Development of Creativity Using 3D Dynamic Geometry System InMA

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Abstract: *Experience of teaching mathematics with Interactive Mathematical Art (InMA) software is described in this article. InMA project has been used for teaching mathematics in Russian schools with in-depth study of mathematics since 2005. Courses in algebra and geometry have been developed for secondary school based on the traditional textbooks. Now about 1500 students and 60 teachers use InMA software, 20 teachers act as experts, i.e., periodically make suggestions to improve certain topics. Teachers have a special interest in interdisciplinary methodical sets, examples of which are demonstrated in this article.*

InMA project includes computer algebra system (CAS), single-stepping system (SSS), graphing tools for all kinds of functions and two kinds of dynamic geometry systems (DGS). That allows creating interactive 2D and 3D geometric images, interactive graphics and text with changeable parameters. InMA is a tool for creating electronic textbooks and methodical sets. All the constructions in the geometric part of InMA project are carried out on the screen. However, teachers prefer to use ready-made teaching packages on the topics, only adding some more lines if it is need to answer questions of curious students.

Many complex geometric problems can be represented as logical chains and a process of constructing of such a logical chain helps to develop student`s creativity. Dynamic geometry systems help to build logic chain, allowing step-by-step to provide a common part of overlapping solids. In this paper we consider the samples of methodical sets created using dynamic geometry system GInMA, the part of InMA project.

1. Introduction

Experience of teaching mathematics with Interactive Mathematical Art (InMA) software is described in this article. InMA project is used for teaching mathematics in Russian schools with in-depth study of mathematics since 2005. Courses in algebra and geometry have been developed for secondary school based on the traditional textbooks of several popular Russian authors. Now about 1500 students and 60 teachers use InMA software, 20 teachers act as experts, i.e., periodically make suggestions to improve on topics. The teachers have a special interest in interdisciplinary methodical sets, examples are demonstrated in this article.

The project developer's team was guided by the idea that the use of computer tools for educational purposes requires an integrated approach. This should be a sequence of steps: concept creation, computer program development, electronic textbook creation with the use of this program and printed textbook data base, use of the electronic textbook by school teachers, the joint modification of the software by programmers, mathematicians and teachers.

The underlying concept is the desire to maximize the interactive features of the computer in teaching, to make imaginative teaching, which provides maximum visualization of algebraic concepts, ample opportunity to change the settings, vivid geometric constructions. This product should be convenient for the teacher, both from a methodological point of view and in terms of application in the classroom during the lesson and at home schooling.

Interactive Mathematical Art (InMA) project includes computer algebra system (CAS), single-stepping system (SSS), graphing tools for all kinds of functions and two kinds of dynamic geometry systems (DGS). Programs are written in languages D (using Gtk) and C # (Net

Framework, Windows Forms). InMA programs are now collected under OS Windows, but the transfer on Linux and Mac is not complicated.

InMA allows creating interactive 2D and 3D geometric images, interactive graphics and text with changeable parameters. Interactive image is close to that used by teachers on the blackboard in the educational process. The formulae are developed in such a way that the brackets, roots, denominators appear and disappear automatically when a change of interactive factors. Single-stepping system interactive formulae ensure the conclusion of the chains of conversions in the customary form, the figures in the customary notation. Program ensures the functioning of interactive points and graphics (limitation of motion by curve, by surface; rational coordinates).

InMA project is a tool for creating electronic textbooks and methodical sets. The electronic textbook idea lies in the fact that the text of traditional textbook becomes the basis, on which tasks of the same kind are created. A teacher gets a kind of a semi-finished textbook with the opportunity of selecting a necessary variant from several others. Each file is provided with a text on methodology of teaching. The basic formulae, tasks and control elements of the file, recommended installations and terminology are indicated there. Algebraic part of the project is designed so that the teacher has no way to change the formulae, but can modify a wide range of coefficients of these formulae. Initial formulae are programmed. All the constructions of the geometric part are carried out on the screen. However, the teachers prefer to use ready-made teaching packages on the topic, only adding some more lines if it is need to answer questions of curious students.

Demo versions of InMA Project can be seen on http://deoma-cmd.ru/

In this paper we consider samples of methodical sets created using dynamic geometry system GinMA, the part of Interactive Mathematical Art (InMA) project.

