WE WELCOMED M.C. ESCHER IN TURKEY'S NEW GEOMETRY PROGRAMME

Burcu AKAYDIN, burcu.akaydin@istek.org.tr, burcuylnz@gmail.com
Müjde UYSAL, mujde.uysal@istek.org.tr
Math TEACHERS
ISTEK ACIBADEM SCHOOLS
TURKEY

ABSTRACT

Studying mathematics through patterns is an opportunity for students of all levels to develop mathematical knowledge connecting different subjects like geometry/geometric transformations and algebra. Considered mathematics as the science of patterns (Biggs e Shaw, 1985; Devlin, 2002; Goldenberg, 1998; Mottershead, 1985; Orton, 1999) it was the start point for our work. Patterns gave many opportunities for the study of mathematical concepts and the development of mathematical process as problem solving, communication, reasoning and proof.

We propose a new method of teaching the principles of geometry to tenth grade students. The students focus on a field of design in which geometry is the design: tessellation. The students define their tessellations using the GEOGEBRA software. This procedure enables them to understand the mathematical principles on which graphical tools, are built upon. It moves the abstract concepts of math into the real world, so that the students can experience them directly, which provides a tremendous reward to them.

INTRODUCTION

Geometry is an important area in mathematics education and it is not only a subject in mathematics courses but also a way of understanding the world around us. We live in an environment that includes objects and things having geometrical shapes. To use these objects appropriately and in the desired way requires knowing the shapes and relationship between the shape and function of the object (Altun, 2004).

Learning geometry is a successive process. Preliminary and fundamental concepts of geometry should be taught in early ages, and the more complicated ones should be taught as the students grow up. Therefore, every step in the school geometry should be taken into consideration seriously. Clements and Battista (1992) describe the school geometry as the study of those spatial objects, relationships, and transformations that have been formalized and the axiomatic mathematical systems that have been constructed to represent them (p.420). Baykul (2005) stated the reasons why geometry concepts were included in the school curriculum as,

•Geometry studies are important in development of students' critical thinking and problem solving skills,
• Geometry concepts provide help in learning concepts in other mathematical areas, such as fractions and algebraic expressions,
•Geometry is one of the important areas of mathematics that is used in daily life.
•Geometry helps the students to realize the world around themselves and appreciate the worth of their world.

It has been argued that many students are not learning geometry as much as they need or are expected to learn (Baynes, 1998; Crowley, 1987; NCTM, 1989; Ubuz & Üstün, 2003). This argument is valid for Turkish students and consistent with the results of international studies such as TIMSS (Third International Mathematics and Science Study) (1999) and PISA(Programme for International Student Assessment) (2003). In both studies, Turkey is one of the least successful countries in mathematics, especially in geometry. Turkish students received their lowest rankings in geometry within five specific content areas in mathematics. To speak for geometry case, in PISA, 75% of students were at or under the basic competence level (known as level 2); where the mean of the OECD countries was on Level 3 in a 6 level scale. The results in TIMSS were same with the ones in PISA. According to the TIMSS, Turkish students scored below the international average in the overall mathematics achievement and they got the lowest mean scores from the geometry area of the test comparing to other four content areas of fractions and number sense; measurement; data representation, analysis and probability; and algebra.

As this is the case, improving students' geometric thinking levels should be one of the major aims of mathematics education. New Geometry curriculum in Turkey emphasizes the importance of patterns, algebraic thinking and geometric transformations. There are many changes in the content included in the high school geometry curriculum. Some new contents such as patterns, tessellations, symmetry, and spatial visualization are included in the new curriculum. These changes are in line with curricula in other countries such as the US, UK, Singapore, Ireland, Holland.

New mathematics curriculum has been developed and is being implemented for primary and secondary schools with ongoing changes since 2004 in Turkey. However, in high schools, this reform has just started at the year 2010. The idea behind these curricular reforms is to change the curriculum from a subject centered to a learner centered one and to change the pedagogies from a behaviorism to more constructivism. The purpose of the curriculum reform is to change considerably the focus and content of the whole national curriculum. The basic objectives of the curriculum reform in Turkey are;

- to reduce the amount of content and number of concepts
- to arrange the units thematically
- to develop nine core competencies across the curriculum
- to move from a teacher-centered didactic model to a student-centered constructivist model
- to incorporate information communications technologies (ICT) into instruction
- to monitor student progress through formative assessment
- to move away from traditional assessment of recall, and introduce authentic assessment
- to enhance citizenship education (Board of Education (BoE), 2005)

