Case Studies in Thinking Processes of Mathematically Gifted Elementary Students through Logo Programming

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Background

Many scholars indicate that the education process for gifted students has to place importance on diversity in depth and range for instructional content, and such diverse curriculum needs to be connected to form an integrated education process or project-based learning which would provide intellectual curiosity and promote the integrated thinking to connect knowledge of diverse fields (Sheffield, 1994; Samara et at, 1992; Maker & Nielson, 1995; Van Tassel-Baska, 1994).

This study has aimed to introduce the effectiveness of the project-based learning using LOGO computer language as a part of curricula for the gifted. LOGO project learning is consisted of a series of unit learning in which students independently draws the diagrams required in completing the works of a selected theme given as project tasks. Through unit learning, the command languages are utilized to search for the most efficient method to draw the diagram and to find the independent solution method for completing work with more advanced ideas. The process to set forth the project theme and plan for programming to draw the appropriate diagram would be the integrated contents with implication for knowledge in several curricula in mathematics, art, science and others to promote logical and systematic thinking with the diverse processes to make the procedure for several diagrams needed to organize the works in addition to integrated thinking. In addition, the process to correct and supplement several diagrams needed to compose a work is closely related to such higher level thinking activities as critical and analytic thinking.

In this study, LOGO is incorporated in the dynamic project-based learning that provides students with opportunities to apply and advance their knowledge and engage in diverse creative activities beyond understanding geometric or arithmetic concepts through the integration of several disciplines not only mathematics but also art, science and others as a positive way to foster higher levels of thinking for gifted students. Many previous studies also consider that LOGO can be used in the effective learning environment to improve higher levels of thinking (Clements & Gullo, 1984; Swan, 1989; Keller, 1990; Clements et al., 2008). However, this study will focus on what strategic thinking the mathematically gifted elementary students use to plan, implement and debug in the programming process as a problem solving process.

Research Method

This study was undertaken with three sixth grade students chosen from a group of 40 mathematically gifted students at the Education Center of the Scientifically Gifted of the Seoul National University of Education, and one a fifth grade student chosen from a group of 20 mathematically gifted students at the Education Center of the Gang-Nam District Office of Education in Seoul. The students learned the basic MSWLOGO commanding language to process the recursive procedure and to execute an animation by using variables and conditional phases to

complete the project during a total of 12 experimental classes. The 12 classes made up for the project learning are classified as shown in <Table 1>. Camtasia Studio 6 was used to record the students' learning processes and audio recordings were taken as well. In order to enhance the feasibility and reliability of the study, diverse types of data were collected including interview data, observation data of the experimental classes, and the linguistic and non-linguistic activities of the learner and others.

<Table 1> The 12 classes Project learning using LOGO

Phase	Activities
Theme setting [1]-[3]	The students first learned the basic commanding language and defined the procedures to draw several diagrams by using commanding language in the first 2 classes, then informed the project learning in the third class. Students cooperated to determine the project theme for each group, what diagrams to draw and how to draw them.
Diagram structuring [4]-[9]	In the 4th to 9th classes students cooperated and selected the work required on the project for each group to build up programming of diagrams selected.
Work completion [10]-[12]	In the 10th to 12th classes, students devised activities to integrate several diagrams made by each group. The previously made diagrams were modified or new diagrams were made depending if necessary. [One of the final results are shown in the appendix of the final page of this paper.]

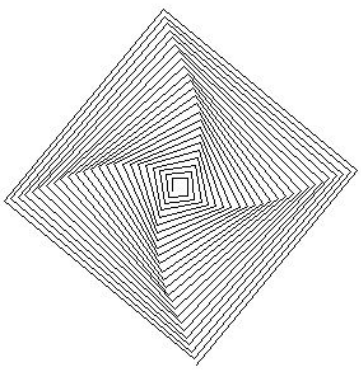
Data Analysis

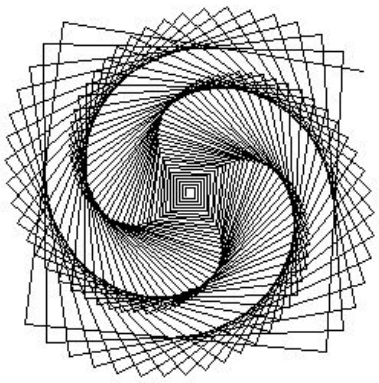
Planning and Implementing Strategies

Analogy

Excerpt 1(6^{th} class)

1 Teacher: How can we draw these two diagrams ?





2 Juseong: The maze of the square shape is rotating. So, it is enough to change the size of the angle in the maze procedure.