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2. Construction of Logical Chains

There are many complex geometric problems, the solution of which requires the integration of imagination and logic. The use of them in the form of logical chains develops student`s creativity. Dynamic geometry system GInMA helps to build logic chain, allowing step-by-step to provide a common part of overlapping solids and deal with it in different ways.

2.1 Logical Chain with (n - 1) Dimensional Objects that Divide the n-dimensional Space

Consider the possible construction of a logical chain, a useful method for studying the mathematical induction and connections between seemingly unrelated phenomena. The task is to find the number of parts into which the objects of dimension (n - 1) divide the n-dimensional space. The initial problems are: to find the number of parts into which n points divide the line or circle. The results are simple, for example N (n+1) = N (n) + 1. The next obvious level is to find the number of parts into which n lines (or circles) of the total or nearly total divide a plane or a sphere. At this step M (n + 1) = M (n) + N (n). Here we have two-dimensional easy to understand drawings. And it is not clear why do we need to use mathematical induction. But at the next step the calculation in space makes clear need for induction and dynamic geometry system, which allows you to disassemble the intersecting objects, to keep the lines of intersection of the solids, to look at the objects from different angles, to look inside the objects. Even such a simple calculation, as the search for the number of intersection points of fours of spheres raises the need for finding the methods of calculation (e.g., double the calculation), because the imagination is clearly not enough. And at the same time the results are obvious and have attracted considerable interest of students. Here the method of transferring information from the previous level is used, where the objects are easily manageable. The last level, where dynamic geometry system is particularly useful, is to find the number of parts into which n planes (or spheres) of the total or almost a generic divide the space. It is useful to examine the symmetry of shapes, studying the equal spheres with centers lying at the vertices of Platonic solids. From the intersection of solids we should pass to the traces on one of them. The samples are shown in the Figure 2.1.