To integrate technology into geometry teaching process, dynamic geometry software systems can provide useful help. Dynamic geometry software refers to interactive software in which students essentially create compass and straightedge constructions, which can then be "dragged," altering the size of the construction, but not affecting the axioms or theorems used in the construction (Mansi, 2003). Healy and Hoyles (2001) argue that "Dynamic geometry systems provide access to a variety of geometrical objects and relations with which users can interact in order to construct and manipulate new objects and relations" (p.235). In such environments students find opportunity to drag, construct, rotate, translate and etc. objects, in order to understand the nature of the phenomena related to particular concepts of geometry. In the dynamic environment, the size and position of the shape is changed while its invariant features remain same. By this way, the construction of geometric knowledge differs in dynamic geometry environments, from traditional, static paper and pencil environments.

With an increase in the availability of technology, Dynamic geometry environments are becoming more prevalent in the classroom. New geometry curriculum in high schools also offers a real integration of technology into the teaching of mathematics and claims the necessity of this integration at all levels.

When students are able to produce numerous corresponding configurations easily and rapidly, they then simply have no (or very little) need for further conviction / verification. (also see De Villiers, 1990; 1991; Schumann & De Villiers, 1993).

Students quickly admit that inductive verification merely confirms; it gives no satisfactory sense of illumination; i.e. an insight or understanding into how it is a consequence of other familiar results. They therefore find it quite satisfactory to then view a deductive argument as an attempt at explanation, rather than verification.

The new program is open for technology usage. The students are encouraged to use calculators for problem solving but not for doing basic operations. It is believed that by this way an opportunity for working on more realistic problems shall become possible and they shall conclude the operations quicker and save from time. With the developing computer technology of our day the education software creates new opportunities for students to learn geometry more meaningfully. There are also sources on the Internet for the teachers

AN INTRODUCTION TO GEOGEBRA

It is a fact that visual communication is universal and is the oldest way of communication, from the prehistory until today. From the other side, the need for multidisciplinary courses is rapidly arising in the last few years, especially in the areas of applied sciences, which are linked to different types of art expression and are using a variety of software. There is a need for such kind of courses in Turkey also. But, in order to use computers as creative artistic tools, it is necessary to know what is the starting point and what is the idea in behind, can we use it for our own purposes.

The last decade has seen enormous strides in the introduction and use of technology in mathematics education. Graphing calculators and PC friendly software such as Geometer Sketchpad has led the way in this pedagogical transformation. A recent newcomer to this sometimes crowded field is Geogebra and it has the potential to make an immediate impact in our teaching of mathematics. Geogebra has one great advantage – it is free. With a simple download at www.geogebra.org you and any of your students will have access to software.

As dynamic mathematics software, use of GeoGebra is getting more common all over the world. In addition to construct geometry dynamically, it also provides, as a key element of learning geometry, visualization, estimation, conjecture, construction, discovery, proof and etc. GeoGebra is found to be very efficient in mathematics education and can be used effectively both in teacher training (Doğan and Karakırık, 2009) and students' learning (Doğan and İçel, 2010).

The software incorporates geometry, algebra, and calculus, which other packages traditionally treat separately, into a single easy-to-use package. GeoGebra runs on virtually any operating system and requires only a Java plug-in and, unlike commercial products, students and teachers are not constrained with licenses to run the software on only a limited number of computers. Moreover, GeoGebra offers the powerful feature for teachers to create interactive online learning environments by supplying not only interactive worksheets, but also the entire software package for their students through the Internet.

Geogebra offers integrated applications so that dynamic geometry is seamlessly linked to scientific calculator capability and a function grapher. It also allows for some use of text and digital images. These facilities allow the teacher to:
– give high quality, attractive presentations linking to real world situations that interest students,
– provide support for students to engage in problem solving related to real world situations,
– set tasks that allow students to explore mathematical regularities and variation within one mathematical representation,
– set tasks that allow students to explore mathematical ideas by linking different mathematical representations of mathematical object

The three examples of curriculum materials presented in this paper were developed as part of our project and focus on some of the affordances of Geogebra. The project aimed to investigate the use of tessellations with the assistance of Geogebra to enhance high school students' engagement and achievement in geometry. Students explored how a variety of new technologies could be used, especially in tessellations.

