On the Effective Use of GeoGebraCAS in Mathematics Education

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Abstract: *The computer-based mathematics education was studied enthusiastically and the effective use of Computer Algebra System was a part of this attempt in Japan. The incorporation of Computer Algebra System in mathematics education in Japan, however, has not become able to become an indispensable tool in teaching mathematics. On the other hand, we have seen that the use of Dynamic Geometry Software in mathematics education has got many results in recent years. GeoGebraCAS is an interactive geometry software that includes Computer Algebra System functions. In this paper, we show the effective use of GeoGebraCAS and consider how the incorporation of Dynamic Geometry Software in addition to Computer Algebra System can make positive impacts in mathematics education.*

1. Introduction

With the evolution of software and hardware programs, it has become easy to use the CAS technology in classes. However, the proportion of teachers, who use computers during their classes, is still quite low and it is hard to say that technology has started to be commonly used in classes. Although several reasons may be considered, there is the fact that the value of technology as a teaching tool is not recognized as one of them. One reason why the use of technology is not advancing is the issue of the necessity of time for preparation.

In mathematics education, what is the purpose of using technology? That is to assisting the developing of students' mathematical thinking. In particular, the CAS (Computer Algebra System) has big possibility in learning of the mathematics. However, we did not perform a clear study about its possibility till now. It is because we thought the CAS to be the expert engineers' tool. The present condition has changed. We must consider about the effective use seriously, now. Then we need to consider "a tool theory" in the cognitive science. We can explain new use possibility of the CAS by being based on the theory.

According to Norman's definition, a cognitive tool is a tool for embodying the image of an outer object that appears in the consciousness on the basis of the human perception, Norman divided the human cognition, when using technology, into two categories: experimental cognition and reflective cognition ([1]).

1) Experimental cognition: to cope without conscious to the outside world change

2) Reflective cognition: to deeply understand for thinking of the meaning of each thing and referring back to experience.

In a learning environment using technology, the human cognition is divided into experiential one and introspective one. The tools for the experiential cognition have to be able to exploit a rich sensory stimulus. The tools for introspective cognition must be supporting the search of ideas.

These cognitions need a different support. It does not function well if we give experiential cognition an introspective tool. Vice versa also will not work. Therefore, teachers must distinguish which type of cognitions the tools are supporting. And for that purpose, the teachers must provide tools that offer appropriate support for a certain activity. In order to use effectively technology, the teachers must consider which one of the two cognitions is suitable for its learning activity. If the consideration for the cognition mode is neglected, the effective use is impossible. If learners enjoy the experiences when they must do introspective searches, they misunderstand its activity as introspective activity.

According to Norman ([1]), there are three learning categories that are useful in determining which of the two cognition modes is suitable for its learning activity:

1) Accretion: to accumulate facts
2) Tuning: to adjust it in a way to use skills involving introspection with an experiential mode.
3) Restructuring: to form an appropriate conceptual structure by introspection.

In many cases, accretion and tuning are seen as experiential modes, restructuring is seen as an introspective mode. To make technology appropriately work as a cognitive tool, teachers must determine what responds to a certain learning activity of students.

On the other hand, Nakahara suggested that “the representation method in the mathematics learning has various things. The representation can become an aim of the mathematic learning, contents and method. It is important.” ([2]). He classified representation in five types: realistic, manipulative, illustrative, linguistic and symbolic.

When it changes a representation from a certain style into other styles, he calls it "translation". "Translation" in the studies of Nakahara is application of representation style in the classroom mathematics. Not only the classroom lecture, in the case of the problem solving, learners become easy to understand if learners can translate the expression methods properly. An advantage of CAS use is that an algebraic calculation is possible. The symbolic representation is an important aim of the mathematics learning. Furthermore, in the learning, it plays an important part for the promotion of communication about the mathematical information and the thought. By using CAS, in changing the coefficient of the polynomial, the learners can think about the phenomenon for a search. The learners can also inspect the mathematical conclusion experimentally that they got as results of thought.

2. Cognitive tool in learning

2.1 Tow kinds of cognition and three kinds of learning

There are many modes of cognition, many different ways by which thinking takes place. The two modes particularly relevant to analyses are called “experimental” cognition and “reflective” cognition.

Reflective thought is very different from experiential thought. Experiential mode involves data-driven processing. Something happens in the world, and the scene is transmitted through our sense organs to the appropriate centers of mental processing.

Reflective reasoning does not have the same kind of limits on the depth of reasoning that apply to experiential cognition, but the price one pays is that the process is slow and laborious. The reflective mode is that of concepts, of planning and reconsideration. Reflective cognition tends to require both the aid of external support (books, computational tools) and the aid of other people.