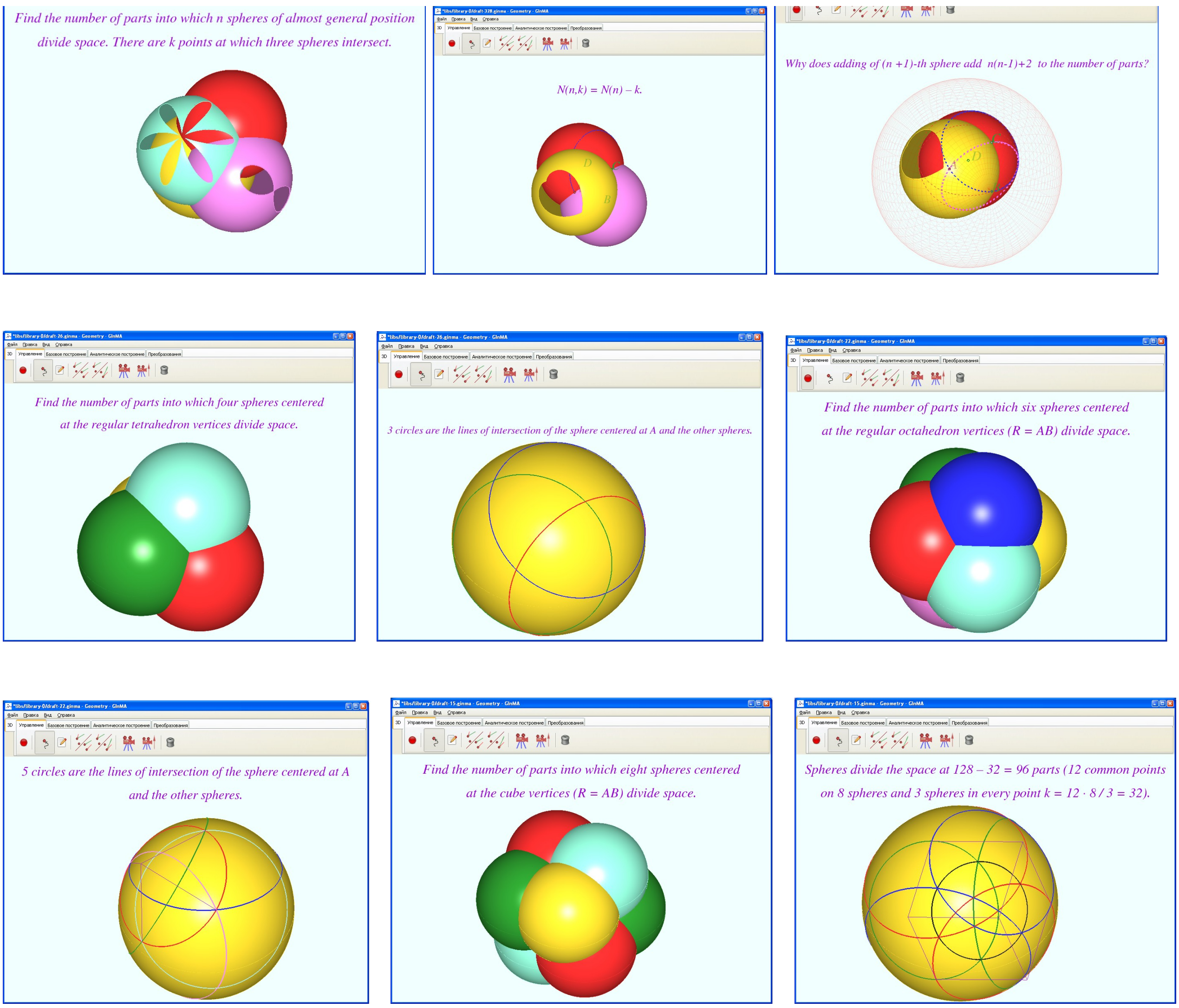

Figure 2.1 Problems Involving the Division of Space by Spheres

2.2 Logical Chain Based on the Cavalieri Principle

Logical chain based on the Cavalieri principle may be useful for the development of calculation engineering of double and triple integrals. The initial problem is to search a volume of a sphere. Relatively simple tasks are: to search a spherical segment volume, a watermelon wedge, a ring and a hoof (the part of a cylinder that is cut off from the cylinder by a plane). Further, the 3-D shapes obtained as a result of rotation of a flat shape about an axis of rotational symmetry are being investigated. We can replace the solid of rotation and change the axis of rotational symmetry. Relationship with the physics is effectively demonstrated by the problems involving a volume of

rotating solid, as the distance from the center of mass to the axis of rotation is given in the formula. Further we study the rotation of 3-D shapes, for example, rotation of the cube about the diagonal and about the edge diagonal. The next step is the search for the configuration of the solids that are a common part of two identical cylinders intersecting at right and at arbitrary angles; a common part of three or more identical cylinders, which intersect in various ways. Further, the solids arising from the intersection of sphere and cylinder are studied. At this step we can consider different variations of the problem. For example, we may take the problem of Viviani curve, connecting points on the Earth's surface at the same latitude and longitude.
Samples are shown in the Figures 2.2.1 - 2.2.3.