METHODOLOGY

The study of transformation geometry normally begins with the learning of isometries, which include translations, reflections, rotations and glide reflections. As a result, an object and its image under an isometry are congruent. The traditional treatment of these topics is by means of formal definitions, examples, basic properties of isometries and their applications. From our own teaching experience, we discover that some students cannot learn the concepts very well. They only memorize the definitions, the properties, the main results and examples discussed in class, without a thorough understanding of the basic concepts involved. When they are asked to solve a modified problem, which they have not seen before, they usually fail to solve it correctly. In this year, we have been trying to adopt a more exploratory approach in teaching in order to enhance their understanding of the basic concepts of isometries, via Geogebra.

After discussing about the translations, reflections, rotations and glide reflections, We talked about the tessellations. They searched the meaning of it and found some examples. Then we tried to have them discover what shapes tessellate and prove how they know that they do actually tessellate. They discovered that the angle measure at any point must be 360 degrees. An example of this was to have students find out if a regular pentagon would tessellate or not. They could calculate that the total angle measure of the pentagon is 540 degrees, which means that each angle of the pentagon is 108 degrees. Once the students know that each angle was 108 degrees they could calculate whether or not 360 degrees is evenly divisible by the 108 degrees. Since 360 is not evenly divisible by 108 they could say that a regular pentagon does not tessellate. They were asked to use this same process for other regular polygons to discover which ones would tessellate and which ones would not.

The students easily discovered the appropriate polygons as equilateral triangle, square and hexagon for regular tessellations.

Then they were asked to make an investigation about M. C. Escher. At this time we want them to discover nonpolygonal tessellations. During their investigations, they found his drawings and recognized that they were tessellations. At that time we started to talk about the techniques which he used and how we could do the similar things by using Geogebra.

All of Escher's tessellating creatures are modifications of tessellating polygons. Considering his tessellation of winged horses (Figure 1), a pattern with translational symmetry alone. To find the mother polygon, the students were asked to go around a single complete tessellating shape (one pegasus), looking for points at which more than two shapes meet. When all such points had been located, they were instructed to join them in cyclic order. A square would be outlined.

Figure 1 : Tessellation 105 - M. C. Escher

When the students studied a pegasus in its parent square (Figure 2), they discovered how Escher modified the square to obtain his creature. Each "parts "on the upper/lower side is compensated for by a congruent "hole" on the lower/upper side. The same is true of the left/ right sides. Corresponding modifications are related by translation by a vector (Figure 3). The area of the mother square is maintained.

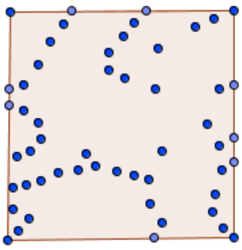

Figure 2 : Screen shot of our first study about Pegasus in parent square by using dots

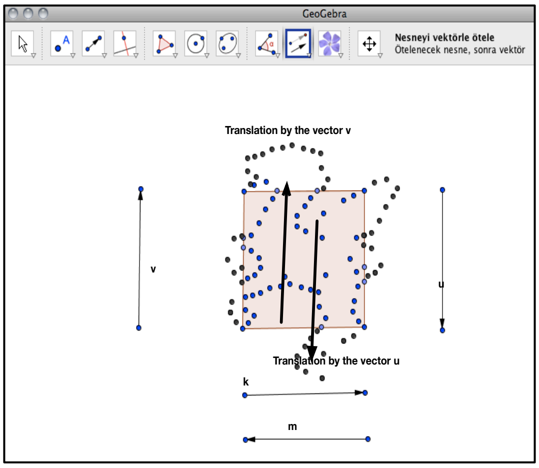

Figure 3 : Screen shot of translated holes by using the vectors u, v, k, m

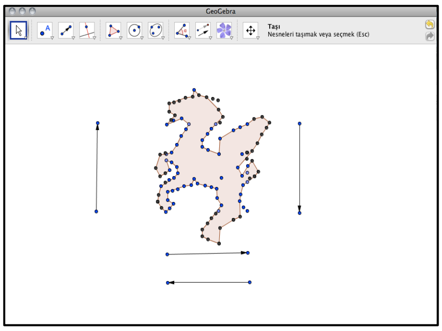

Figure 4 : Screen shot of obtained Pegasus

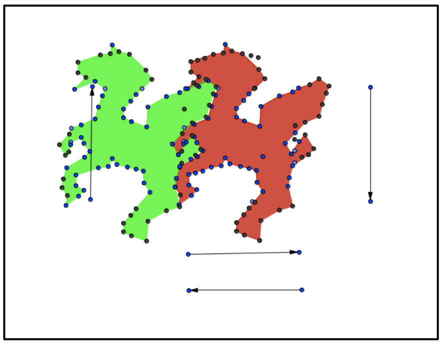

Figure 5 : Screen shot of tessellated Pegasus

Once students had experienced with creating tessellating art, they were ready to learn other ways to create these symmetrical patterns. To begin, we reviewed the regular tessellations and introduced other tessellating polygons like rectangles, parallelograms, and quadrilateral kites.

