quadratic curves was insufficient and that effective application of ICT to mathematics education was not attempted actively. The author feels that ICT is not used extensively in present-day mathematics education. As a future task, we mathematical teachers should apply ICT aggressively. In this case, hardware and software should be modified so that our students might become familiar with them. Teachers should use them to a greater degree. It is desirable that they are able to perform basic mathematical programming of Cinderella and K$_E$TCindy introduced in this report. To do so, the author intends to develop and distribute valuable educational materials for mathematics using K$_E$TCindy.

Acknowledgements

Appendixes

1. For figure drawing by Script in Examples 1, 2 and 3 in the Encyclopedia of Geometric Solutions [3], reference is made to Item 4 Trajectory of quadratic curve, Section 4 Problems relating to circles, Chapter 5 Trajectory. When results of this problem are used, circle P can be drawn by adding the quadratic curve concept to geometric construction by rulers and a pair of compasses as stated in this paper. Problem numbers 1440, 1444, and 1443

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[4] Under the SSH program, in collaboration with universities and other institutions, high schools focus on science and mathematics in conducting experiential learning, research projects and curriculum development. The program aims to foster students with a high level of creativity and a passion for science and technology. [6]

in Encyclopedia of Geometric Solution referred to in this paper are incorporated here as references[5].

— Problems from Encyclopedia of geometric solution —

1440. Fixed straight line ℓ and xed point F outside this straight line are given. Obtain the trajectory of center P of the circle which passes through F and is tangent to ℓ.

1444. Obtain trajectory of center of circle P which passes through xed point F inside xed circle F′ and is tangent to xed circle F′.

1443. Obtain the trajectory of center of circle P which passes through xed point F outside xed circle F′ and which is tangent to xed circle F′.

The trajectory of circle P is represented by a parabola having focal point F and directrix ℓ, ellipsoid having focal points F′ and F, and hyperbola, respectively. These are drawn by KETCindy.
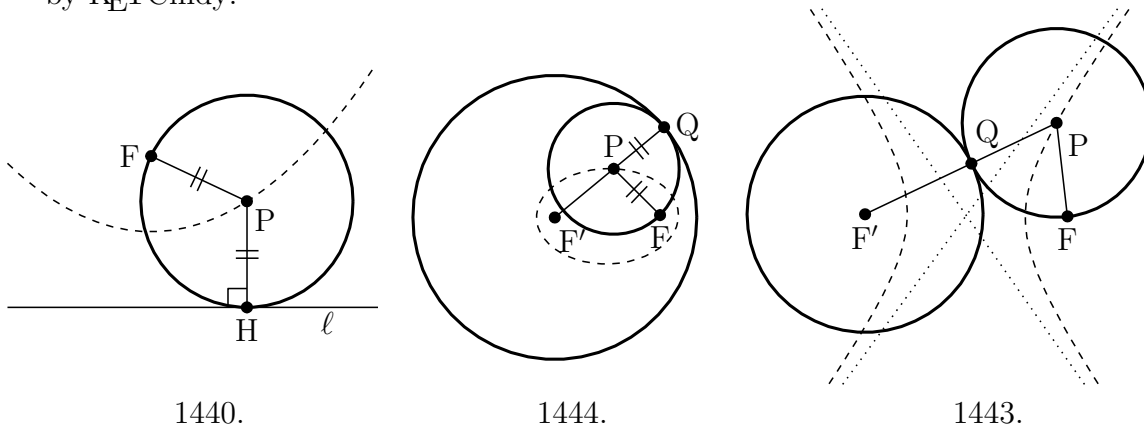

| 1440.	1444.	1443.
Point H is the point of contact of circle F and ℓ	Point Q is the point of contact of circle F′ and circle F	Point Q is the point of contact of circle F′ and circle F

