

Interactivity and flexibility exemplified with Cabri

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Abstract: This presentation will support an approach in the development of digital technology, in which student use and teacher use are taken into consideration. This kind of technology enables the development of students centered resources fostering exploration and inquiry by students but also enabling teachers to adapt them in relation with their pedagogical aims and the needs of their students. Such technology scaffolds a progressive process of integration and instrumentation of technology by teachers while teachers may in a first step experiment ready-to-use resources in their classes and then make changes increasing over time as they are more confident and gain experience.

1. Two kinds of users of technology in the mathematics classrooms

The use of digital technology in mathematics classroom deals with the two main facets of mathematics education: teaching and learning. Studies often have addressed one of these facets: how students use technology or how teachers integrate technology into their practice. This presentation will address both points of views on the way digital technology may favor the use of tasks in the teaching/learning processes. As Thomas and Lin [1] (2013) claim, “the digital technologies may change or challenge traditional student learning trajectories, it can and will also challenge teaching strategies”.

Tasks play indeed an essential role in the teaching/learning processes. “Tasks generate activity which affords opportunity to encounter mathematical concepts, ideas, strategies, and also to use and develop mathematical thinking and modes of enquiry. Teaching includes the selection, modification, design, sequencing, installation, observation and evaluation of tasks.” ([2] Watson et al., p.12, 2013).

This presentation focuses on two critical aspects of technology with regard to tasks: interactivity for students and flexibility for the teachers. Interactivity plays a critical role on the students’ solving processes when they are faced with a task in a digital environment. Information received from the environment may assist students in the use of technology or in the solving process. Flexibility allows the teachers to easily design technology based tasks appropriate for the specific conditions they encounter in their curriculum or in their classrooms.

2. Interactivity for the student

The word interactivity is used to denote interactions between a digital environment and the user. It became very popular and every digital resource, game or environment is told to be highly interactive. Unfortunately the meaning of interactivity became loose and more than often it simply refers to the possibility of pressing keys or buttons or touching objects on the screen.

Here we would like to analyze the notion of interactivity in mathematical and cognitive ways. Interactivity in a mathematical digital environment is a reaction of the environment to the user’s actions related to mathematics. The user without a cognitive additional burden must perceive these reactions and get information on his/her actions. To play a cognitive role, interactivity must take

place both for simple actions of the user such as use of a tool or of a feature of the environment and for more complex cases.

2.1 Interactivity at the interface

An example of simple but important interactivity is the readability and mathematical correctness of the tooltips, particularly for learners.

Compare for example the actions and tooltips when creating a simple triangle without names for the vertices in two cases:

In the first case, the user is clicking three times: when clicking for the first point, a message appears “Triangle passing through a new point” for the second point “this new point” and for the third point “and this new point”.

In the second case, the user clicks a first time, no message appears but a point is created with name *A* if no other point was created before or with the *n*th letter in the alphabet if *n*-1 points are already existing. The user then clicks a second time, no message but a point denoted with the *n*+1th letter is created, the user clicks a third time, no message but a point with the *n*+2 the letter is created. Surprisingly the triangle is not yet obtained. The user has to click again on the first point to get the triangle.

In the first case the actions to be performed and the reactions of the environment are consistent with both the intention of the user and the mathematical definition of a triangle. The messages are even doing more. They assist the user by informing him/her about the progress of the creation process. The ‘and’ in the formulation “and this new point” expresses that it is the last point to create.

In the second case, the reactions are not consistent with the user intention. By choosing the tool “Triangle”, the user transmits to the system that he/she wants to create a triangle, i.e. three points connected by segments or three segments, each pair of segments sharing a common endpoint. But in this case the interface expects the user to perform an additional action giving for the second time an information item already given. One vertex of the triangle has to be clicked twice. In this case, the reactions do not assist the user, they are surprising with regard to his/her intentions: four clicks for creating a triangle and letters not intended and not chosen are displayed.

Then when hovering over the triangle or clicking on it, the tooltips are:

- “This triangle” in the first case
- “Segment a: Segment AB of triangle poly 1” in the second case

In the latter case, the tooltips are far too complex for learners. They derive directly from the programming of the software and the underlying logic is related to the internal functioning of the software that users and particularly novice users do not have to know. Why does appear ‘segment’ twice in the tooltip with two different names? What does mean ‘poly’? In the former case, the tooltip is consistent with the intention of the user.

