Effects of spreadsheet towards learners' usage of mathematical language

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Abstract: *Language has been an essential element in mathematics education as it is a tool used between individuals to convey the information of mathematical concepts. Some researchers discovered that inconducive language usage impedes mathematics learning. This papers reports on the effect of spreadsheet to the language used by a group of 32 pre-university students in defining "limits". The data revealed that (i) spreadsheet changes learners' commandment of language in three aspects which are usage of examples, types of expressions, and accuracy of terminology; (ii) spreadsheet does change learner' choice expression while learning mathematics regardless their academic ability; and (iii) through spreadsheet environment, learners are able to find a suitable and conducive expression to learn mathematics. This study is significant in such a way that it gives insights to education researchers, educators or curriculum-makers about the roles of spreadsheet in language which then influences mathematics learning.*

1. Introduction

The importance of learning *limits* has been acknowledged among researchers and educators. There is a range of contexts requiring the concept as pre-requisite knowledge such as area of a region, volume of a solid, centre of mass, surface area, convergent series, kinematics, maximum and minimum problems, tangent lines, asymptotes and so on [2, 6]). In this paper, *limits* refer to *limits of a sequence*, which is defined by $\varepsilon - N$ definition as in [2]:

"We call a sequence $\{a_n\}$ to be convergent to a real number A if for any positive number ε, there is a natural number N such that $|a_n - A| < \varepsilon$ for all $n \geq N$" (p. 70).

Because of the complicated structure of this definition, educators and learners often find that the correct concept of *limits* is hard to grasp [6]. A range of technological interventions have been developed by researchers or educators to remediate the difficulty of learning *limits* such as BASIC programming [11], DERIVE [14], spreadsheet [8], Flash Animation [4], and etc..

This paper is about a research of using spreadsheet in teaching and learning *limits*. In particular, the research question addressed is "what are the effects of spreadsheet to the language used to define *limits*". The remaining part of this paper is divided into several sections. The first section is Literature Review in which a conceptual framework about misconception related to language is presented. The second section is Methodology in which the research procedure to answer the research question is discussed. The third section is Result and Discussion in which the data obtained from the procedure is discussed and elaborated with past researches (if there is any). The last section is Conclusion in which the results are summarised and types of future researches are suggested.

2. Literature Review

Throughout the history of human's civilisation, language has been an important tool in a society for communication. Language allows interaction between individuals, including transferring information between one another which is then "translated" into own understanding, or in a simpler term "learning". Mathematics knowledge cannot be shared in a group if there is no suitable language to convey one's thinking in a concrete form which can then be "captured" by the other members. [13] described that expression of mathematics could be carried out in three forms being "everyday language" (language used in daily life), "mathematics specific language" (language used only in mathematics) and "mathematics symbols" (notation representing a concept). Some examples of these mathematical expressions are displayed in Table 1. Mastery of these expressions is essential to enhance one's proficiency in mathematics as each of them demands different mathematics literacies and serves for different purposes [1, 16]. For instance, the use of everyday language in mathematics relates real-life applications, whereas mathematical symbols enable one to manipulate theorems involving one or more mathematical concepts in a more efficient way.

Table 1 *Types of Expression Used in Mathematics and Some Examples*

Type of expression	Example
Everyday language	Limits, volume, order, natural numbers, etc.
Mathematics specific language	Parallelogram, tangent, hypotenuse, secant, etc.
Mathematics symbols	$\sum_{r=1}^{n} r^2$, $\sqrt{x}$, $\log_a b$, $\tan x$, etc…

However, problems of learning mathematics always arise due to language factors such as vocabularies with multiple meaning in various contexts, language syntax, specialist terms and symbolism. According to [15], forming vocabularies of mathematics by deriving from everyday language could result in problem for learners due to change in meanings. Taking the word "volume" as an example which is a common term in general, it refers to "capacity of a solid" in mathematics but it can also refer to "level of sound" in everyday English. Different definitions of the same word might result in ambiguity and confusion because the familiarity with the everyday English meaning has rooted deeply among learners [1, 16].