3 Jihan: No, the length of the side is changing too.

4 Juseong: The length of the side in the maze was different too. So, it is enough to change only the size of the angle in the maze procedure.

5 Teacher: In the regular polygon maze, the size of the angle was consistent but only the length of the side was changed.

6 Jihan: The angle size is related to 90 degrees.

7 Juseong: Right. The left diagram has the size of the rotation angle smaller than 90 degrees and the right diagram is about 91 degrees. I can make it now. In the maze procedure, if the variable value is 91 and 300 we can draw the right diagram.

8. Jihan: In the case of 89 and 300 we can draw the left diagram. Fantastic!

The students who learned the recursive procedure to draw the maze thought the diagram was similar with the maze procedure in the sense that the length of the side is changed and the diagram was different to the maze procedure in the sense that the size of the internal angle of a regular polygon is changed (2, 4, and 6). Based on the similarity and the difference, they could draw the diagrams presented by the teacher (7 and 8). They understood that the angle size for drawing the two diagrams is related to the internal size of the regular square (6) and with such basis, the left diagram has a rotation angle smaller than 90 degrees(8) and the right diagram has the size of rotation angle larger than 90 degrees (7).

Generalization

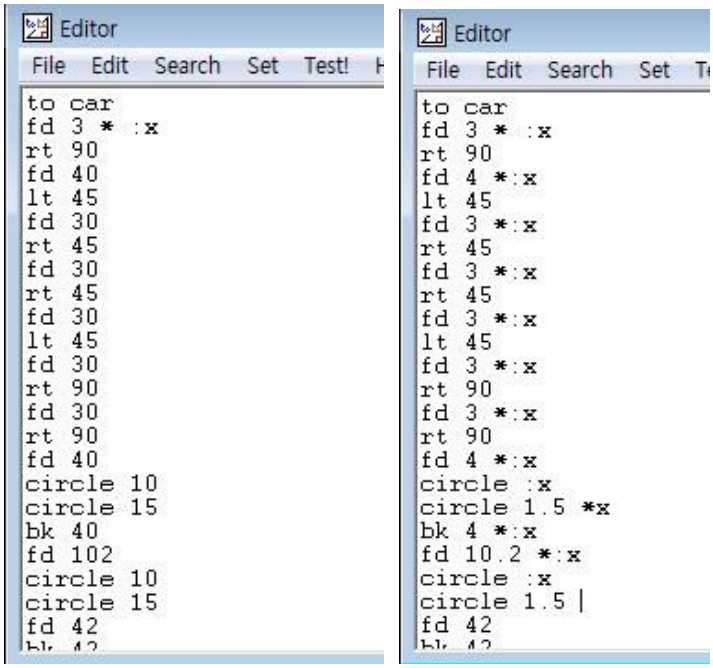
Excerpt 2(5^{th} class)

1 Soohyun: Yes, let's input a variable in the vehicle procedure that I made. Then the vehicle size can be controlled as I want.

2 Teacher: The lengths in the segment making variable are all different. So, it might be difficult…

3 Soohyun: The variable can be inserted in order that when its value is 10, this size of vehicle can be drawn. The angle size has no change so it's not difficult.

4 Jihan: The procedure to adjust the vehicle size…, that's fantastic. This is why formulae are so good.



In the class to draw various figures using circles and several regular polygons, Soohyun drew the vehicle to place in the project. After drawing a special sized vehicle, she determined the method to generalize the procedure for drawing the vehicle (1). The special sized vehicle seemed to be difficult to make generalization as the circle and polygon were mixed, but one can detect that Soohyun considered the generalization in mind from the beginning (3). The students understood that the generalization of the procedure is convenient to draw several shapes and they had a clear grasp of the meaning of generalization as a formula in mathematics (4).

Critical thinking

Excerpt 3(5^{th} class)

1 Juseong: If a circle is drawn by using a regular polygon, it's difficult to draw some kinds of shapes. No, it might be impossible.

2 Teacher: What shapes are those?

3 Juseong: When the regular polygon procedure is used, the length of the radius cannot be accurately known. In other words, a shape combining a circle and straight line cannot be drawn. For example, in drawing a semi-circle, we do not know what the length of the straight line has to be. So a gap forms.