Just as there are several kinds of cognition, there are several kinds of learning (accretion, tuning, and restructuring).

Accretion is the accumulation of facts. This is how we add to our knowledge, learn new vocabulary or perhaps the spelling of an already known word. When you already have the proper conceptual framework, accretion is easy, painless, efficient. Little or no conscious effort is required under these circumstances. But, when there isn't a good conceptual background, then accretion is slow and arduous. In this case, it can be difficult to learn the material.

In between the initial stages of novice performance and the skilled, smooth performance of the expert are hours and hours of practice. Practice tunes the skill, shaping the knowledge structures in thousands of little ways so that the skill that in the early stages required conscious, reflective thought can now be carried out automatically, in a subconscious, experimental mode. Experiential thought is tuned thought. Tuning is a slow process.

The difficult part of learning is forming the right conceptual structure. Accretion and tuning are primarily experimental modes. Restructuring is reflective. Restructuring is the hard part of learning, where new conceptual skills are acquired.

To make GeoGebra appropriately work as a cognitive tool, teachers must determine what responds to a certain learning activity of students.

2.2 Differences in cognitive mode using computer learning of mathematics

This section, we compare the two problems that use the same operation on a computer.

For example, by manipulating the slider, students are able to observe the changes of graph, and discover a mathematical law (Fig 2.1). This kind of mathematics class using a computer is typical in Japan. The changes of relationship between graphs and sliders are as follows:

Slider-a	(a-1) the parabola changes a direction
	(a-2) the apex moves obliquely
	(a-3) the parabola changes the width
Slider-b	(b-1) the apex moves like parabola
Slider-c	(c-1) the parabola moves parallel to y-axis
	(c-2) intersection of parabola and y-axis is changed

Even in the same operation, different mathematical problems lead different results of learning. It can be said that the cognitive mode depends on the content. We compare the following two problems.

(Problem A) Operate sliders a, b, c. Find a changing regularity of graph. (Fig 2.1)
(Problem B) Operate sliders a, b, c. Overlaid graph A to Graph B. (Fig 2.2)

Problem A is not lead students to regularity from (a-1) to (c-2). Reflections are different on each student. In other words, some students find mathematical laws by themselves, but the other students might not find. One student success a mathematical learning, another student fails.

An aimlessly operation is not learning ([3]). Student misunderstands that he will be able to understand mathematics by computer operations ([4]). On problem A, some students may lose the mathematical goal. Problem A tends to guide students the experiential mode, not reflective mode.

On the other hand, problem B is not easy operation at first. And soon, students succeed by repeating sequence of (a-3)→(b-1)→(c-1)→(a-3)→(b-1)→・・・. Where operations (a-3)→(b-1)→(c-1) is the basic knowledge of a parallel moving of a graph. Problem B grows mathematical knowledge in students. Problem B is said to be more reflective than problem A.

In mathematic class that is using computer, good problems are necessary for students to grow a reflective cognition. On cognitive viewpoint, a teacher has to consider contents of problems.

