Professional Development for Teachers in Mathematical Modelling

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Abstract:

The importance of mathematical modelling and its value in mathematics education has been discussed and emphasized by various researchers in the field. However, it is widely acknowledged that mathematical modelling can be demanding for students and teachers. Many teachers, including experienced practitioners in Singapore, do not have any formal training in mathematical modelling as a student either at school or in the university, and teaching it has been very challenging. Very often, a teacher's decisions, from planning of a mathematical modelling task to its execution in the classroom, depend on his orientation, resources and goals. Unless a teacher has been properly trained, adequately prepared and well resourced, his decisions are unlikely to result in a successful mathematical modelling lesson.

In this paper, we discuss a school-based professional development (SBPD) programme aimed at preparing teachers to teach mathematical modelling in the classroom. This programme has been successfully carried out in several schools in Singapore over a period of about two years. A case study will be presented to illustrate the key principles of the programme. Examples of student work and modelling tasks designed by participating teachers will be presented. In addition to the observation that there is a definite paradigm shift in the teachers' orientation and goals in teaching mathematical modelling, it is also evident from these examples that technology had played a crucial role in the success of the modelling lessons designed by the participants of the SBPD programme.

Introduction

In recent years, mathematical modelling has become an important area of growing interest in mathematics education. Many researchers have discussed the relevance of teaching mathematical modelling and the value of learning mathematical modelling in the classroom [1], [2], [3], [4], [5]. In Singapore, the Ministry of Education (MOE) has also supported the teaching of mathematical modelling, seeing it as a "worthwhile learning experience that will benefit students and prepare them well for the future" [6].

However, despite the emphasis and constant attention, mathematical modelling has yet to become a common or popular approach of mathematical study in the Singapore classroom. One reason could be that mathematics teachers here have limited knowledge and hence little confidence in mathematical modelling. At the same time, there is a lack of relevant resources and tested exemplars of mathematical modelling for local teachers to tap on [7].

It is widely acknowledged that mathematical modelling can be demanding for students and teachers [5]. Therefore, for mathematical modelling to succeed in the classroom, teachers will need to be properly prepared and suitably trained. Other researchers have emphasized the importance and nature of pedagogical content knowledge (PCK) in effective teachers [8], [9], [10]. Hence, to be an effective teacher of mathematical modelling, one not only needs to know mathematical modelling, but also possess the pedagogical acumen to deliver it at the classroom level.

Unfortunately, most of our teachers do not have any formal training or experience in mathematical modelling as a student, whether in school or at the university. It is understandable then that teaching something one has no experience in or knowledge of can be challenging and daunting. Moreover, the unpredictable nature of a mathematical modelling lesson or class has further deterred and discouraged teachers from trying to implement mathematical modelling lessons in the classroom [11].

This problem of limited knowledge and experience in the subject area is compounded by the fact that there is a general lack of relevant and usable resources, as well as support for the teacher who wishes to plan, design and implement modelling lessons [7], [12]. Meaningful mathematical modelling tasks involve problem solving in an unfamiliar setting, often requiring cross-disciplinary knowledge. Thus, the teacher needs to be really well prepared, well informed, and well resourced.

With this in mind, a teacher-centric, school-based professional development (SBPD) programme in the teaching of mathematical modelling was designed and implemented in a few pilot schools. The objective is to help teachers develop the appropriate orientation, adopt a positive mindset and acquire the requisite competencies to teach mathematical modelling in the classroom.

The SBPD

The SBPD programme consists of three dimensions, namely, Content, Process and Context, that contribute towards developing teachers' confidence and enhancing teachers' decision making in planning, designing and teaching mathematical modelling. The essential components are illustrated in Figure 1 below.