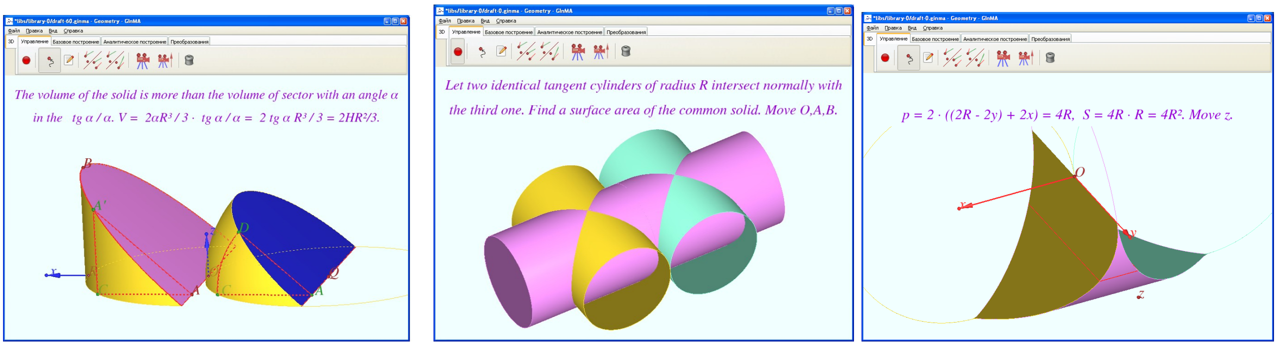

Figure 2.2.1 Spherical Segments and Common Parts of Intersecting Cylinders

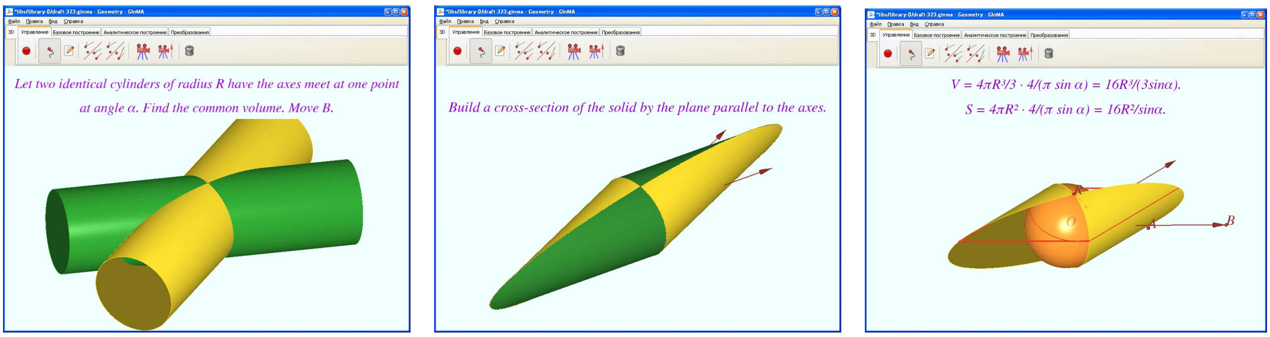

Figure 2.2.2 Problem involving Intersection of two Cylinders

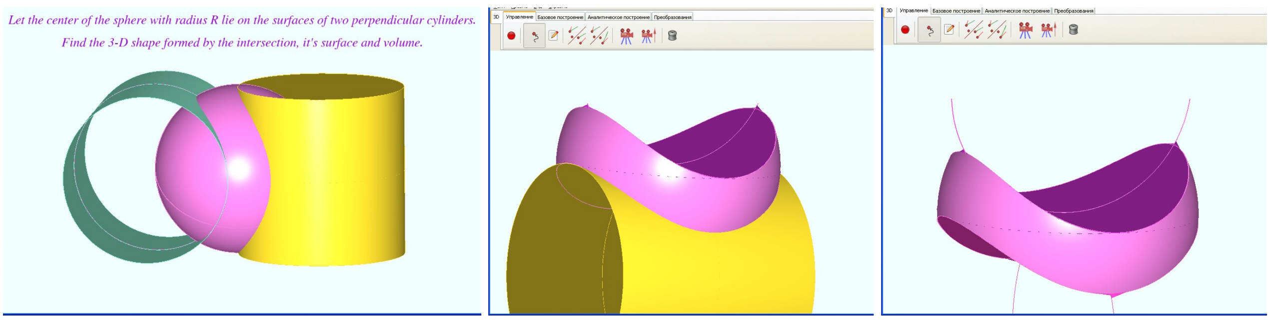

Figure 2.2.3 Problem Involving the 3-D Shape Formed by Intersection of the Solids