2. The following illustration is used frequently in Wasan. Circle O and circle O′ tangent to common line of tangency g which is tangent to circle F can be drawn. However, point F is the focal point of the parabola shown by dashed line; ℓ shown by dashed line is directrix.
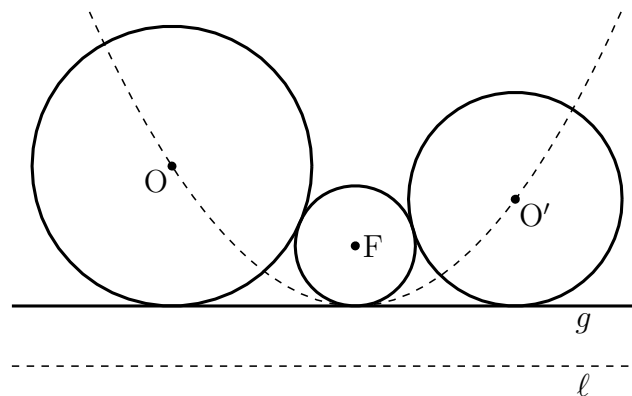
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[5] Although these problems ask a student to obtain a trajectory, these are modi ed to meet the contents of this paper. Figures are of extended geometric construction by Cinderella and ketcindy, and vertex and points are uni ed to those used in this paper.

3. The author shows how to insert the tpic le in the LaTeX under the layer environment. It is possible to determine the position of the gure as follows.


```
\begin{layer}{150}{55}
\putnotese{90}{0}{\input{Parabola.tex}}
\putnotese{110}{51}{Fig.1.}
\end{layer}
```
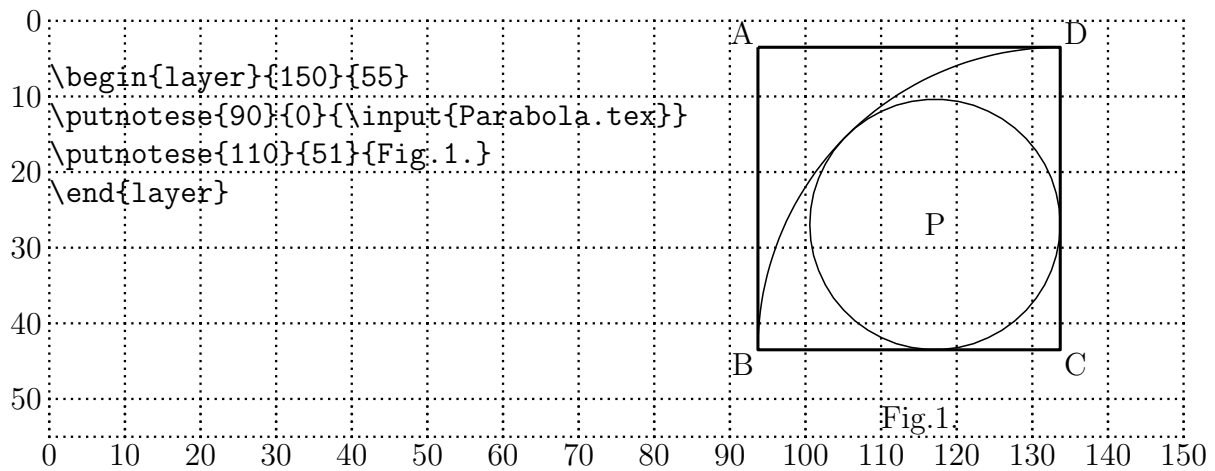
Fig.1.

The rst line means that you display the plane in a grid pattern from point $(0,0)$ to point $(150, 55)$ by x, y coordinates. The second line means that the le of "Parabola.tex"– Fig.1′s upper-left corner – is placed on the southeast of point $(90, 0)$. When you are satis ed with the position of the gure, you should turn o the grid as below.

```
\begin{layer}{150}{0}*
\putnotese{90}{0}{\input{Parabola.tex}}
\putnotese{110}{51}{Fig.1.}
\end{layer}
```

Therefore, it is possible to obtain the gure of Example 1.