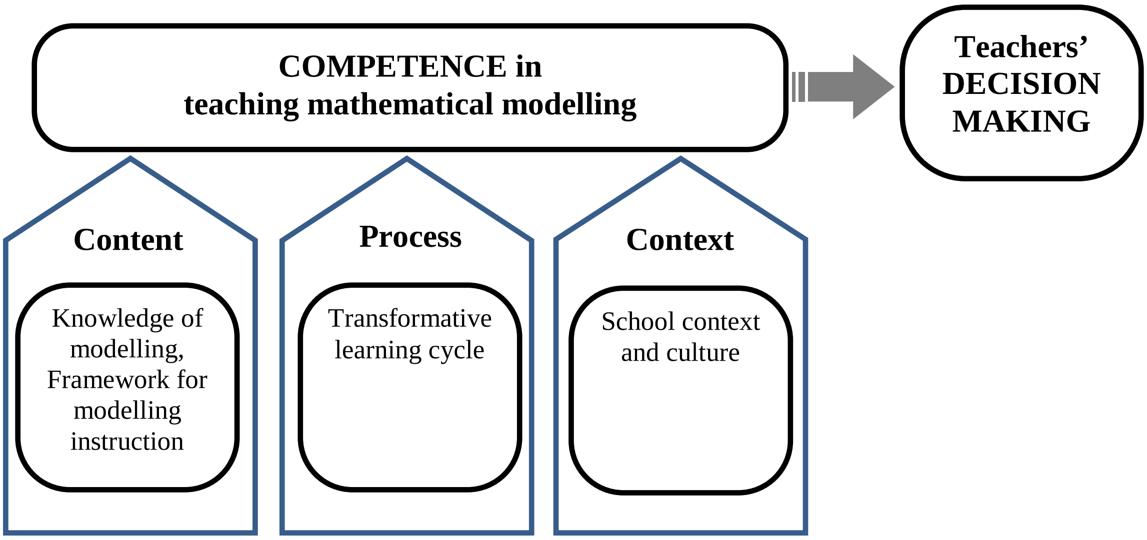


Figure 1. Essentials of the SBPD programme

The content of the programme will include the subject matter (that is, mathematical modelling, approaches to modelling, the modelling process and so on). More importantly, pedagogical content knowledge of modelling will also be discussed and this would include knowledge of modelling task space, knowledge of the broad spectrum of tasks, diagnosing students' learning difficulties during modelling process, strategies of intervention modes, appropriate beliefs and so on. In addition, a new framework for the teaching of mathematical modelling will be introduced and used as a tool in translating teachers' ideas into modelling tasks or lessons [13]. This framework essentially consists of five questions that will guide teachers as they develop and design their modelling tasks (see [14]).

As noted by Thomson and Zeuli, the process of transformative learning in professional development promises to bring about "changes in deeply held beliefs, knowledge, and habits of practice" [15], which is more desirable and impactful. To achieve this effect of transformative learning, the project team will work closely with teachers, providing advice and support while teachers plan and design mathematical modelling lessons for their students. This active engagement in planning, designing and classroom enactment was video-recorded for both review and analysis. Review was necessary to provide the basis and platform for teachers to discover and resolve cognitive dissonance, if any, in the process of applying their new knowledge. To further strengthen and deepen the learning, the whole process of applying was repeated.

It is not uncommon to hear teachers say that while they may have learnt something from some professional development courses, they find it hard to apply what they have learnt in their school. This is especially so in mathematical modelling. Customized support, as well as adequate allocation of professional development time, can foster the coherence and meaningfulness of teachers' professional learning experiences in a particular school context and culture [16]. The SBPD comprises a planning sequence that incorporated goal setting, planning, applying and reflecting. These sub-processes were influenced by various aspects of a teacher's work in school. In order to align these processes with the school's multiple areas of focus, placing the programme within the school context, both physically and philosophically, had brought about support from the school leaders and participating teachers. It had made the entire exercise more relevant and useful for teachers, their students and their school.

The SBPD programme consists of three phases (Training, Applying and Reflecting), with two learning cycles spread over a period of between six and eight months. The activities in these phases are summarized in Table 1 below.

Table 1: SBPD Programme for teachers in mathematical modelling

Programme Milestones	Activities
Baseline Checking	Pre-programme discussion to ascertain baseline knowledge of participating teachers
Phase 1: Training	Learning to be a modeller Framework for teaching of mathematical modelling
Phase 2: Applying	Planning and designing modelling lessons Implementing modelling lessons
Phase 3: Reflecting	Analysing modelling lessons Post-lesson discussions

A Case Study

The school chosen for the case study reported here is an *autonomous* secondary school. School leaders of an autonomous school have some level of freedom in providing programmes outside of the national curriculum. In this case, it was decided that mathematical modelling would be planned to be integrated into the Secondary 1 and 2 (Grades 7 and 8) mathematics curriculum.

Ten teachers (four male and six female), each of whom had between one and 23 years of teaching experience, participated in the SBPD programme. At the preliminary interviews, although the participants had expressed concerns about their lack of understanding in teaching mathematical modelling, they welcomed the opportunity to develop their competencies in the SBPD programme.