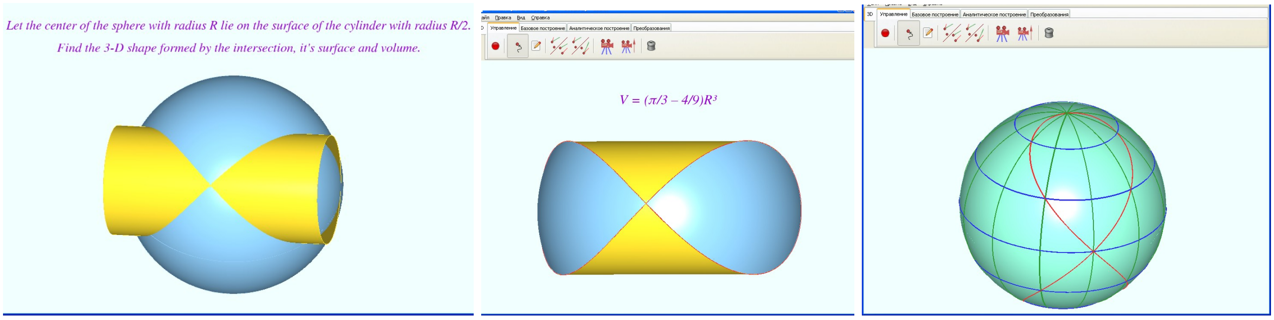

Figure 2.2.4 3-D Shape Formed by Intersection of the Sphere and the Cylinder

2.3 Logical Chain for the Method of Mirrors

Consider the possible construction of a logical chain for the method of mirrors. The basic task is to build the shortest path between two points, passing through the line if the line and the points lie in one plane. The development of this problem is to find the shortest path within the triangle (a rectangle, a diamond), which contains points on each side. An important feature, to which attention should be paid, is that the triangle has a unique way, and the rectangle has many ways of equal length, and the center of the rectangle is the center of symmetry of each path. The next step is to find a similar path in a regular tetrahedron, an arbitrary tetrahedron, a cube, a cuboid and a regular octahedron. Constructing similar paths, we get plenty of opportunities to discuss: one path is in the tetrahedron, the set is in the cube; the plane of the pairs of adjacent parts of the trajectory is perpendicular to the facet plane (property of the reflected beam); the trajectory has central symmetry, etc. The examples are shown in the Figure 2.3.

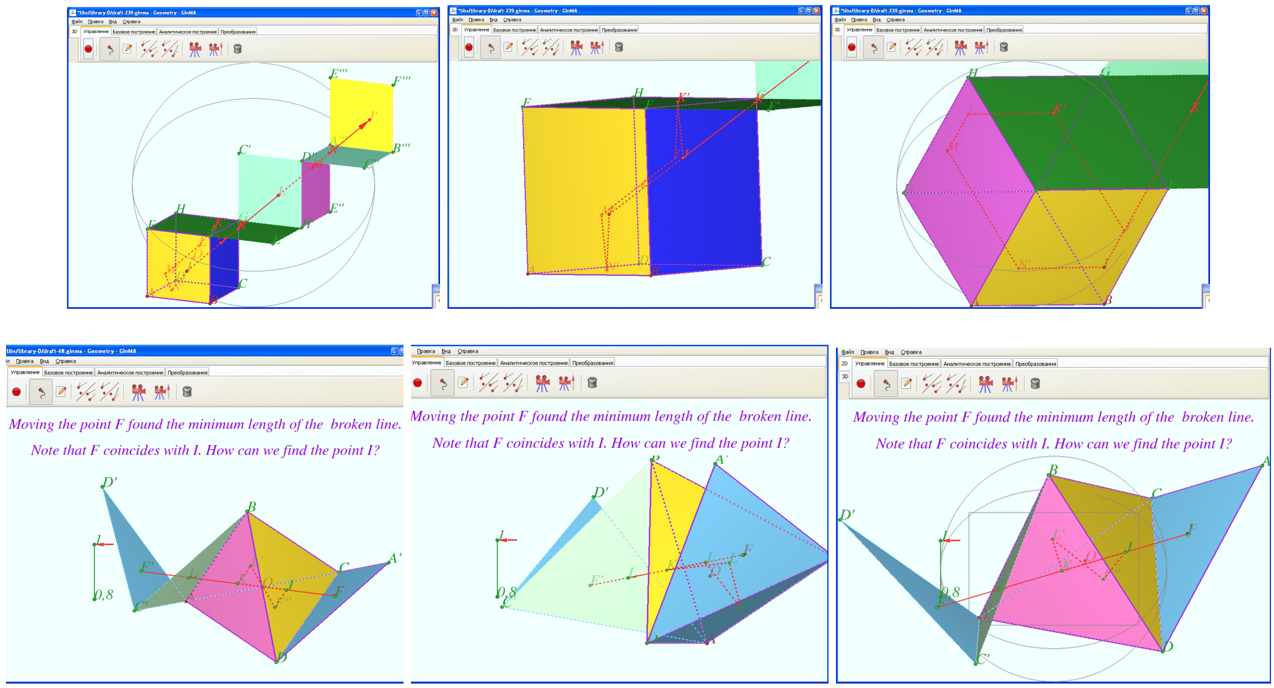

Figure 2.3 Construction of a Shortest Path between the Points within the Solids

2.4 Logical Chain Based on Finding a Volume of Polyhedral Intersection

The logical chain based on finding a volume of polyhedral intersection may begin from the intersection of regular tetrahedrons, one obtained from another by rotation about an axis passing through the middle points of opposite edges. This task allows you to make an elementary calculation of the total volume for an arbitrary rotation angle. The problem involving a volume of the solid in the case of a cube cut by dihedral angle, when the angle edge lies on the cube diagonal, is reduced to finding equal-area cross-sections of the cube being cut by half-planes. If the angle between the half-planes is 60°, then the largest and the smallest volumes are of the same.