These reactions may seem to be micro reactions and not be important for the user as soon as he/she masters the use of the software. We would like to claim that a learning environment is aimed at helping the students to learn mathematics. Their interface must not puzzle them and not be counterproductive from a cognitive aspect. They must foster the focus of students on mathematics and not be conflicting with a fragile knowledge they are in the process of acquiring. Not only the usability of the software (users’ friendliness) is low but also it results in a loss of utility (the extent

to which the needs of the user are met; in the case of educational technology, utility is related to the learning potential of the interface).

2.2 Interactivity in the mathematical activity

When students solve problems, the role of feedback to their answers has been stressed by research as a catalyst for changing their solving strategy in case of an incorrect strategy. Students are not only convinced that their answer is wrong but they can extract information from feedback that can be used for searching a more appropriate solving strategy ([3] Brousseau, 1997).

Let us give a simple example in Cabri 3D [4]. The task for the students is to construct a box containing tennis balls (Fig.2.1).

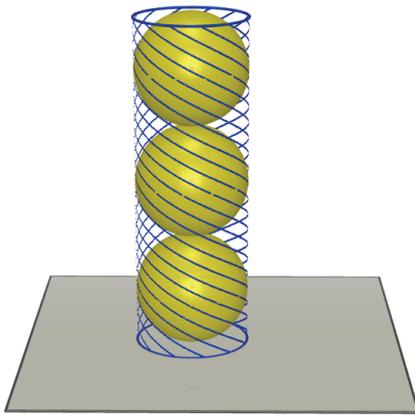


Figure 2.1 The box

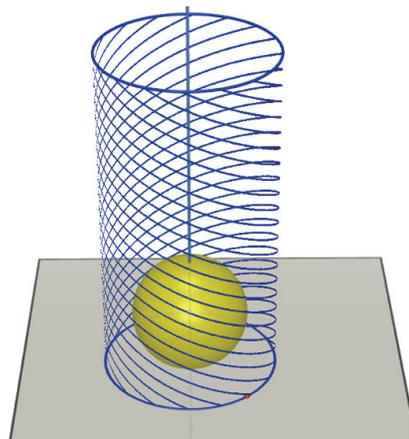


Figure 2.2 The cylinder is enlarged

A first strategy is to construct the cylinder, to put a point on the axis of the cylinder and then to create a sphere with center this latter point and passing through the center of the cylinder base. Then the center of the sphere is dragged until the sphere seems to become tangent to cylinder.

However when changing the radius of the base of the cylinder, the sphere is no longer tangent. The construction is not robust against dragging (Fig.2.2). The drag mode in dynamic geometry environments is a very powerful source of feedback. Information brought in the case of the box is that the center of the ball must be constructed and not simply dragged. It leads to the need of a geometric analysis of the position of the center of the ball.

From a cognitive perspective, feedback gives rise to a conflict between what students expect and how their construction behaves, leading in Piagetian terms to an equilibration process aimed at cancelling the conflict. Overcoming the contradiction is a way of wiping out the conflict.

Let us give another example in 3D geometry in Cabri [5] where the source of feedback is different. Students are faced with the task of constructing the net of a pyramid without folding it. They have permanent access to the pyramid (Fig. 2.3) but must complete the net in a 2D environment (Fig. 2.4). The aim of the task is to require from students an analysis of the nature of the faces of the polyhedron and of the relationships among them.

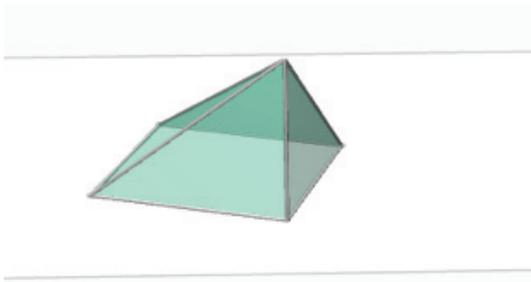


Fig. 2.3 The pyramid

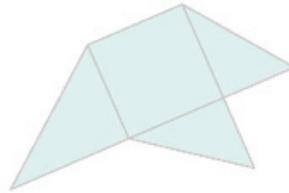


Fig. 2.4 The net to be completed



But after students have completed the construction of the net, they are allowed to check their net by folding it. Folding the constructed net shows whether it is correct but also in case the net is incorrect may reveal some geometric reasons for the inadequacy of the net. An example of a wrong answer is displayed in Fig.2.5. When folding, the student not only perceives immediately that his/her answer is wrong but also may see (Fig. 2.6) that the angles are not appropriate: the acute angle at the bottom is too small. The right angle is too large.