4 Teacher: Then, what should we do?

5 Juseong: We need to use the radius to draw the circle.

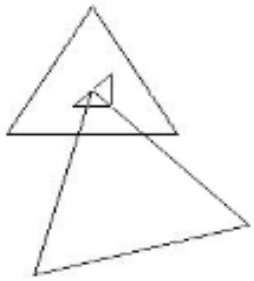
After learning how to draw a circle by changing the procedure they drew a regular polygon, the students were able to draw different types of shapes by using the circle. However, Juseong pointed out that the mixed figure of the square, regular triangle, and circle was difficult to draw because the length of the radius was inaccurate (1). He analyzed and evaluated the existing commanding

language to solve the problem and in order to draw a diagram integrating a circle and straight line, the length of the radius has to be known (3). He recognized that a new commanding language was required to draw the shape (5), and such an analysis motivated them to investigate the procedure to draw a circle using radius, which means that their role had changed into that of a designer making new programming language through the critical thinking. .

Progressive thinking

Excerpt 4 (10^{th} class)

1 Juseong: What is this? What happened?



2 Jihan: A twisted tree, oh! How about we call it the tree of Pisa?

3 Juseong: If so, shall we try drawing the triangle to become smaller while slowly rotating?

4 Jihan: Then it could have the shape of a golden spiral.

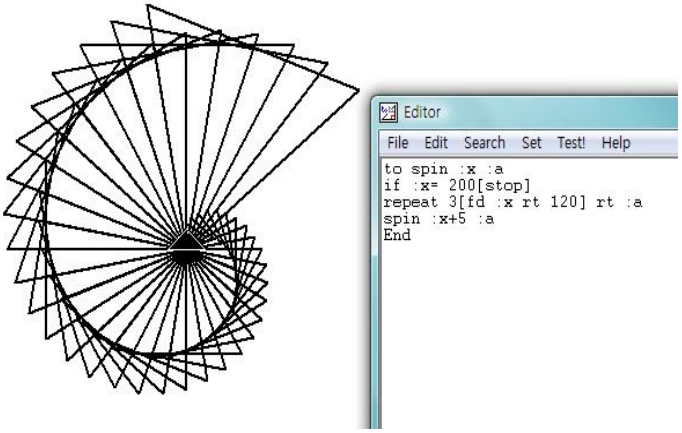
5 Juseong: I have a good idea. Let's draw the golden spiral with the triangle located in the center. What about a golden spiral tree?

6 Juseong: Then, how can we draw the golden spiral?

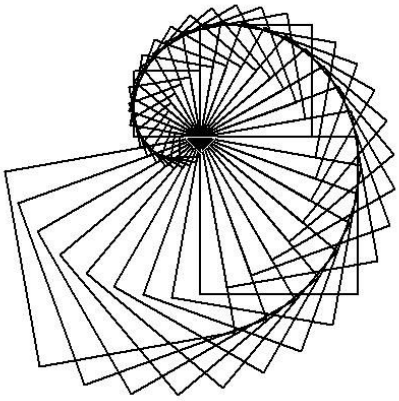
7 Teacher: If you use command language to change the rotational angle that you've learned, you might draw a similar shape to the golden spiral…

8 Soohyun: The conditional phase can be used.

9 Juseong: I have made it with the hint from you. Take a look.



10 Soohyun: It might be possible to use a square instead of a triangle. It is a complete a golden spiral!



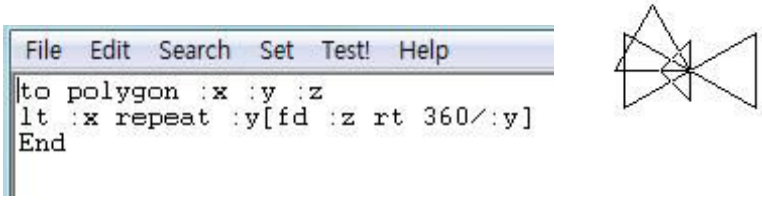
Juseong intended to draw the stair-shaped trees using the regular triangle for the project and tried to establish an appropriate procedure. However, because the directions to draw the first triangle and the second triangle differed the desired diagram could not be drawn. However, rather than having a negative attitude the students were able to grasp this as an opportunity to make a new shape(3, 5). With the help of the teacher who asked about how they could draw the golden spiral tree (7) the students were able to understand that the diagram of the golden spiral shape could be drawn by using the conditional procedure to change the length of the side and the size of the angle (8). Juseong drew a golden spiral shape in which a triangle rotated and became gradually smaller (9). Soohyun extended it by using a square instead of a triangle(10). Interpreting the diagram in error from a different point of view became a god opportunity for the unintended learning.