In Phase 1 of the SBPD programme, three 4-hour training sessions were carried out. Participating teachers engaged in independent modelling experiences involving three tasks in the first two sessions. These tasks were based on common modelling approaches suggested by Ang [17]. The framework for teaching mathematical modelling (as described in [13]) was introduced in the third and last of these training sessions.

In Phase 2, participating teachers worked their groups (or pairs) to plan and design modelling lessons for their students. One focus group discussion (FGD) was held to discuss their design for a Level 3 task, as part of the transformative learning cycle.

Phase 3 consists of implementation of the task, and meetings and discussions with participating teachers. A FGD was held to discuss lesson issues that arose from their enactment of modelling lessons. This was followed by a post-programme interview with all participating teachers.

Sample Modelling Tasks

A number of ideas for good modelling tasks arose during Phase 2 of the programme. Participating teachers were able to find real and relevant problems that may be suitably constructed into modelling tasks for their students. In this section, two such tasks are highlighted.

The Darts Game

The task was posed as follows.

> Our school is organizing a carnival to raise funds. Our class has been assigned to be in charge of a “Darts” game stall. Instead of the usual bulls-eye type of dartboard, design a new dartboard and provide rewards that will be attractive enough and yet strike a balance between players winning and the class earning a profit.

This is a real problem and students were given a $100 budget to run the stall in a real carnival. As a trigger, students were given the proposed design shown in Figure 2, but this was not attractive, and rather arbitrary in terms of choice of rewards.

Students worked on the problems in groups, and were asked to upload their work on Google Docs for sharing and discussions. Sharing their work on Google Docs had facilitated discussions and the level of participation was heightened. In addition, students quickly learned from one another, and were able to modify or justify their own designs.

This task could also involve running a simulation using the proposed designs, to determine the experimental probabilities of each landing space, and hence the relative value of the rewards.

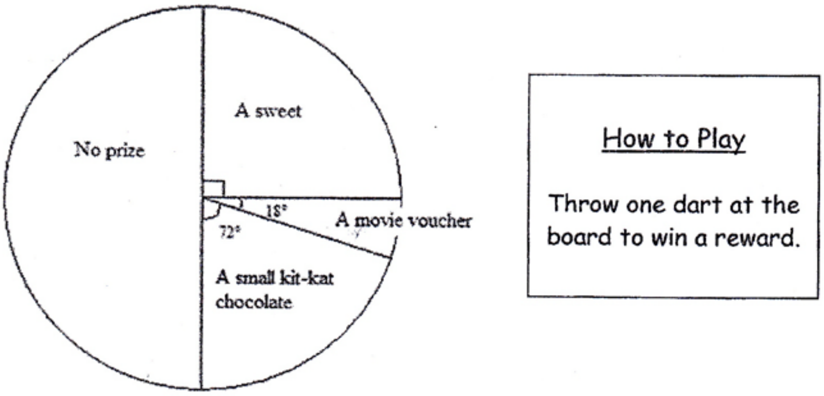
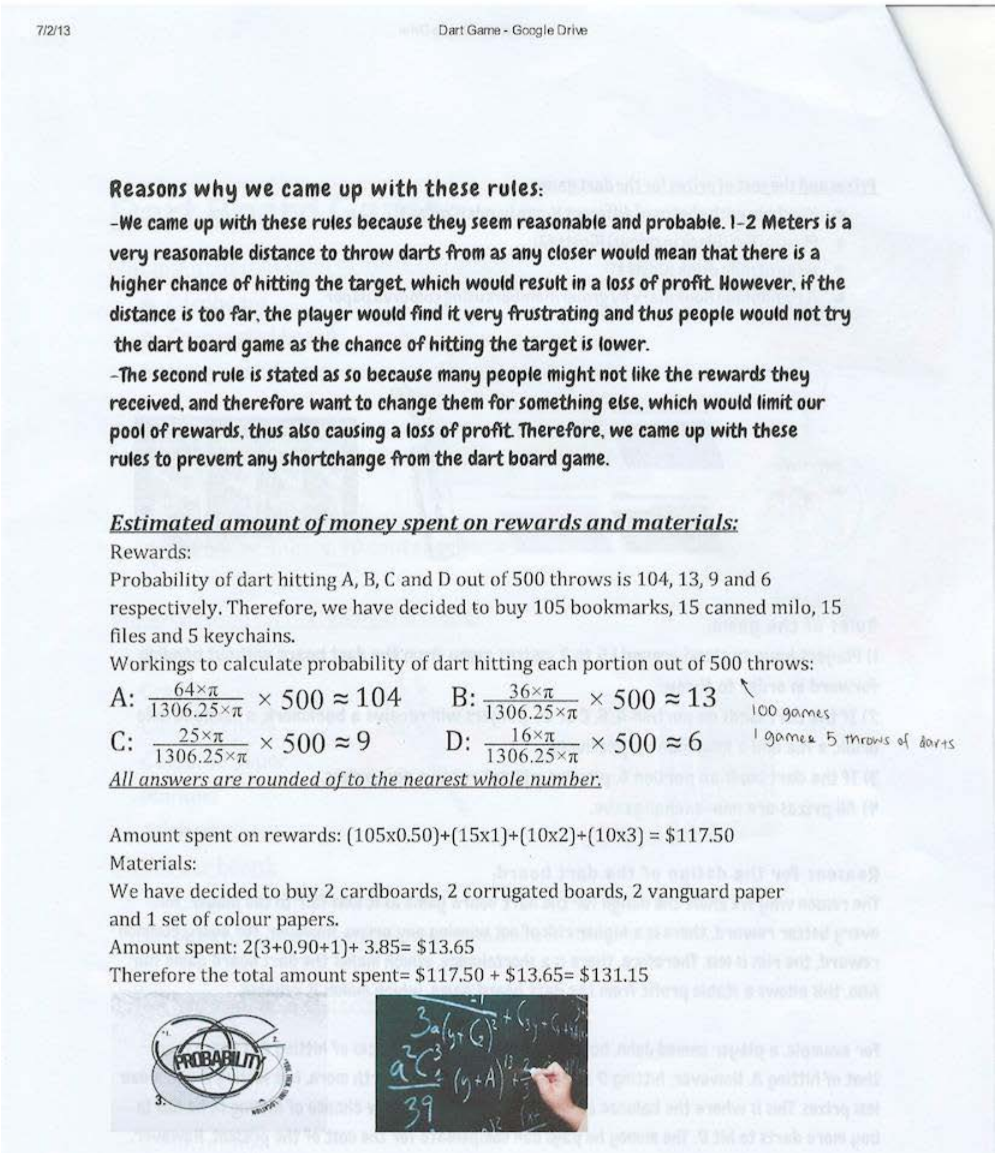