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Teaching Mathematics using Augmented Reality

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Abstract

Information technology enables us to develop innovative learning/teaching tools for mathematics education both in the classroom and out-of-school activities. This paper shows a brief potential and challenges of using Augmented Reality (AR) in mathematics education. The learners can view geometrical objects in 3-dimention having better understanding of the structures. Mobile phones or computer tablets can be used to view the 3D geometrical objects using special application software. Autodesk Maya software is used to draw geometrical objects and some AR viewing software can be used to view the objects in 3D. This paper will describe how to develop a simple AR system for the improvement of abilities of learning mathematics. Sample AR materials used for mathematics education at high school as well as university level will also be discussed.

1. Introduction

Augmented Reality in education is relative new but developing rapidly. Sometimes mathematics, especially geometry, in the classroom is difficult to understand because the students have to imagine in a three-dimension way. Virtual Reality (VR) can be used to arouse curiosity and raise motivation of students to enhance the learning process with a high potential.
Azuma [1] gave a good definition of Augmented Reality (AR). AR is a variation of VR. AR allows the users to see the real world with virtual objects composited with the real world. The users can also see geometrical virtual objects like a cube or a cone in 3D superimposed with the pictures of a cube or a cone in 2 dimensions in a textbook. This will help the students to visualize for better understanding.

2. Related Work

For better understanding of mathematical models, Virtual Reality (VR) can be used to raise interest of the students as suggested by several authors [9, 11, 12]. Information technology enables us to develop a new approach for mathematics education both in the classroom and out-of-school activities. The important purpose of an educational environment is to introduce social interactions among users in the same physical area [12]. Construction of 3D objects combines four research areas: geometry, pedagogy psychology and augmented reality. There are several researchers developed Augmented Reality from Virtual Reality [2, 3, 4, 13, 6 and many others]. The educational dynamic geometry applications such as Geometer's Sketchpad [5], Cindarella [10], Euklid [8] and Cabri Geometry [7] support two- dimensional geometry only. Augmenter Reality is a rapidly developed with connections of computer graphics and user interface research.

3. Applications

Mobile phones and computer tablets become common tools in daily life. The people always use mobile phones as calculators. Information technology can enables us to develop applications on mobile phones for mathematics education purposes. In order to create 3D objects for mathematics education approaches, we need software for making 3D geometrical models and for scanning or viewing the objects.

There are several software applications using for creating 3D objects in the market. These applications can run on mobile phones or on computers. In this paper I used software Autodesk Maya to create 3D geometrical objects for AR applications. This software can be purchased from personal or commercial uses from website: www.autodesk.com. There is also a student version for educational purposes only.
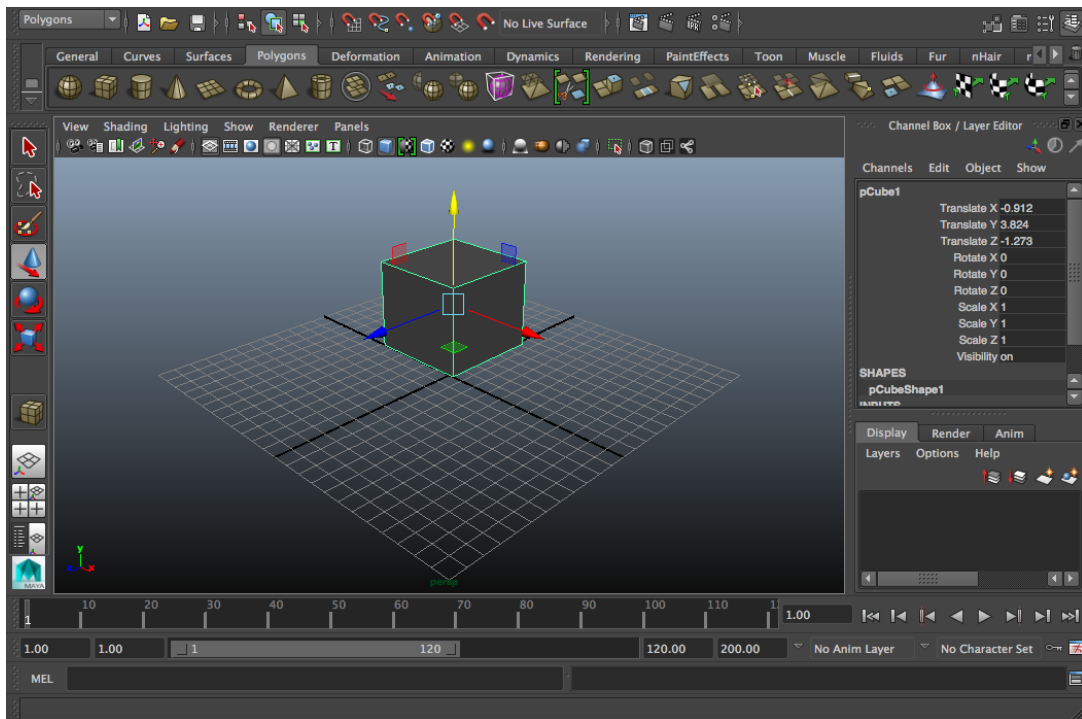
Fig.1 Working screen of Autodesk Maya for making a model of a cube.