Flexible thinking

Excerpt 5(3rd class)

1 Jeho: Normally a polygon is made by using two variables that shows the number of side and length of side for the procedure, but I added one more variable that displays the rotation angle. If the first variable X has a value of 150 while the second variable Y has a value of 3, the ▽ -shaped

triangle is drawn.



2 Teacher: It means that the direction to draw triangle differs depending on the variable X.

3 Jeho: Yes. So, the hexagon can be drawn by 6 rotations of the triangle with the rotation angle of 150 without separate definition of the hexagon procedure.

4 Teacher: The basic drawing can be drawn in several directions.

5 Jeho: The procedure can be conveniently used when drawing the butterfly shape. Just change the rotation angle to draw several shapes in symmetry.

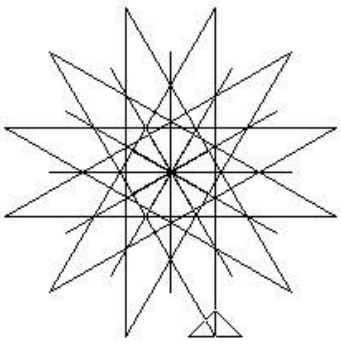
Presenting polygons with diverse shapes in the diverse locations is one of the important mathematical learning principles. Under the LOGO environment, the turtle shape is toward the 12 o'clock direction so that under the usual procedure of triangle, it has the ▷ -shaped triangle to draw. However, Jeho did not stop at drawing ▷ -like the triangle on the determined location but constructed the procedure to draw triangles in the diverse direction (1). He recognized that the newly defined procedure was effective in displaying polygonal shapes in the desired direction like symmetry (5).

Debugging Strategies

Visualization

Excerpt 7(2nd class)

1 Jeho: Teacher! I tried to draw the windmill rotating in the opposite direction of the windmill to rotate toward the right direction, but it came out like this. But, I think it also looks nice.



2 Juseong: I can see what you have done wrong. When returning after making LT 90, it has done RT 90 but you did still LT 90?

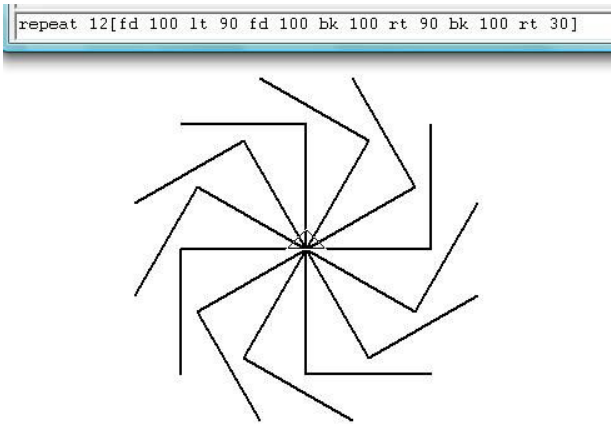
3 Teacher: Wow! Great. How could you know by just looking at the diagram?

4 Juseong: I used BK to draw the windmill, and when I returned after drawing a wing, the rotation direction was the opposite to the rotation direction.

5 Teacher: It would be hard to find the problem in rotation direction just by looking at the drawing…

6 Juseong: Looking into Jeho's diagram, it is wholly connected. So I could think that the rotation direction could be wrong._

7. Jeho: I made it but I could not think of it…. We're alright now.



The sunflower shape and the windmill shape are examples of the several shapes to draw by using the regular square procedure. Before drawing the sunflower shape, Jeho drew the windmill diagram rotating in the opposite direction of the sunflower shape. Therefore, he thought that if the rotation direction could be changed from RT to LT in the procedure the windmill shape rotating in the left direction can be drawn easily (1). However, it made the wrong shape like sunflower. As watching this shape, Juseong found out that the unexpected diagram was connected in its entirety and based on this visualization the rotation direction is changed to RT when BK returned to the original position (2, 6). Juseong clearly understood how the moving segment of turtle changes depending on the direction of the turtle. In particular, he found that the returing direction has to be RT when the flier direction is LT based on the fact that the shape was changed depending on the commanding language on the position of the turtle,.