Next, we presented Escher's several tessellations, each generated by its own unique modifying rule or rules. Each student was provided with a set of corresponding worksheets, and instructed to add the parent polygon as demonstrated with the Pegasus tessellation. [Remember: Join points where more than two tessellating shapes meet.] The students studied each tessellating shape in its parent polygon, looking for corresponding "bumps" and "holes", and deduce the transformations used to modify the polygon.
In the Lizard of Escher's, a modification to either the top or bottom side of the parent square was rotated 90 degrees to an adjacent side. Each rotation is about a vertex of the square between the related sides, inevitably alternate vertices of the square.

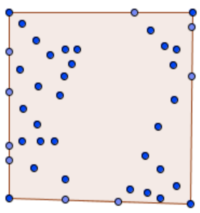

Figure 6: Screen shot of first study about Lizard in parent square by using dots

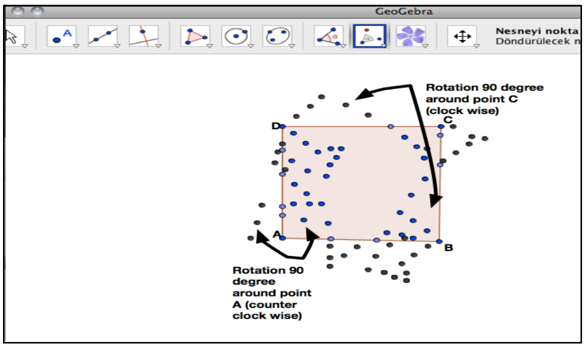

Figure 7: Screen shot of rotated holes around corners.

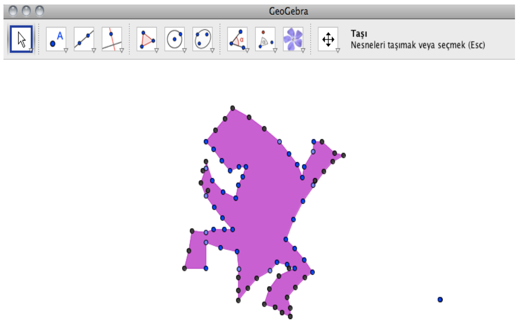

Figure 8: Screen shot of Lizard

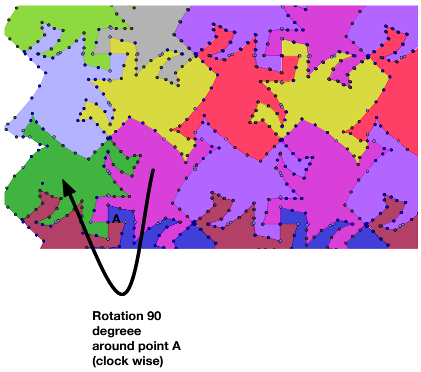

Figure 9: Screen shot of tessellated Lizards

In Escher's fish tessellation, a modification to one half-side of the parent equilateral triangle is rotated 180 degrees about the midpoint of that side to the adjacent half-side. Then a modification to one of the other sides is rotated 60 degrees to the third side about the vertex between them.

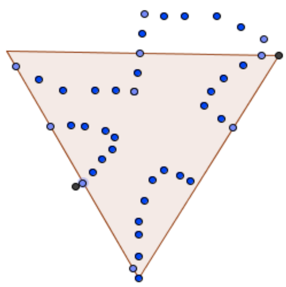

Figure 10: Screen shot of our first study about Fish in parent square by using dots

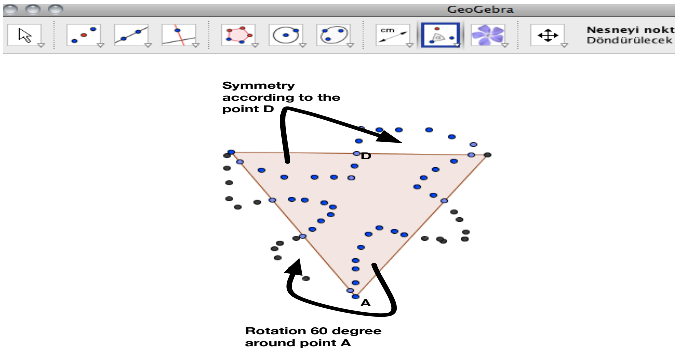

Figure 11: Screen shot of getting the Fish by using symmetry and rotation.