We have to run the programme Autodesk Maya on a computer to create a model. A type od model can be chosen by clicking a button on the model menu on the top left of the screen. The size and position can be adjusted by dragging a mouse to create the chosen model. The Fig. 1 shows the cube model created by Autodesk Maya. After the model is created, it must be saved using an export command to store the model in a working folder. The model must be saved in the file type

DAE. In this case, the model is saved under the name CUBE.DAE. This model (a cube) will be used to display in a 3D floating on a background in a real world.

In order to construct an AR of the model we created from Autodesk Maya, we need another software to link the model to the background or tracker. The software for Augment Reality can be purchased from http://www.augmentedev.com/. A student version can also be obtained for educational purposes only by contacting the software provider.
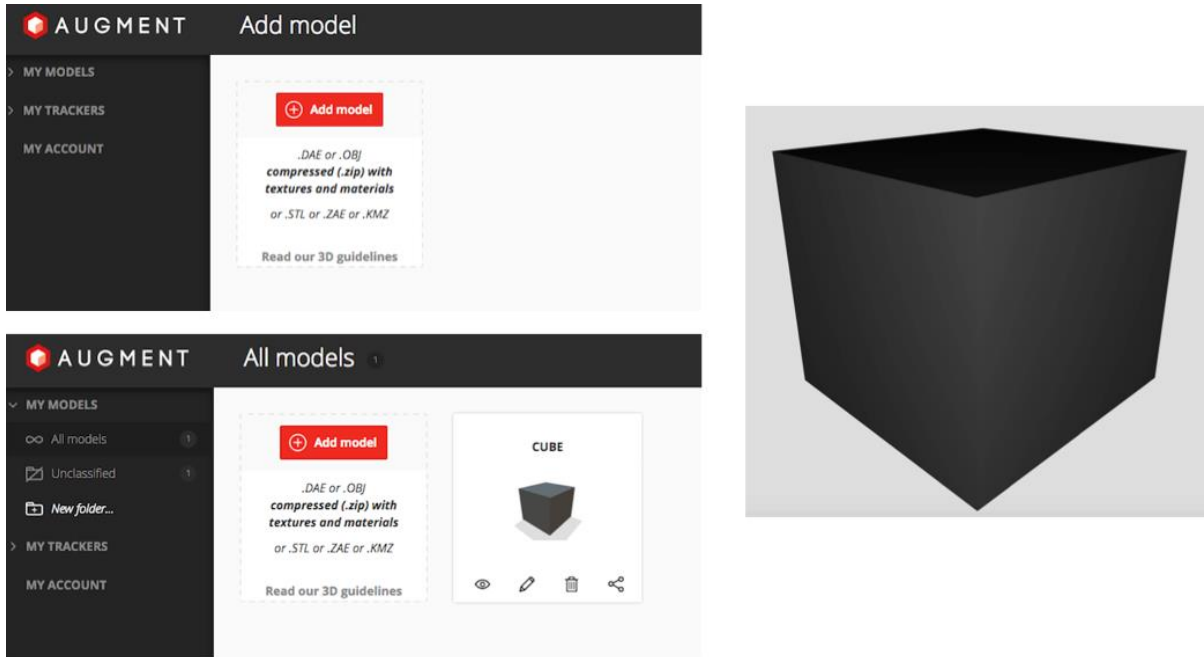

Fig.2 Working screen of Augment for making an AR of a cube model.