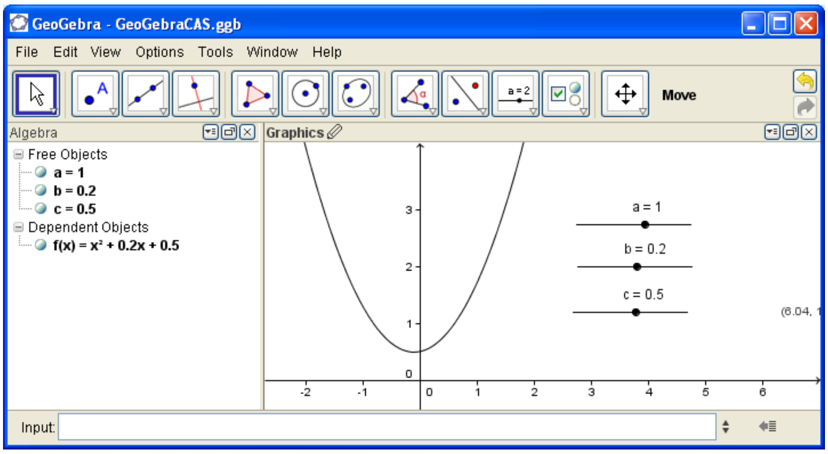

Figure 2.1

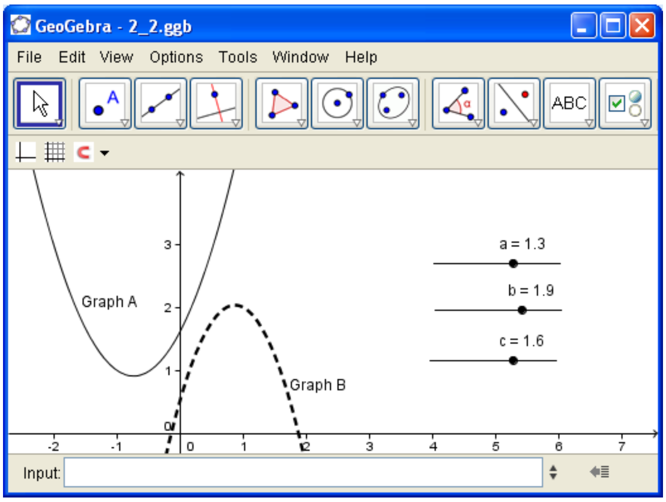

Figure 2.2

3. Translation of Representation in Mathematics Education and GeoGebra

3.1 Studies of Nakahara and Translations of Representation using GeoGebra

In Japan, Nakahara has been conducting research of representation in mathematics education. His studies are based on Bruner. He divided representations into the five styles as follows ([2]),

1) Realistic Representation:
   real-world situations, real representations, including experiments with concrete objects.
2) Manipulative representation:
   concrete representation of operational activity, human processing, concrete objects have been modeled.
3) Illustrative representation:
   pictures, diagrams, graphs, etc.

4) Linguistic representation:
like Japanese in Japan, English in England and America. representations by ordinary language.
5) Symbolic representation:
representations with Numbers, letters, symbol of operation, mathematical symbols.

Nakahara shows five representation systems. (Fig3.1) The direction of upper and lower shows the Bruner's stages of development. Bi-directional arrows are meant to translations of representation.

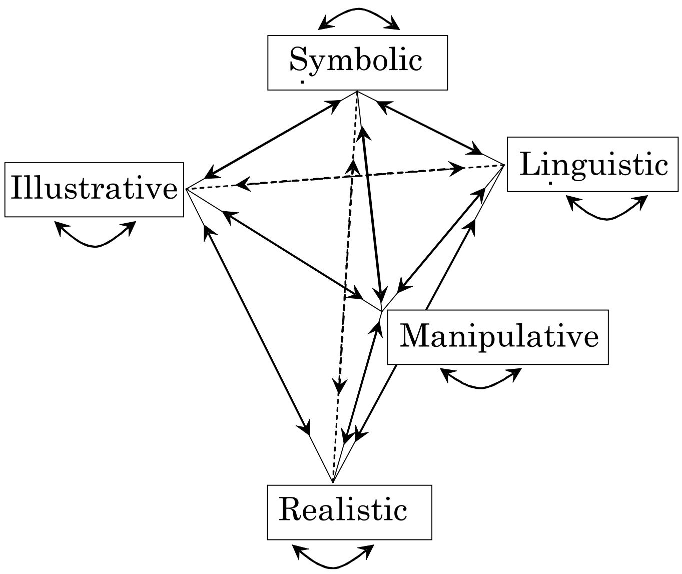

Figure 3.1

Lower stage is more concrete and upper stage is more abstract. Mathematics learning does go on to abstract from concrete. Nakahara insists on that understanding of mathematics is deepening through the translation of representation.

One of the goals of learning mathematics is understand of symbolic representation. GeoGebra has CAS ability, and has ability to convert representations among Graphic, Numeric and Symbolic.

Graphic, Numeric and Symbolic are 'Trinity' in computer. The significance of computer use is in the 'Trinity' ([5]). In mathematics class using a computer, Shimizu shows the effects of thinking associated Graphic and Symbolic. ([6])

This means that if teacher used GeoGebraCAS effectively in class, students can understand mathematics deeply.

3.2 An instance of using features on GeoGebraCAS

In this section, I show an instance of transformation of representations in mathematics class using GeoGebra. Specifically, it is translation of representation in class of introduction of integral.

Suzaki has said that, Many Japanese high school students can calculate the integral, but students don't understand the meaning of integration ([7]). One reason is that the definition of integration. In Japanese high schools, the definition of integration has become the inverse of differential. When teaching an introduction to integrating the partial sums, there are two difficulties. On quadrature by parts, the first difficulty is the calculation of the partial sum algebraically.

For example, at the sectional measurement of the function f(x) = x^2, calculation $1^2+2^2+3^2+\cdots\cdots+k^2$ is needed. It is difficult for high school students to calculate. If they want calculate this, it needs a tricky calculation. And if they change a function f(x) (i.e. x3, x4, x5 etc.), the calculation is more and more difficult.