Another reason of complication felt by mathematics learners could be the structure of the sentence i.e. syntax. [7], [12] and [15] identified the following misconceptions or problems: (a) learners commonly regarded conditional statements "if p, then q." and "if q, then p." to be logically equivalent; (b) learners got confused easily by statements consisting of negation that changes the truth value such as "not", "in-", "un-", "without" and etc.; (c) learners misinterpreted lengthy compound sentences joined by connectors such as "which", "that", "because", "but" and etc.; and (d) learners found statements in passive form difficult to understand due to subject-predicate inversion. These misconceptions or problems might impede construction of correct concepts among learners because of misunderstanding in the language.

[15] also reported that specialist terms which were used in mathematics classroom only did not encourage learners to explore a concept further. Words such as "tangent", "hypotenuse", "quadrilateral", and etc are rarely used in daily life, and learners might find no point in studying concepts relating to these terms as they are not able to apply in other context. This might lead to shallow understanding which is not conducive for learning abstract theorems.

Symbolism makes mathematics distinct from other fields. Symbols act as a stepping stone in enabling axiomatic formulation from a set of specified properties to define mathematical structures or deduction of other properties [9]. Each symbol functions to carry the meaning of both process and concept simultaneously. For examples, the *limit* notation $\lim_{n\to\infty}\frac{n+1}{n}$ could mean the process of converging the value of the function by substitution of various infinitely large numbers, or it could also mean the concept of the horizontal asymptote of the function. Yet, learners often overemphasise the process part of the symbol causing them to learn computation more than the concept itself [6]. According to [3], the way of pronouncing a symbol also affects learners' acquisition of proper understanding of a concept due to some missing information. Taking $\frac{2}{3}$ as an instance, reading it as "two thirds" or "two over three" might give different interpretation to fractions and thus learners might understand fraction in different ways. The second pronunciation losses the "rationality" of the fraction which might lead to misconceptions like $\frac{a}{b}+\frac{c}{d}=\frac{a+c}{b+d}$.

3. Methodology

3.1. Sample and Location

This study was carried out in a private college in Malaysia. A class of 32 students of the college took part in the research. The groups had mixed academic capabilities which were determined through formative assessments such as monthly tests, quizzes, assignment and etc.

3.2. Spreadsheet Module

A spreadsheet module was designed, centring on $\varepsilon - N$ definition, to remediate the difficulty in learning *limits*. It consisted of 11 worksheets, each addressing different learning outcome(s). The purpose of the module was to enable learners to interpret the abstract structure of $\varepsilon - N$ definition in a concrete environment. They were expected to learn the concept of *limits* from various perspectives such as (a) the relationship between *limits* and cluster points and asymptotes; (b) convergence and divergence of a sequence; (c) uniqueness of *limits*; and (d) distributivity of operations involving *limits*. The content of each worksheet in the module is summarised in Table 2. For readers' convenience, a sample of the worksheets is provided in the Appendix, accompanied with a screen shot which illustrated how spreadsheet was used in this study. The module was inter-rated by three senior lecturers from the college having at least 20 years of experience in teaching mathematics. Generally, they gave positive feedback in terms of content, instructions in the worksheets, levels of difficulty and variety of the questions in the worksheets.

3.3. Data Collection and Analysis

This is a qualitative research and is exploratory in nature. Firstly, a pre-test was conducted by asking the participants to answer the following question:

"Describe what is meant by the limit of a sequence a_n is some number *A*. (You can describe in few sentences, diagram, tables or any illustration that suits your understanding)."

This open-ended question was to capture language(s) used by the participants to define *limits* mentally. It required the participants to use their own words to express the meaning of *limits*. Then, they were assigned to take the spreadsheet module for 8 weeks, 2 days each week and 30 minutes each day. In the end of the treatment, a post-test was conducted using the same question. All written responses in the module as well as in the instrument and observation throughout the treatment were transcribed and analysed for the change of language used to understand *limits*. Several informal interviews were also carried out to justify some anomalies.