When we go to Augment website, we can see the working screen as shown in Fig. 2. The cube model we saved in the working folder of Autodesk Maya has to be added to the AR folder by clicking the button ADD MODEL on the screen. After upload the cube model to Augment folder, a picture of a cube will appear on the screen.

In order to view 3D cube model of AR using a mobile phone or a computer tablet, we need a background picture to be scanned. The background is called tracker.
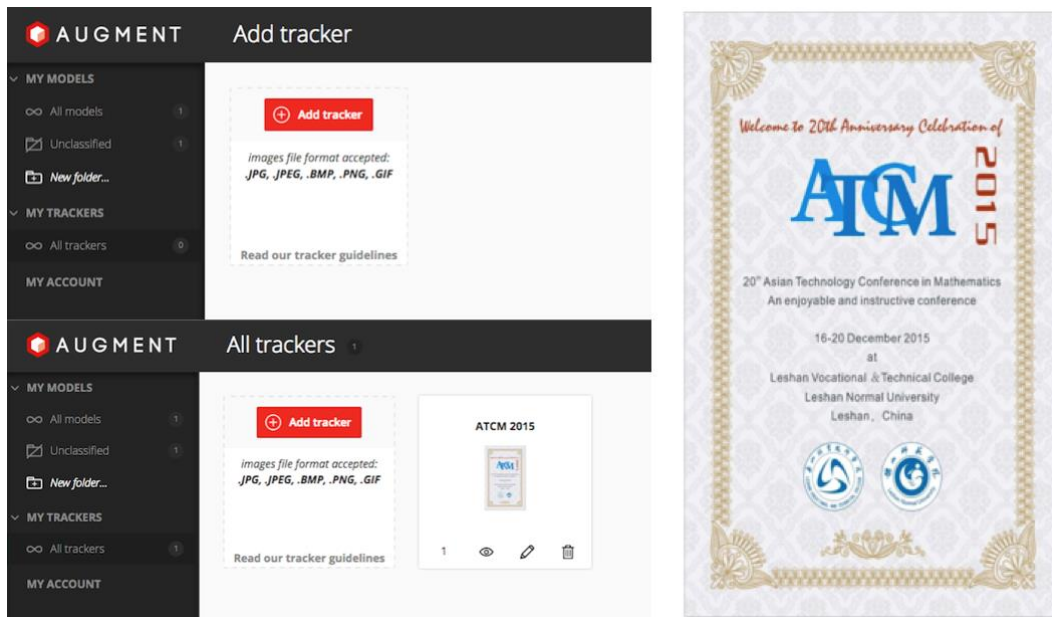
Fig. 3 A working screen of Augment for uploading a tracker to AR folder. In this case an ATCM 2015 poster is used as a tracker for AR of a cube model.