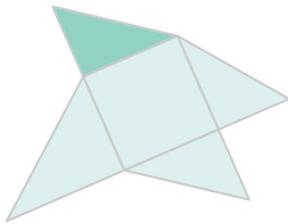


Fig. 2.5 A wrong answer

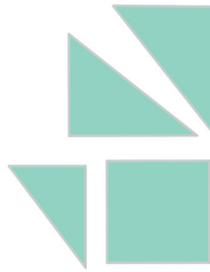
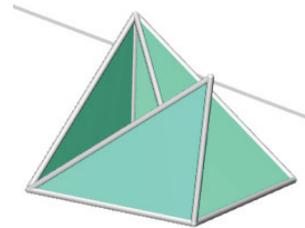


Fig. 2.6 Folding the wrong net



In this perspective, interactivity comes from an interplay between the problem students are faced with and the possibility offered to students to check their answers. A problem is posed to the student with some constraints avoiding an easy solving process resorting for example to pure perception and calling for mathematical solving process. After students have given an answer,

-they are given the possibility of checking their answer through manipulations in the computer environment (like in the example of the net)

-or technology enables them to easily see what results from their answer (like in the case of the drag mode).

Interactivity originates from the coordination of the possibilities offered by the environment and from the choice of the mathematical question or problem to solve. Technology enables

-to design a setting for the mathematical question: simulation of a problem in real world, numerical, graphical or geometrical setting;

-to play on the conditions in which students can solve the task: available tools, possibility or not of dragging;

-to offer means of feedback by enabling students to vary some elements of the question, to transpose it in another setting linked to the initial setting or to free some conditions of the problem setting.

Clearly the design of interactivity is based on didactical choices. But these choices are largely conditioned by the possibilities of the digital environment. Didactical choices and features of the environment are strongly intertwined in the design of interactivity.

3. Flexibility for the teacher

If many innovations taking advantage of technology in mathematics teaching took place during the past thirty years, despite institutional support the use of technology did not widespread, as the enthusiasm of the pioneers would have hoped.

More than ten years ago several studies focused on teacher practice using technology in ordinary classrooms. Observations of teachers showed that integrating technology was not an easy task for teachers. Instead of following textbooks, the teachers must design worksheets [6] (Monaghan, 2004); they must adapt the management of several kinds of time in their classrooms and the relationship between, on the one hand, old and new knowledge and, on the other hand, paper-and-pencil techniques and computer software techniques (Assude, 2005) [7]. Monaghan [6] also showed how technology could affect the emergent goals of the teacher during the lesson. Haspekian's (2005) [8] findings about the management of complexity when a teacher integrates spreadsheet converge supports the research previously quoted: the teacher must simultaneously take into account "the tool features, the instrumented techniques, and the concepts involved." This explains why teachers often preferred to use themselves the power of such technology by demonstrating properties and theorems in front of the students (Ruthven and Hennessy, 2005) [9] and let not students using themselves technology.

Over the ten past years the number of digital resources and applets has grown dramatically freeing the teachers from designing themselves the activities. In the same manner as exercises and problems are available in textbooks for teachers, these latter can now find a huge number of digital resources in repositories. However weaknesses of these repositories have been identified (Trgalova et al. 2011) [10]: difficulty of finding resources fitting to a specific educational context due to the lack of metadata describing accurately the resource content, lack of quality guarantee, difficulty of resource appropriation by teachers. The aim of the European Intergeo project (2007-2010, Kortenkamp et al., 2011) [11] was to set up an international multilingual repository of resources for dynamic geometry environments. The project developed in addition a questionnaire meant for teachers who wanted to assess the quality of repository resources. This questionnaire turned out to be an assessment tool but also a means for teachers to appropriate the resources, to be able to analyzed them and possibly to modify them (Trgalova et al. 2011) [10].

Indeed between on the one hand, open environments giving complete freedom to teachers for designing activities but requiring time and a careful preparation and on the other hand, ready-to-use resources non totally adapted for students, an intermediate approach is possible: providing teachers with flexible resources that they can easily modify and adapt for use in their classrooms.