Figure 2: A proposed design for the dartboard



7/2/13 Dart Game - Google Drive

Reasons why we came up with these rules:

-We came up with these rules because they seem reasonable and probable. 1-2 Meters is a very reasonable distance to throw darts from as any closer would mean that there is a higher chance of hitting the target. which would result in a loss of profit. However, if the distance is too far, the player would find it very frustrating and thus people would not try the dart board game as the chance of hitting the target is lower.

-The second rule is stated as so because many people might not like the rewards they received, and therefore want to change them for something else, which would limit our pool of rewards, thus also causing a loss of profit. Therefore, we came up with these rules to prevent any shortchange from the dart board game.

Estimated amount of money spent on rewards and materials:

Rewards:

Probability of dart hitting A, B, C and D out of 500 throws is 104, 13, 9 and 6 respectively. Therefore, we have decided to buy 105 bookmarks, 15 canned milo, 15 files and 5 keychains.

Workings to calculate probability of dart hitting each portion out of 500 throws:

A: $\frac{64\times\pi}{1306.25\times\pi} \times 500 \approx 104$ B: $\frac{36\times\pi}{1306.25\times\pi} \times 500 \approx 13$ 100 games

C: $\frac{25\times\pi}{1306.25\times\pi} \times 500 \approx 9$ D: $\frac{16\times\pi}{1306.25\times\pi} \times 500 \approx 6$ 1 games 5 throws of darts

All answers are rounded of to the nearest whole number.

Amount spent on rewards: (105x0.50)+(15x1)+(10x2)+(10x3) = $117.50

Materials:

We have decided to buy 2 cardboards, 2 corrugated boards, 2 vanguard paper and 1 set of colour papers.

Amount spent: 2(3+0.90+1)+ 3.85= $13.65

Therefore the total amount spent= $117.50 + $13.65= $131.15

Figure 3: A sample of a shared work uploaded on Google Docs

Relay Race Problem

This problem was posed as follows.

In a 4 x 100m relay, a preceding runner has to pass the baton to the succeeding runner within a certain range of distance. At what point should the preceding runner be when the succeeding runner starts his sprint so as to reduce the overall timing.