Visualization is useful for understanding the Newton's extremality principle and allows comparing the features of the integration method and the elementary computation methods.

The samples are shown in the Figures 2.4.1 – 2.4.3.

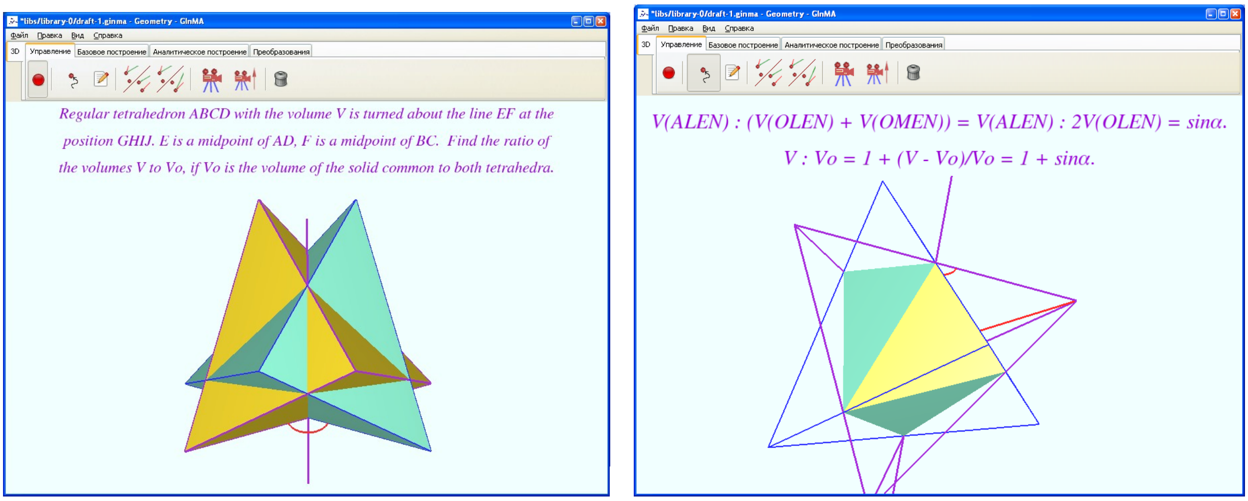

Figure 2.4.1 Problem Involving the Intersection of Regular Tetrahedrons

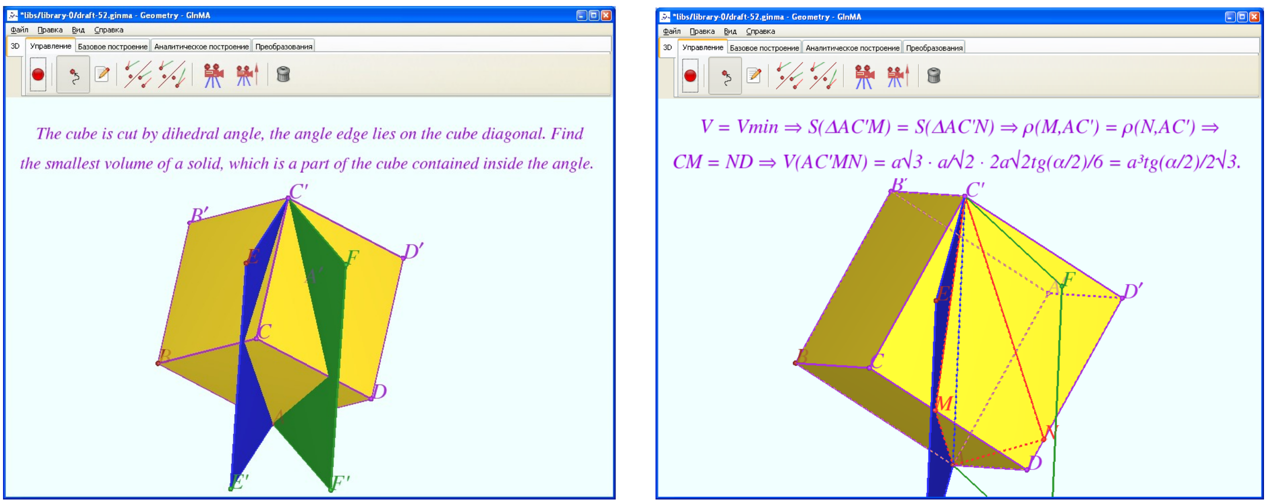

Figure 2.4.2 Problem Involving the Cube Cut by Dihedral Angle

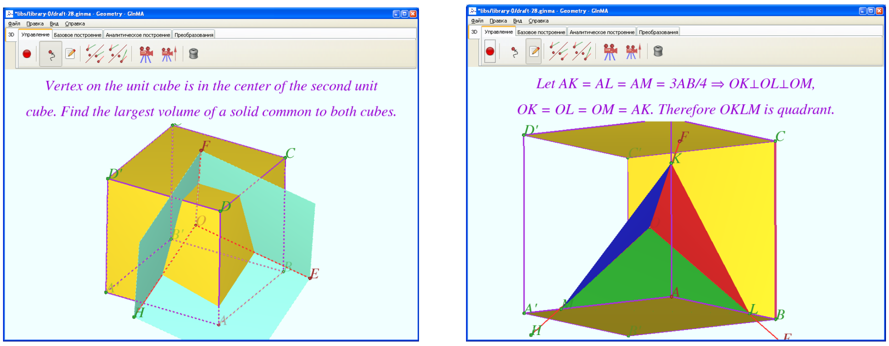

Figure 2.4.3 Problem Involving the Intersection of the Cubes

3. Development of Geometric Imagination

Dynamic geometry system gives a great opportunity for research. For example, let us consider the problem involving the location of three regular tetrahedrons inside the cube when the tetrahedron edge is equal to the cube edge. This problem allows the teacher to find the student`s difficulties in space image understanding. It becomes obvious by studying the actions that students take