Table 2 *Content of Spreadsheet Module*

Worksheet	Learning outcome
1	Judge the validity of *limits* with reference to the formal definition.
2	Identify the significance and relationship of ε and N in the formal definition.
3	Prove the implication that if the *limit* of a sequence exists and is known, then the *limit* is the horizontal line clustered by infinitely many points.
4	Predict the *limit* of a sequence geometrically.
5	Prove the implication that if there exists no horizontal line of clustered points, then the *limit* does not exist.
6	Disprove the implication that if $y = A$ is a horizontal line of clustered points, then the *limit* of the sequence is A.
7	Prove the distributivity of sum of *limits*.
8	Prove the distributivity of scalar product of *limits*.
9	Prove the distributivity of difference of *limits*.
10	Prove the distributivity of product of *limits*.
11	Prove the distributivity of quotient of *limits*.

4. Results and Discussion

The effects of spreadsheet towards the language used by participants in this study are discussed in three aspects being (a) usage of examples; (b) types of expressions; and (c) accuracy of terminology. Due to high variety in the participants' responses and the space constraints, only several samples of responses are presented for each of these aspects.

4.1. Usage of Examples

Some participants tended to use only examples in explaining what a *limit* is instead of giving a more general description before the treatment (see Table 3). Most of them computed *limits* of different functions and did not elaborate further. Examples seemed to be the only way they could think of in defining an object. Among some of them, says Participant *A*, "examples-only" definition was still practised even after undergoing the treatment. However, the "sense" of the explanation was more generic because he was trying to relate $\varepsilon - N$ definition with the example given in his statement, as highlighted in the capital letters.

Table 3 *Response of Participant A*

Before treatment	After treatment
Let's say $a_n = \frac{1}{n}$. As $n \to \infty$, $a_n \to 0$. Hence, $A = 0$.	Let's say $a_n = \frac{1}{n}$. We know that the limit of $a_n = \frac{1}{n}$ is 0 as $n \to \infty$. This means that the DIFFERENCE between $a_n = \frac{1}{n}$ with 0 is LESS THAN any number for EVERY SINGLE n.

It was also found from some responses that *limits* was defined in terms of its existence, indicated by words or phrases like "exist", "has *limits*", "does not exist" and etc. However, the explanations were accompanied by examples of functions of which *limits* do not exist, examples of functions of which *limits* exist and the values of the *limits*. A sample response of Participant *B* is displayed in Table 4. After the treatment, *B* attempted to generalise a suitable definition, but still in terms of existence of *limits*. With reference to the responses of *A* and *B*, the effect of spreadsheet seemed unable to change their usual practise of defining a term but they tried to apply $\varepsilon - N$ definition wherever possible.

Table 4 *Response of Participant B*

Before treatment	After treatment
• Limit can exist or not exist. For example, the limit of $a_n = 0$ exists and equals 0. • The limit of $a_n = \frac{1}{n}$ exist and equal to 0 because a_n becomes smaller and smaller and finally approaches 0 as n grows larger. • The limit of $a_n = n+1$ does not exist because a_n grows infinitely larger as n grows larger.	• If the limit of a_n exists and is *A*, then $a_n - A$ is less than a number for all *n*. • If the limit of a_n exists but is not *A*, then $a_n - A$ is not less than a number for all *n*. • If the limit of a_n does not exist, then there is no *A* such that $a_n - A$.

Similar to *B*, Participant *C* also used examples to define *limits* in terms of existence before the treatment (see Table 5). Instead of using only one example for each case as by *B*, *C* listed out as many examples as he could for each case, consisting of different kinds of functions. His response was also discovered to contain very few words, which could be due to his limited vocabulary in making sentences. However, differing from *A* and *B*, *C* did not use example(s) in the post-test as his sentence was closer to the generic meaning of *limits*, although not completely accurate. In this case, the spreadsheet module was assumedly suitable to *C*. It is also interesting to point out that *C* defined *limits* using some "actions" involving him as a subject in the sentence, indicated by words "I choose a number…" and "I compare…". This drastic change was uncommon among the other responses.