The succeeding runner should begin running from her position, S, and pick up speed when she sees the preceding runner approaching and reaching position P (see Figure 4). This is so that when she actually receives the baton, she will be well on her way in reaching her maximum running speed. The question is, where should Point P be, relative to Point S?

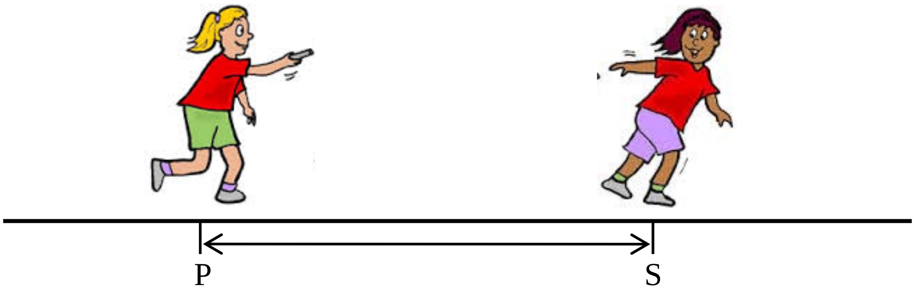


Figure 4: Preceding and Succeeding runners in a relay race

This problem was discussed by Osawa, who noted that in order to solve this problem, "*students were required to have large amounts of mathematical knowledge, and the skills to apply it*" [18]. However, when this problem was posed as a modelling task, participating teachers in the SBPD programme had planned for sufficient scaffolding, and made good use of technology to bridge the cognitive gap.

This task had involved students thinking about the factors that influence or determine the position of Point P. Eventually, students realized that they would need to know the runners' "running profile" – that is, how one picks up speed starting from rest – and to assume that runners are able to maintain their top speed for a period of time (at least a few seconds). A runner's "running profile" is simply the "distance-time" graph over a short distance, and to obtain this, some practical data collection exercise had to be carried out. Cones were placed at regular (10 m) intervals, and each runner is video-recorded as he sprints a distance of about 50m (see Figure 5).

Figure 5: Data collection for relay race problem

Technology and Modelling

In both the tasks described and carried out by teachers in the SBPD programme, some form of technological tools was used.

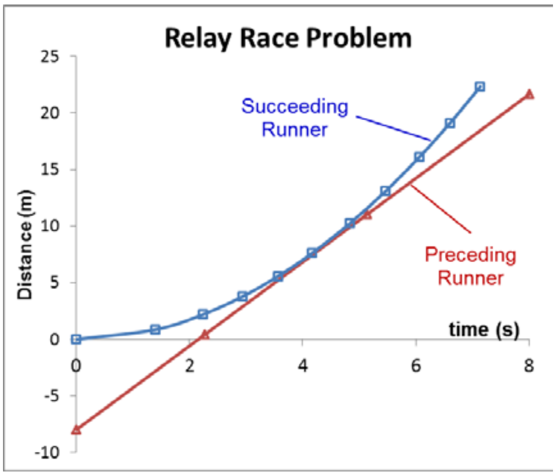
In the first task, students created and edited online documents to facilitate better sharing and discussion. The technological tool had enabled a much richer and more focused discussion amongst students as they worked on the modelling task. Teachers were also able to monitor students' thought process and assess their progress throughout the lesson and activity.

The task also provided opportunities for students to engage in simulating and outcome. Some of the dartboards had designs that contained irregular shapes and it was not easy to calculate the theoretical probabilities of a randomly thrown dart landing in them. An estimate needed to be found and this involved having to run simulations on a computer, which required skills and experience in coding.

In the second task, after collecting data, students needed to construct a function to represent the runner's running profile. A graphing calculator with regression tools may be used to construct the required function. Alternatively, as in this case, MS Excel's Solver tool may also be used to fit a function to the collected data. In the present study, students had earlier been taught the Solver tool technique through examples similar to that described in [19]. A sample of students' work on this problem is shown in Figure 6.