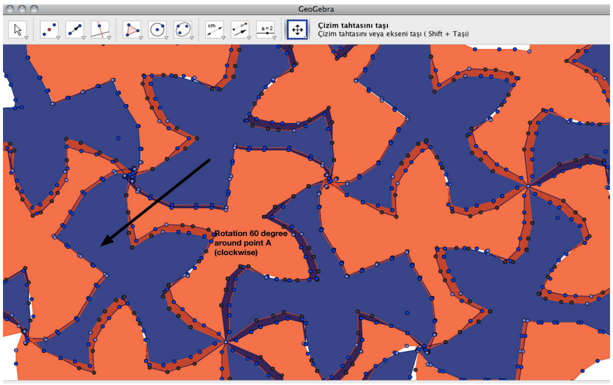

Figure 12: Tessellated Fish

CONCLUSION

As mathematics educators, we always would like to see our students broaden their knowledge bases by expanding on what they already know which will ultimately help them understand mathematical properties in greater depth. In this study we observed that students gained an understanding of what tessellations are and how they can be done by using GeoGebra. Studying tessellations helped children to understand the things they see everyday in a different context. It allowed children to follow the rules to create their own patterns, and this in turn, allowed them to discover what they did and motivated them to make things better. While studying with tesselations, students often forgot that they were doing geometry and commented that this experience was more enriching than their traditional geometry experience.

Geogebra can enhance the tessellation experience by giving students hands-on experience in manipulating objects to create patterns. Especially those students who have a trouble visualizing geometry, will benefit even greater from the use of dynamic geometry software that allows them to "see" what is happening (Gibbon, 2001). This software gives the students a basis for discussions to reflect on what they see. This type of technology can make math come alive and can be the "yes, I got it!" for someone who is struggling in visualizing ideas in their heads.

It is very easy to incorporate tessellations into every subject area of the curriculum because they are found everywhere and they connect geometry and our lives in a way other concepts cannot. Enriching the curriculum for more "talented" kids could be as simple as having them apply more than one transformation of an object to create patterns or prove different ideas. Students who are interested in learning more can have extra computer time using Geogebra to explore and share in a group on their own.

Overall, students of all abilities and levels will benefit from learning about this specific area of geometry often referred to as motion geometry. It is important for students to be interested and excited about geometry on several different levels because geometry can relate to their own lives at any point in time. We are sure that by studying tessellations students can make a connection between geometry and the real world with a deeper meaning and understanding, and do not see geometry as something they "have" to do.

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SAMPLES FROM STUDENTS' WORKS.

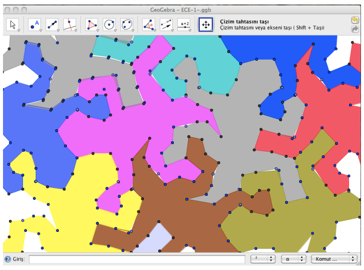

Figure 13: Student work 1 (Rotation)

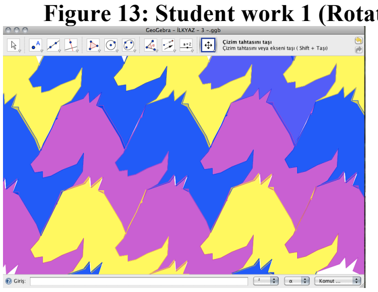

Figure 14: Student work 2 (Translation)

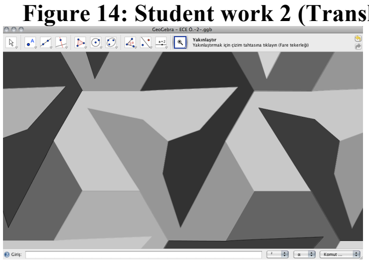

Figure 15: Student work 3 (Rotation – Translation)

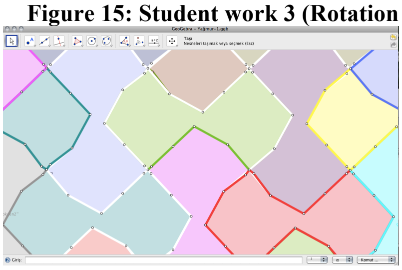

Figure 16: Student work 4 (Rotation – Translation)