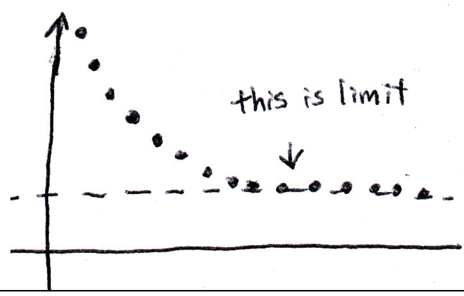
Table 5 *Response of Participant C*

Before treatment	After treatment
• Exist: , $\frac{1}{n}$, e^{-x}, $\frac{\cos n}{n}$… • Not exist: n, e^{x}, $\cos n$…	I first choose a number *d*, and then I compare it with the value of $a_n - A$. If $a_n - A < d$ for all values of *n*, then *A* is the limit of a_n.

4.2. Types of Expression

Table 6 displays Participants *D* and *E*'s responses in the pre-test and the post-test. Before the treatment, some participants did not really provide their ideas about *limits* but instead they "translated" part of the question in symbolical language, particularly "the *limit* of a sequence a_n is some number *A*". It was presumed that they regarded symbols as the main medium in definition, tending to focus on the symbolic part of mathematics and ignore the language which conveys the actual information of a concept. Whenever *limits* were discussed, the use of symbols like "$\lim_{n\to\infty} a_n$" that represents "*limits* of a_n as n grows infinitely large" and "$\to$" that represents "approaching" were prioritised [9]. After the treatment, they made use of more words or some two-dimensional representations to express their thinking although their notions were not the whole meaning of *limits*. They were more expressive in the end of the treatment as they opened themselves for more choices of communication.

Table 6 *Responses of Participants D and E*

	Before treatment	After treatment
D	$\lim_{n\to\infty} a_n = A$.	When *n* grows larger, the values of a_n converge to to *A*.
E	$a_n \to A$ when $n \to \infty$	

Before the treatment, some participants, says *F*, attempted to explain *limits* in terms of existence using mathematical equations (see Table 7). They defined *limits* as "exist" by labelling "= *some numbers*" whereas "not exist" by labelling "$=\infty$". This case is different from *C* who only listed some functions that possess *limits* and that do not possess *limits* without evaluating each. After the treatment, *F* was found to use more words than before. According to [10], this change of behaviour could be due to adaptation in working with spreadsheet that provided them convenience in learning, and hence they related what they have practised to definition.

Table 7 *Response of Participant F*

Before treatment	After treatment
Exist: , $\lim_{n\to\infty}\frac{1}{n}=0$, $\lim_{n\to\infty} e^{-x}=0$, $\lim_{n\to\infty}\frac{n+1}{n}=1\ldots$ Not exist: $\lim_{n\to\infty} n=\infty$, $\lim_{n\to\infty} e^{x}=\infty\ldots$	(a) A value ε is chosen. (b) Find the value of $\lvert a_n - A \rvert$. (c) The value in (b) less than (a) for all values of *n* means the limit is *A*.

Participant *G* was one of the several participants who were good at using words to convey their idea before the treatment. After taking the module, he chose other media to represent *limits* such as diagrams, matrices, concept maps and so on (see Table 8). It could be deduced that the interactive representation in the spreadsheet environment, which involves array visualisation, action of dragging, variable manipulations, and etc., brought some changes to his way of expressing a mathematics concept [5, 8].

4.3. Accuracy of Terminology

In describing *limits*, it was also common to see that participants tended to equate terms which have different meanings in the pre-test. Referring to Table 9, Participant *H* regarded ∞ as "*limit* does not exist" which is a major misconception because ∞ means "unbounded" whereas "*limit* does not exist" is defined by "left-hand *limit* not equal to right-hand *limit*". In other words, if a *limit* does not exist, it does not mean that the functions will increase / decrease unboundedly. His commandment of terminology was improved after the treatment as he related the convergence of functions to the existence of *limits*. On the other hand, Participant *I* regarded *limits* as "asymptotes" (see Table 9), which is another misconception because such statement only describes some properties of *limits*. If A is the *limit* of a function, then the line $y = A$ could be EITHER an asymptote OR a clustered line of the function. Using inappropriate words caused both to fail to grasp the full notion of *limits* [15]. This misconception was remediated after the treatment but minor mistakes were still identified. The idea of "*limits* are equal to asymptote or clustered line" does not completely convey the concept of *limits* because multiple asymptotes and / or clustered lines of a function cannot be *limits* simultaneously due to the uniqueness of *limits* of each function according to $\varepsilon - N$ definition. However, the responses of *I* indicated that spreadsheet has broadened his view about *limits*.