The second difficulty point is the way of representation of dynamically changes. As a changing thin rectangle, it becomes difficult to calculate the area.

But, if students use GeoGebraCAS, they can easily calculate difficult calculations. And they can see the dynamic changes in shape. Students can focus on thinking, not calculation. Through changes in graphs, symbols and numerals, it is possible for them to find mathematical laws and formulas.

Fig3.2 shows CAS view and algebra view on GeoGebra. The CAS view and the algebra view are changed interactively by operating the slider. The "nusum" and "limit" are changed according to change of index "n" of x.

The important point is that the "limit" can be calculated exactly. A software not having algebra calculation system can calculate just numeric calculation. In this kind of software, if you calculate integration of x^2 (from 0 to 1), it shows "0.33333". But, GeoGebraCAS will show "$\frac{1}{3}$" as the answer.

Because of the exact calculation, students can find the integration formula $\int x^n dx = \frac{1}{n+1}x^{n+1}$ from the screen changes. They can understand the representations which is associated with graphs and expressions mathematically.

The relation between symbols and graphs is essence of mathematical learning. By using GeoGebraCAS, it is possible for students to learn the mathematical relations by themselves.

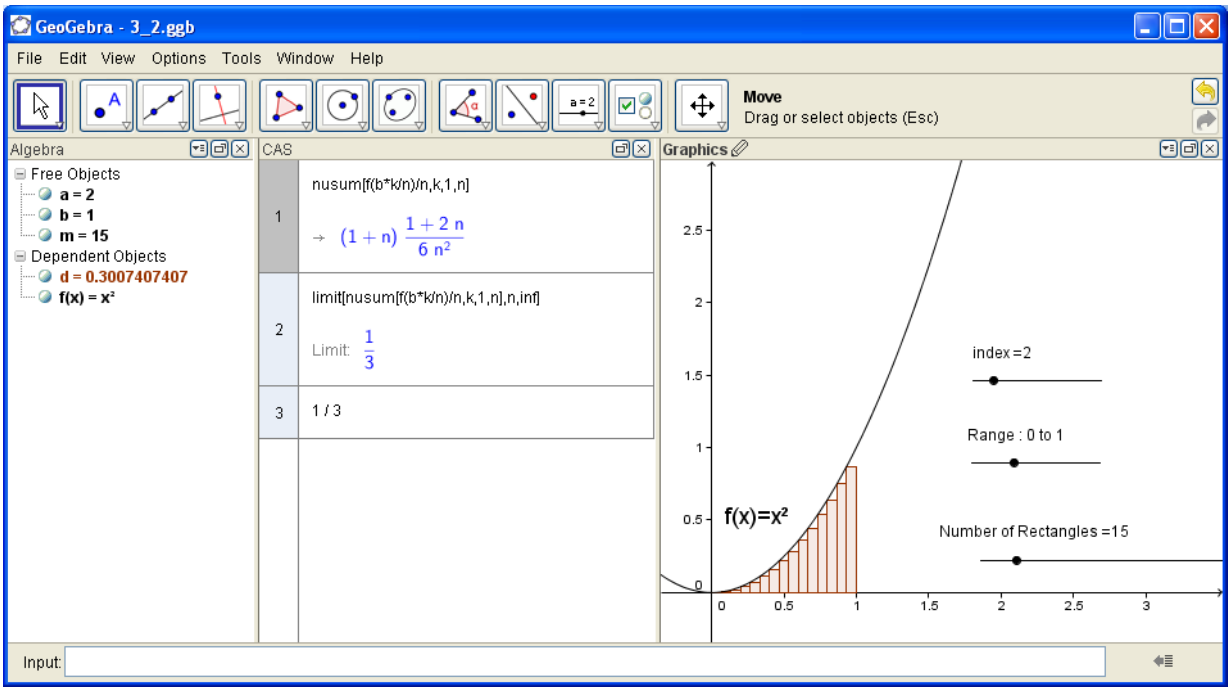

Figure 3.2

4. Observation

Generally, when it comes to making students' calculations “acting” through the introduction of technologies such as the CAS, it is feared that the calculation ability and the algebraic insight will not advance. However, as an objection to that through a careful teaching, students become able to choose by themselves when to use CAS. In other words, by educating in a way to make the students able to judge the use of the CAS depending on the situation, the worry on the insight will be avoided. Through inserting activities developing students' basic calculation abilities and algebraic insight, solving easy problems with hand calculations becomes more efficient. Moreover, through this kind of experience, students can evaluate mathematics from a broader perspective. And students can sense the power of their own mathematics and their relation with technology from the system view and the personal view standpoint. We would like to bring up such students using GeoGebraCAS.

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