An activity must meet two needs:

- it must contribute to specific learning;
- it must be adapted to students abilities and competencies, to the actual state of the progression of the mathematics classroom in which it is used.

In attempting to coordinate these two needs, the teacher may need to customize the resource for his/her classroom.

The teacher may want to change

- the formulations to make them consistent with the ones in use in the classroom;
- the context in case the activity is contextualized and students of the teacher are not familiar with; or the teacher may want to motivate students in setting the activity in a context shared by the whole classroom or related to a situation experienced by all students.
- the length of the activity or the number of questions, depending on organization of the time planned by the teacher;
- the learning aim of the activity by changing the value of task variables or didactical variables (Brousseau [3]) affecting the students' solving strategies;
- in relation with the learning aim, the mathematical difficulty of the activity too high or on the other hand too low with regard to students' abilities;
- the aim of the activity in the teaching process: is it meant for assessment, for practicing, for discovering ? Is it meant for home assignment?

The teacher may also want to tune the difficulty of the activities in relation with different groups of students. The teacher may challenge advanced students in increasing the difficulty of the activity. He/she may motivate weakest ones in proposing an easier activity. In addition of tuning the difficulty, the teacher may want to modify the weight of each question in marking the work of the student on the activity.

3.1. Deriving a variety of tasks from one task

Some examples of possible modifications are illustrated below in *Cabri Factory* [12] a collection of mathematics resources for middle school.

In the activity book entitled “In the right place”, students must drag a point and place it at the position of the fourth vertex of a particular quadrilateral (parallelogram, rectangle, square, rhombus) in four diagrams. The only available tool is the *Angle* tool that displays the measure in degrees of an angle determined by three points.

In the case of placing the fourth vertex of a parallelogram (Fig. 3.1), the task requires using twice the property of supplementary interior angles to make opposite sides parallel. Students must display the measure of three angles and complete the task in two steps: making a first pair of sides parallel and then the second pair of sides parallel without changing the angle determining the first pair of sides.

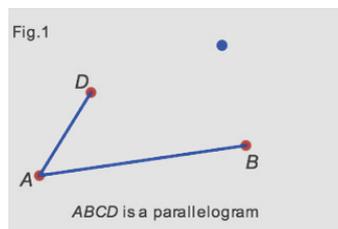


Fig. 3.1 – The blue point must be placed on hidden vertex *C*

This strategy is not spontaneous for students for two reasons:

- they generally do not think to this property as they are attracted by the shape parallelogram in absence of display of lines. They want to achieve the task only by making two opposite angles congruent. The task makes operational the property of supplementary interior angles implying parallelism.

- they try to place the point in only one step by making the opposite angles congruent and the sides visually parallel.

It usually results in a wrong position for the point. In this case, an error message telling the student that the point is not at the right place is displayed as *Cabri Factory* enables the author of the activity to check students' answer whatever the positions of given points A , B and C are. In addition students can access a help page in which they can explore the impact of measures of angles made by two lines and a transversal by dragging one of the two lines. Feedback to students combines information about the wrong position of the point and the possibility of exploring the link between angles and position of lines.

The learning aim may be changed by the teacher. The teacher may want students to practice the congruence of opposite sides for obtaining a parallelogram. In that case, in the teacher mode, the teacher has just to drag the tool *Angle* out of the student page and to drag the tool *Distance* inside the student page. If the teacher prefers to foster the tool *Circle* to reinforce the use of a circle as measurement transferring, he or she makes this latter tool available instead of the tool *Distance*.

If the aim of the teacher is to assess the availability of properties at students after the teaching properties of parallelograms is completed, the two tools *Angle* and *Distance* may both be offered to students. The most popular method will emerge from the students' work. That work is continuously saved and when the work is completed, the teacher may have a look and see which property prevails among students.

The task may be too difficult for some students not having acquired the property of supplementary interior angles for making lines parallel. The teacher may want them first to practice on only one pair of sides. He can draw a parallel line to segment AB and passing through D , and redefine the blue point as a point belonging to that line.

The teacher may organize a sequence of tasks from this only resource, each activity being derived from the preceding one by changing the value of one or several task variables in order to allow a progress in the appropriation of the relation between parallelism of lines and angles. The two first tasks deal with four points and ask for placing point D as the fourth vertex of a parallelogram. In the first task, only one angle must be varied to place the point, in a second task two angles must be varied. The third task deals with a crossed quadrilateral (Fig. 3.2): students must place point D so that $ABDC$ is a parallelogram. Alternate angles or interior angles can be used to solve the task but alternate angles are more visible since in this case no sides are displayed for one of the interior angles.