We have to prepare a tracker so that it will be used to scan and link with the model that we have in the model folder. An ATCM 2015 poster is used as a tracker for AR of a cube model. In order to upload a tracker photo, we have to click an ADD TRACKER button on the Augment screen. The ATCM tracker is uploaded in to the tracker folder as shown in Fig. 3.
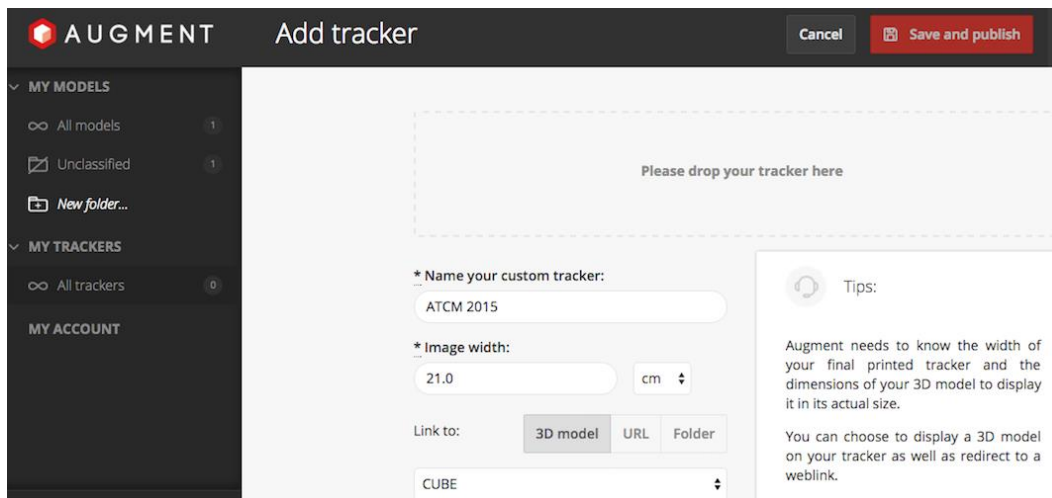

Fig. 4 Press Save and publish to form the link between the model and the tracker.

The last step to view AR 3D image of the model is to download a special software application for scanning the tracker. In this paper I used application "AUGMENT" for iPhone. When running the application AUGMENT, the menu of the functions will appear as shown in Fig. 5. Press scan button to scan the ATCM 2015 tracker on the left hand side of the Fig. 5.
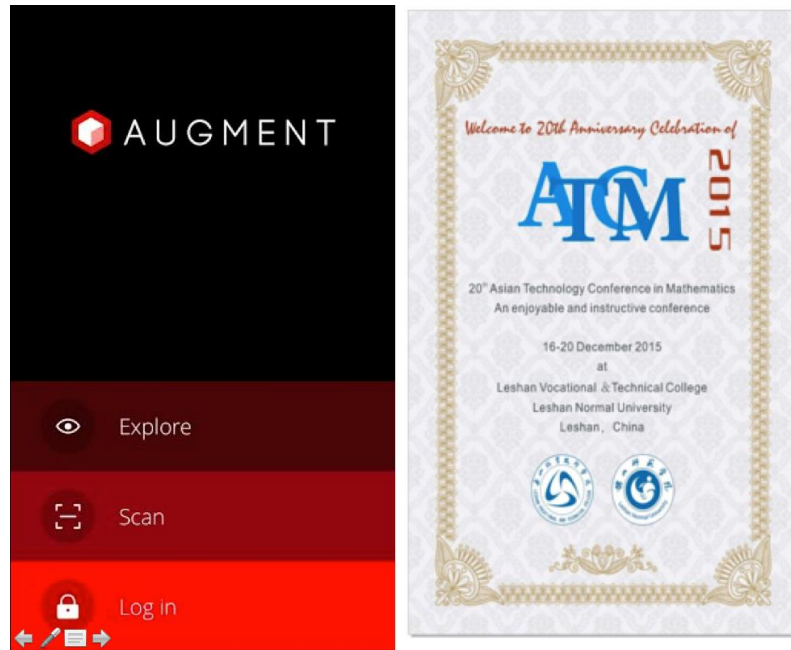

Fig. 5 Using AUGMENT application to scan the ATCM tracker to view 3D model.

Fig. 6 A cube model viewing from different angles.

After scanning a tracker, a cube model will appear in front of the ATCM tracker when using a mobile phone to scan. 3D images of a cube model can be seen from different positions like a real object floating in the air with tracker background.