	A	B	C	D	E
1	Preceding			c	-8
2				m	3.70406
3	Time (s)	Distance(m)	Model	SqDiff	New Model
4	0.00	0	0.00000	0.00000	-8.00000
5	2.27	10	8.40823	2.53374	0.40823
6	5.14	20	19.03889	0.92373	11.03889
7	8.00	30	29.63251	0.13505	21.63251
8	10.74	40	39.78165	0.04768	31.78165
9	13.77	50	51.00497	1.00996	43.00497
10			Average	0.775026	
11	Succeeding				
12				a	0.43792
13	Time(s)	Distance(m)	Model	SqDiff	
14	0.00	0	0.000	0.000	
15	1.40	2	0.858	1.303	
16	2.24	4	2.197	3.250	
17	2.94	6	3.785	4.905	
18	3.57	8	5.581	5.850	
19	4.17	10	7.615	5.689	
20	4.84	12	10.259	3.033	
21	5.47	14	13.103	0.805	
22	6.07	16	16.135	0.018	
23	6.60	18	19.076	1.157	
24	7.14	20	22.325	5.405	
25			Average	2.8560	

(a) MS Excel worksheet utilizing Solver tool



(b) Graphical output showing that baton should be exchanged when preceding and succeeding runners are running at the same speed.

In this case, Point P should then be 8 m from Point S

Figure 6: Sample of students' work for the Relay Race problem

Discussion

In order to evaluate the SBPD programme, all training sessions, meetings, lessons and post-lesson discussions were video-recorded for analysis. The videos were transcribed, and category coding of the teachers' teaching decisions, orientation, resources and goals in the modelling classroom was carried out. An analytic tool adapted from Schoenfeld's framework for goal-based decision making [20] was used to record and analyse the observed data.

In this tool, a lesson segment that corresponds to each stage of the modelling process will be parsed into lesson episodes. Each lesson episode will subsequently be parsed into identifiable lesson issue. For this purpose, the three broad types of lesson issues are (i) positive teaching and learning moves from the teacher; (ii) learning opportunities lost; and (iii) negative teaching and learning moves from the teacher.

Due to space constraints, details of the tool and the analytical methods will not be discussed here. Instead, Table 2, which contains a selection of the observations made in the present case study, is presented to demonstrate the impact of the SBPD programme.

Table 2: Sampling of observed changes in participating teachers in various dimensions

Dimension	Before SBPD programme	After SBPD programme
Modelling Competencies	Teachers' motivation is focused on students' learning of skills in certain mathematical topics.	Teachers' focus shifted to providing learning opportunities for students to develop mathematical modelling competencies.
Orientation	Teachers generally adopt a "*modelling as vehicle*" orientation where the objective is to teach a specific topic in the syllabus through examples in mathematical modelling.	A "*modelling as content*" orientation is prevalent amongst participating teachers, who emphasize the various stages of the modelling process.
Resources	Teachers have limited knowledge and experience of modelling, and lack the common understanding of principles of task design, resulting in tendency to use resources that may not be relevant or appropriate.	Improved understanding of modelling process, design of tasks, anticipation of learning opportunities, and even facilitation skills, resulting in overall increase in confidence.
Goals	Teachers tend to have no specific goal or objective, and view lessons in mathematical modelling as a typical lesson for a mathematical topic.	Teachers have clear goals for their students, ranging from development of competencies, to demonstration of clear mathematical reasoning in their models.

The list in the Table 2 is neither complete nor exhaustive. Nonetheless, it serves to provide a sample of what was observed before the intervention (in this case, the SBPD programme) and after. In general, it is clear that the programme has had a positive impact on the orientation, resources and goals of the participating teachers. More details and results are reported in [21].

In addition, it is noted that in most of the activities and tasks designed, planned and carried out by participating teachers, technology had played an important role in the success of the activity. The types of technology used had included videography and platforms for document sharing, as well as the usual computer software such as spreadsheets, dynamic geometry software, graphing tools and so on. It is clear, therefore, that technology has played a pivotal, albeit supportive, role in mathematical modelling. One reason is that mathematical modelling may sometimes involve mathematical skills that are not yet accessible to the student, and in such cases, technology serves to make the mathematics more accessible ([22], [23]).

Conclusion

A school-based professional development programme to help teachers build their capacity in the teaching of mathematical modelling is presented in this paper. The focus of the programme is on transforming a mathematics teacher into a confident teacher of mathematical modelling, and not just raising awareness or providing resources. Observation and results from the case study indicate that the programme has had some positive impact, and these are encouraging signs. A more complete translation project is currently being considered. In addition, there are plans to build a digital repository consisting of video cases, exemplars of modelling lessons, instructional videos and electronic resources to provide teachers with a continuum of professional development possibilities in the area of mathematical modelling. It is hoped that together with these resources and support, the programme will make a significant difference to the capacity and confidence of teachers in the area of mathematical modelling in Singapore.

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