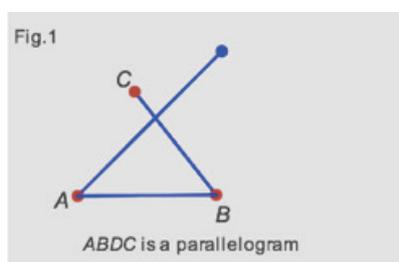


Fig. 3.2 A crossed quadrilateral

The teacher has just to delete segment AD , to create segment BD , change the label D into C , and to join point A and the blue point with a segment.

Then the number of points can be increased making less visible the sides to make parallel and the related angles. For example, the teacher can ask students to make a hexagon invariant through reflection in a point with the only available tool *Angle* (Fig.3.3).

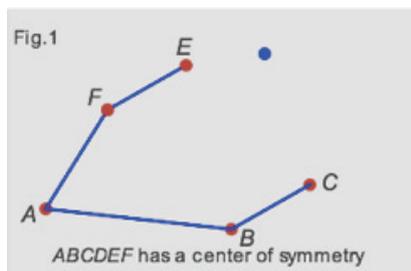


Fig. 3.3 The blue point must be placed on hidden vertex *D*

The teacher has just to introduce a new point *B* and its reflected point *E* with respect to the midpoint of *CF* and change labels of the existing points. The hidden target point is the same as earlier, *ACDF* being a parallelogram.

All these transformations of the activity are possible because of the availability of three modes in *Cabri Factory*:

- the author mode
- the teacher mode
- the student mode.

When designing the activity, the author provides the teacher with many geometrical tools (point, segment, line, parallel line, point symmetry, angle, distance...) in order to enable the teacher to modify the activity to a large extent without taking care of feedback evaluating students' answers that is preserved during the transformations done by the teacher. In addition, the teacher mode offers editing facilities, such as editing texts, changing fonts and colors, inserting images, hiding or displaying objects.

All subjects of mathematics give rise to possibilities of change for the teacher. An example about graphing linear functions is presented below.

A coordinate system is displayed as well as a blue point. The expression of a linear function is given. Students must place the blue point on the hidden line representing the function (Fig.3.4). If they succeed, they receive a message congratulating them. If they fail a first time, they have access to a help page (Fig. 3.5) in which the hidden line a point *P* and its coordinates are displayed. Students are asked to calculate $f(x)$ for a given x coordinate of *P* and to look where the y coordinate of *P* is equal to $f(x)$. If they fail a second time, they have access to a solution given for another function. They have a third try for allowing them to reinvest what they understood from the solution.

The expression of the linear function is randomly generated within intervals for the y intercept and the slope that can be changed by the teacher. No tools are available for the student in the activity page.

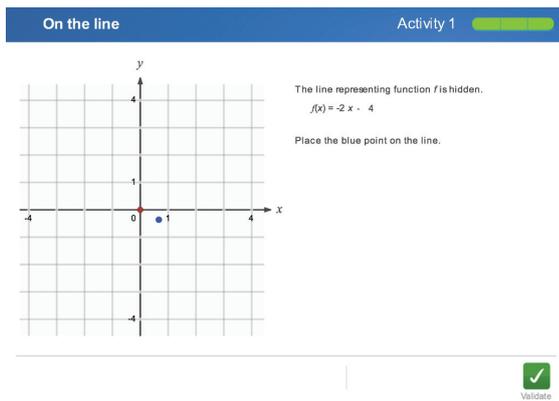


Fig. 3.4 Placing a point on a hidden line

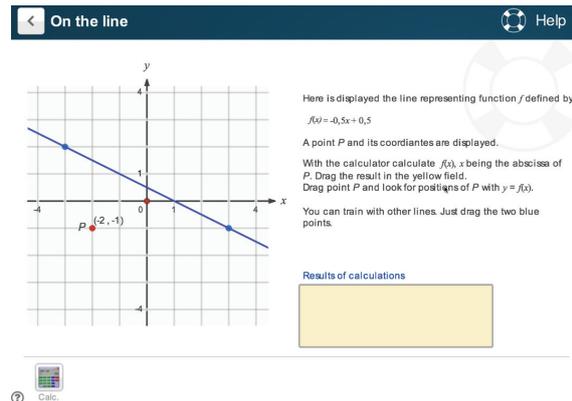


Fig. 3.5 Help page

The learning aim is dual:

- being able to use the expression of the function to place a point on its graph;
- identifying a point with given coordinates in a coordinate system.