Table 8 *Response of Participant G*

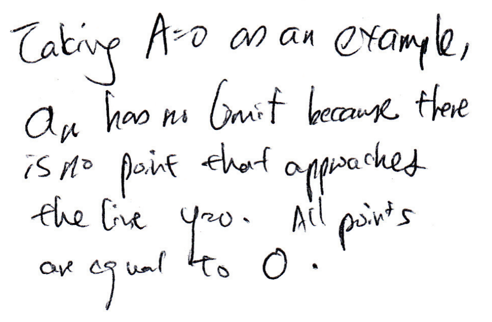
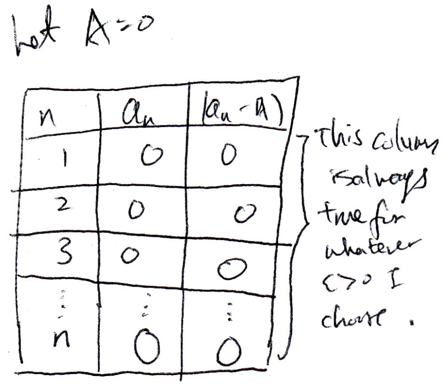
Before treatment	After treatment
	

Table 9 *Responses of Participants H and I*

	Before treatment	After treatment
H	It means A is a number that can be defined. If the limit does not exist, then A is ∞.	The limit exists only when a_n is convergent, and there is only one and only one limit for each convergent a_n. Likewise, the limit does not exist only when a_n is divergent.
I	$y = A$ is the asymptote of a_n.	Asymptotes and line clustered by points.

Participant *J*, a high-achieving learner, was one of the very few participants who had got correct understanding about *limits* before the treatment (see Table 10). His response perfectly matched $\varepsilon - N$ definition which could possibly be due to his academic background. However, he was discovered to switch from using mathematical language to spreadsheet language in defining *limits*, indicated by the technical terms of spreadsheet in his response such as "generate", "true output", "dragging", "table" and etc. This result is coherent to a qualitative study done in [5].

A number of participants preferred to use daily language in describing *limits* instead of using mathematical language. Referring to Participants *K* and *L*'s pre-test responses in Table 11, words such as "approaches", "moving to infinity" and "grow smaller and smaller" were widely used. [6]

pointed out that daily language did not help learners understand the genuine mathematical meaning of *limits* due to the limiting ability of the words in explaining the abstract structure of a mathematical concept. After the treatment, they seemed to have used less daily languageas the word “convergent” is common only in science and mathematics classroom. It was predicted that spreadsheet provided a conducive environment for them to use more mathematical language as these are very rare in daily life. Commandment of mathematical language enables one to convey a mathematic concept more accurately.

Table 10 *Response of Participant J*

Before treatment	After treatment
A is the limit of a_n if there exist *N* in such a way that $\|a_n - A\| < \varepsilon$ for all $n \geq N$.	A is the limit if we are able to identify a value of *N* such that $\|a_n - A\| < \varepsilon$ which generates all true outputs as we drag downwards the table for any ε.

Table 11 *Responses of Participant K and L*

	Before treatment	After treatment
K	Points approaches *A* as they move to infinity but does not equal to *A*.	All points of a_n converges to only one and only one value *A*. If there is no convergent value, then a_n does not have limit.
L	It means a function grows smaller and smaller to *A*.	Convergent to *A*.