in order to establish the tetrahedron in the desired position relative to the cube. Another example is the problem involving the number of circles of given radius with given center that can be accommodated touching the faces of the quadrant. The samples are shown in the Figure 3.1.

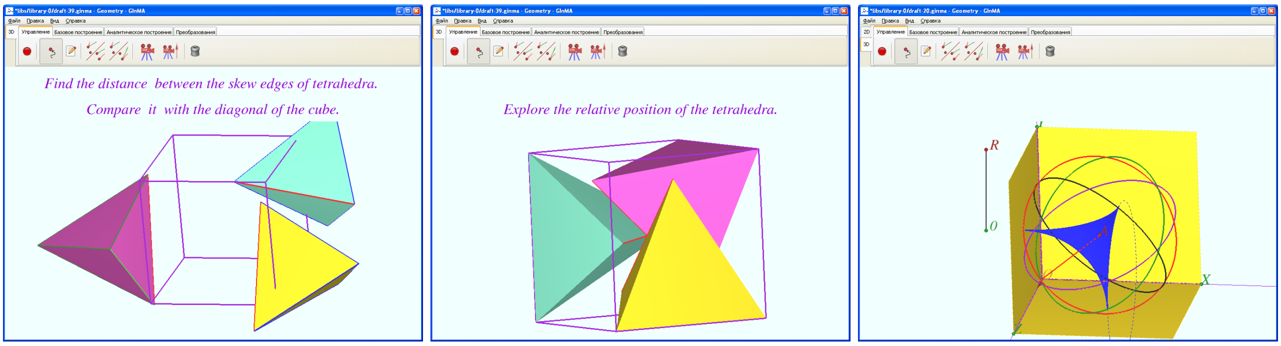

Figure 3.1 Regular Tetrahedrons inside the Cube and the Circles inside the Quadrant

4. Conclusion

Dynamic geometry system GInMA allows to create and to explore the problems and the logic chains that use the geometric configurations. It develops students' creativity, thinking and calculation methods, and helps to understand a multiple integrals. The established system is increasingly being used in Russian schools.

References

[1] Shelomovskiy, V. (2010). *InMa program for learning math.* Workshop. Proceedings of CADGME 2010, Conference on Computer Algebra and Dynamic Geometry Systems in Mathematics Education, Hluboká nad Vltavou, Czech Republic.