After playing several times, students may notice that it is very efficient to use the y intercept of the graph.

When students do not have any longer difficulties with the y intercept, the teacher may want students no longer use the y intercept but rather the slope of the line or the coordinates of a point with x coordinate different from 0 and 1. Each axis can be moved even outside the display of the system as in Fig. 3.6. In the following page of the activity book, the point to be placed can only move on a line parallel to the x axis (Fig. 3.7).

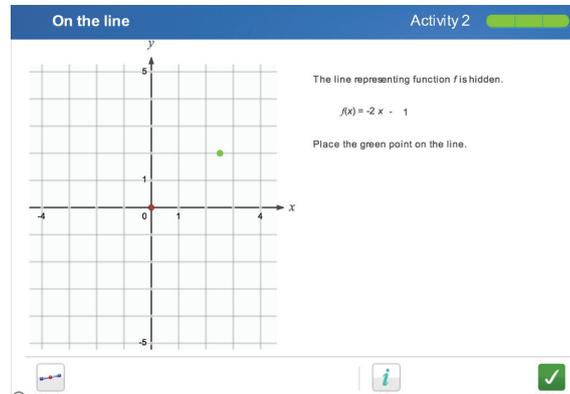
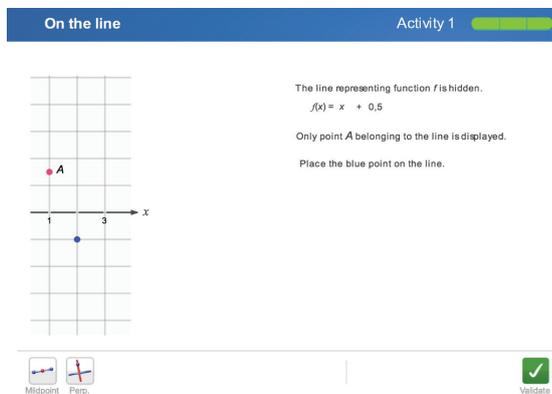


Fig. 3.6 Avoiding the use of the y intercept **Fig. 3.7** The green point moves on a hidden line

This new task requires from the student to calculate the x coordinate of a point of the graph when the y coordinate is known. The teacher can change the parallel line and place it on the x axis. In this case students must find the x intercept.

The teacher can then challenge students by redefining the green point on line $y = x$. Either students have already been introduced to algebra and they can find out the solution x_0 of the equation $x = f(x)$ and then estimate the position of x_0 .

Or they have not yet been introduced to equations and they can solve the task by varying the green point and for each x coordinate of the green point equal to an integer calculate the y coordinate of the point of the graph of the function. It is so possible to determine whether the graph is below the

green point or above and to identify a small region where the green point should be. Then it is possible to find two close points of the graph of the function in this region and to estimate where a segment joining these two points will cross the line $y = x$.

As seen on the previous examples, flexibility is due to the Cabri technology itself and to the didactical design by the author. Cabri offers the possibility in the teacher mode of easily modifying the choice of the available tools for students, of changing the value of parameters and even of changing questions by preserving the evaluation of students' answers by the environment. In case the question is changed, teachers have only to indicate in a specific field the expected answers. The didactical design is the second factor of flexibility: the author of the prepared activity must carry out a didactical analysis of the task variables and consider the possible variations in relation with the learning outcomes of the activity.

Textbooks offer prepared exercises that teachers extensively use, as they have little time for carefully designing new exercises. The role of the teacher lies in the choice of the adequate exercises and their management in the classroom. Technology may offer the same service but with greater flexibility.

3.2 Living resources extending teachers' experience

Providing teachers with already prepared but flexible resources that can be modified fulfills several functions:

- the teachers may adapt the activities to their pedagogical choices;
- the resources are *living* resources (Geudet et al. 2013) [13]: they are enriched by teachers' experience and extended in various ways according to teachers' choices. The starting activities play the role of a seed.
- The starting activities scaffold in Bruners' [14] sense the experience of teachers novice in the use of technology and contribute to professional development.