4. Using Augmented Reality in the Classroom

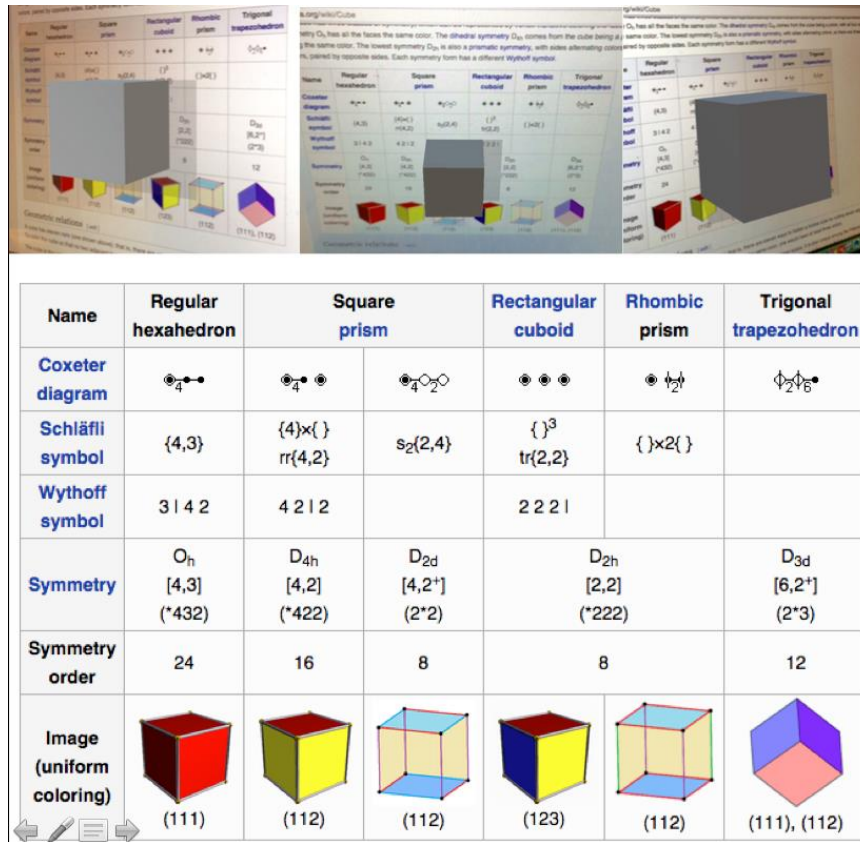| Name	Regular hexahedron	Square prism		Rectangular cuboid	Rhombic prism	Trigonal trapezohedron
Coxeter diagram						
Schläfli symbol	{4,3}	{4}×{ } rr{4,2}	s_2{2,4}	{ }³ tr{2,2}	{ }×2{ }	
Wythoff symbol	3 | 4 2	4 2 | 2		2 2 2 |		
Symmetry	O_h [4,3] (*432)	D_{4h} [4,2] (*422)	D_{2d} [4,2⁺] (2*2)	D_{2h} [2,2] (*222)		D_{3d} [6,2⁺] (2*3)
Symmetry order	24	16	8	8		12
Image (uniform coloring)	(111)	(112)	(112)	(123)	(112)	(111), (112)

Fig. 6 Different views of a cube model using a page of textbook as a tracker related to the content in the book.

With the help of AR the teachers can raise interest and motivation of students to enhance the learning process with a high potential for better understanding. It will be more interesting if the teachers can create various AR geometrical models using some pages of the textbook or worksheets as trackers related to the contents being studied. The students can use their own mobile phones or computer tablets to view 3D objects they are studying coming out from the pages and the Virtual objects can be seen from different angles in a 3-dimensional way. The sample of this idea is shown in the Fig. 6.

5.   Conclusion

Author conducted several mathematics workshops for teachers and camps for students and was looking for new ways to review math terms for better understanding. Author tried the way using QR codes as well as GIF animator. These ways could make more fun in the classrooms. Augmented Reality (AR) is another way that author tried to make classroom more interesting and fun. This could raise interest and motivation of the learners. Using geometrical pictures in the textbook as trackers to create AR objects, the learners paid more attention to the classroom and they studied more from textbook. Author asked students to create their own AR objects related to the topics they are studying. Some of the students used their own AR objects linking with QR codes as well as GIF animator for their work. The learners shared their AR works among groups. This shows a very strong impact in improving learning environment of mathematics classroom or even self-study anywhere.

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