5. Conclusion

This paper addresses the effects of spreadsheet to learners’ language in learning mathematics, having seen the important roles of language in grasping the correct notion of mathematical concepts [1, 3, 7, 12, 15, 16]. Three main notable implications of this research were identified: (i) spreadsheet changes learners’ commandment of language in three aspects being usage of examples, types of expressions, and accuracy of terminology; (ii) spreadsheet does change learner’ choice expression while learning mathematics regardless their academic ability; and (iii) through spreadsheet environment, learners are able to find a suitable and conducive expression to learn mathematics. Despite that spreadsheet is suitable for only some mathematics learners in terms of remedy for misconception, the language skill learnt from an interactive environment does give a difference to mathematics learning. Due to the limitation of the research in terms of the methodology and the conceptual framework, why and how spreadsheet gives such changes cannot be answered in this paper. Hence, more exploratory researches are required to answer these questions.

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Appendix

Worksheet 6

In the end of this activity, you should be able to
(a) identify the relationship between the limit of a sequence and the terminal value.
(b) explain the existence of the limit of a sequence by using $\varepsilon - N$ definition.

> ***Question of the Day***: From previous activities, we know that if <u>the limit of a sequence is *A*, then the horizontal line *y* = *A* would be clustered by infinitely many points</u>. How about if *y* = *A* is a horizontal line of clustered points, can we say that the limit of the sequence is *A*?

Instruction: You are going to work with Microsoft Excel to answer the above question.

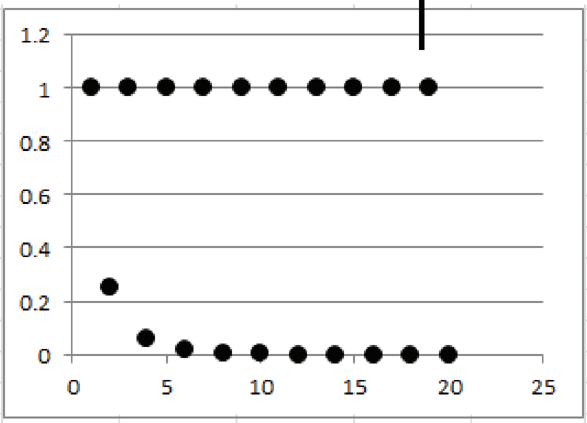
1. A sequence a_n is such a way that if *n* is odd, then $a_n = 1$; and if *n* is even, then $a_n = (0.5)^n + 1$.
   (a) Sketch the scatter plot of a_n for $n = 1, 2, 3, ..., 20$.
   (b) Predict the limit of a_n from your scatter plot. If you cannot get any, expand the scatter plot for more *n* values until you get a value.
   (c) Verify your answer in (b) by using $\varepsilon - N$ definition with $\varepsilon = 0.1$. If all are "FALSE", expanding the table for more *n* until you get **ALL** "TRUE". [You can stop if you have reached $n = 1000$.]
   (d) Write down the value of *N* such that all outputs are TRUE when $n \geq N$.
2. Repeat No. 1 for various a_n. Then, discuss the ***Question of the Day*** in your group.

(1) Build the table of the sequence.

(7) A is the limit of the sequence if we are able to find a value from the column n such that ALL outputs after that value are TRUE.

(2) Sketch the scatter plot of the sequence.

n	a(n)	abs[a(n) - A]	
1	1	1	FALSE
2	0.25	0.25	FALSE
3	1	1	FALSE
4	0.0625	0.0625	FALSE
5	1	1	FALSE
6	0.015625	0.015625	FALSE
7	1	1	FALSE
8	0.003906	0.00390625	TRUE
9	1	1	FALSE
10	0.000977	0.000976563	TRUE
11	1	1	FALSE
12	0.000244	0.000244141	TRUE
13	1	1	FALSE
14	6.1E-05	6.10352E-05	TRUE
15	1	1	FALSE
16	1.53E-05	1.52588E-05	TRUE
17	1	1	FALSE
18	3.81E-06	3.8147E-06	TRUE
19	1	1	FALSE
20	9.54E-07	9.53674E-07	TRUE



A	0
epsilon	0.01

(3) Predict the possible value(s) of the limit from the plot and put it here.

(4) Choose a suitable value of epsilon according to the *ε-N* definition.

(5) evaluate the difference between a(n) and A according to *ε-N* definition.

(6) FALSE if the difference > epsilon and TRUE if the difference < epsilon