Let us discuss the latter point. Several studies show that the ability of designing technology based tasks requires a strong mathematical knowledge for teaching, a good instrumentalisation and instrumentation of the technological tool [1]. Goos (2005) [15] identified a good range of factors influencing teacher technology use that include the availability of appropriate teaching materials and opportunities to learn. Novice teachers find it very difficult to design from scratch activities in the digital environment: they need to resort to existing activities for preparing other ones. Prepared activities and the comments and suggestions for modifications fulfill the function of providing a model of an activity and a didactical analysis of the activities. The tools and parameters made available to teachers fulfill several scaffolding functions [14]:

- enrolment of the teachers: they perceive that they can exert their pedagogical freedom;
- reduction of the steps required to design a new activity: the design of other activities is simplified with regard to a design in an open ended environment;
- maintaining the activity of teachers and controlling their frustration: often novice users have too ambitious goals when starting designing activities and for these reasons may fail to reach their goals. The limitations of the possible changes avoid such a danger.

In this presentation, we wanted to stress how much the development of digital technology for mathematics education must take into account two kinds of users, students and teachers. In particular we have discussed two factors playing a critical role in the usability and utility of a digital

environment: interactivity for the students and flexibility for the teachers. Analyzing these two factors we have seen that the development of technology is strongly intertwined with mathematical, cognitive and didactical choices.

References

- [1] Thomas, M., Lin C. (2013). Designing Tasks for Use With Digital Technology. In: *Task Design in Mathematics Education, Proceedings of ICMI study 22* (C. Margolinas, Ed) (Vol. 1, pp.111-118), <http://hal.archives-ouvertes.fr/hal-00834054>
- [2] Watson, A. et al. (2013). Introduction. In: *Task Design in Mathematics Education. Proceedings of ICMI study 22* (C. Margolinas, Ed) (Vol. 1, pp.9-16), <http://hal.archives-ouvertes.fr/hal-00834054>
- [3] Brousseau, G. (1997). *Theory of didactical situations* (N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield, Eds. & Trans.). Dordrecht, The Netherlands: Kluwer Academic.
- [4] Cabri 3D v2.5 (2010) Grenoble, France: Cabrilog. www.cabri.com
- [5] Cabri v.2.1 (2013) Grenoble, France: Cabrilog. www.cabri.com
- [6] Monaghan, J. (2004). Teachers' activities in technology-based lessons. *International Journal of Computers for Mathematical Learning*, 9(3), 327–357.
- [7] Assude, T. (2005). Time management in the work economy of a class—A case study: Integration of Cabri in primary school mathematics teaching. *Educational Studies in Mathematics*, 59(1/2/3), 183–203.
- [8] Haspekian, M. (2005). An “instrumental approach” to study the integration of a computer tool into mathematics teaching: The case of spreadsheets. *International Journal of Computers for Mathematical Learning*, 10(2), 109–141.
- [9] Ruthven K., Hennessy S., & Deaney R. (2005). Incorporating dynamic geometry systems into secondary mathematics education: Didactical perspectives and practices of teachers. In *Moving on with dynamic geometry* (pp. 138–158). Derby, UK: Association of Mathematics Teachers.
- [10] Traglova, J., Soury Lavergne S., Jahn, A.-P. (2011) Quality assessment process for dynamic geometry resources in Intergeo project, *ZDM Mathematics Education*, 43, 337-51
- [11] Kortenkamp, U., Laborde C. (2011) Interoperable Interactive Geometry for Europe: an introduction, *ZDM Mathematics Education*, 43, 321-23
- [12] Cabri Factory v.1 (2014) Grenoble, France: Cabrilog. www.cabri.com
- [13] Gueudet, G., Pepin, B., Trouche, L. (2013) Textbooks' Design and Digital Resources. In: *Task Design in Mathematics Education, Proceedings of ICMI study 22* (C. Margolinas, Ed) (Vol. 1, pp.327-336), <http://hal.archives-ouvertes.fr/hal-00834054>
- [14] Wood, D., Bruner, J., Ross, G. (1976) The role of tutoring in problem solving. *Journal of Child Psychology and Child Psychiatry*, 17, 89-100
- [15] Goos, M. (2005) A sociocultural analysis of the development of pre-service and beginning teachers' pedagogical identities as users of technology. *Journal of Mathematics Teacher Education*, 8, 35-59