Empirical Inference

Excerpt 8

1 Soohyun: Shall we have the procedure to change the flower number only fixing the flower size? No. It would be better to change even the flower size.

```
File  Edit  Search  Set  Test!  Help
to flower :x :y
circle :x *23 repeat :y[repeat 2 [repeat 90[fd :x rt 1] rt 90]rt 360/:y]

End
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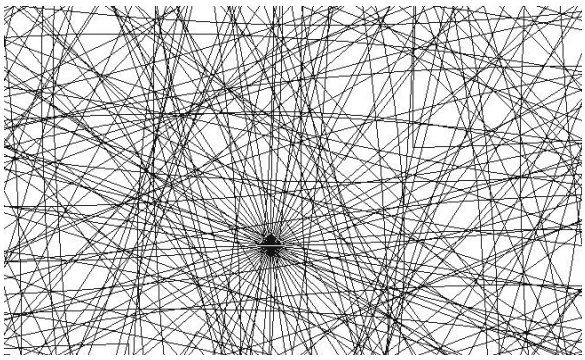
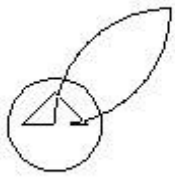
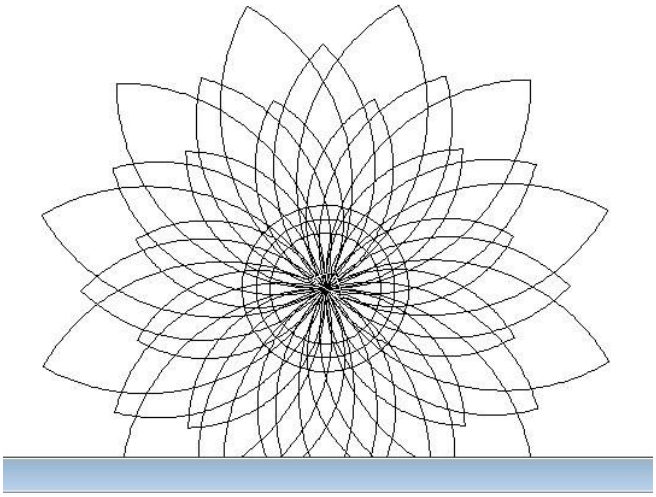
2 Soohyun: (After running “flower 50 30”) Why do we have the shape like this? It should be a flower…. The value of the variable does not seem right. (Murmuring while looking at the erroneous diagram) What should the variable be?

3 Teacher: The repeated frequency was too many.



4 Soohyun: I have a good idea. Let’s execute in sequence from “flower 1 1”. Oh, I know what is wrong. The input of radius and rotation angle were too large. Now I have it completed. Let me show you a great flower (After inputting flower 2 12 RT 15 flower 2.5 12 RT 15 flower 3 12) Take a look!





Soohyun constructed the general procedure to draw a flower with different sizes of petals by using circles and arcs(1). In order to draw a flower, a specific value of the two variable was inputted on the general procedure, but it did not work. But the error was occurred by the selected values of variables rather than the procedure itself (2). Therefore, “flower 1 1” with the smallest value was selected as a part of the way of finding the proper values for the flower (4). Through the shape with the variable value 1 1 that is, on the basis of her experience, she drew an inference of the rotation numbers of other petals, and the radius sizes of the desired shaped flower.

Conclusion

Through the analysis of the strategic thinking types of gifted elementary students displayed in project learning using LOGO, we are able to obtain the following implications for education. First, project learning using LOGO can be utilized in an integrated curriculum customized for students gifted in mathematics. The work for each theme made with diverse types of graphics can provide a meaningful learning experience that integrates logical elements of mathematics and aesthetic elements of art in completing a final product in each class, therefore it is an overall integrative, cohesive and systematical gifted education program. Thus, it can be applied as an alternative to current educational curricula for the gifted in mathematics in that it allows comprehensive and meaningful learning of the internal value of logo language, as well as fostering an in-depth understanding and applicability of mathematical concepts like properties of polygon, conversion of shape, understanding on angle, the concept of variables, etc.

Second, LOGO project learning can be facilitated as an efficient educational program to further improve the creative problem solving skills of gifted students. Analogic thinking, generalization, critical thinking, progressive thinking, flexible thinking, visual inference and empirical inference demonstrated in the LOGO programming process represent higher-level thinking capabilities. This implies that learners' experience of programming in various way diagrams presented by their teacher or independently planned unique diagrams have a close relationship with such thinking activities.

Third, analysis on higher thinking skills displayed in the LOGO project learning can be used as the basis for developing curriculum materials for teaching and learning for gifted student in mathematics. Materials used in this LOGO project learning can used as the instructional model for considering what type of strategic thinking should be emphasized for LOGO learning project and how learning and teaching process should be accomplished in school mathematics.

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Appendix: The final results of LOGO project learning by students