Redefining school as pit stop: It is the free time that counts

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The evolution and revolution of the relationship between man and technology (i.e. instrumental genesis) together with a redefined conception of teaching and learning have given opportunities to learn or even make mathematics in more learner (community) centred, and more distributive (i.e. free from time, location and formal modes) environments. On the other hand, it has been recognised that the general lack of enjoyment of institutional mathematics teaching is one of the basic reasons behind the bad reputation of mathematics in society. Increasing students' motivation to make mathematics through enjoyment and playing, especially in their free time, might therefore be a relevant research focus. The article discusses this position from seven challenges represented in first author's ATCM2008 plenary. It gives some examples of environments which can stimulate modelling processes, for which school could take the role of a pit stop to orchestrate technology-based investigation spaces which allow students to explore spontaneously the facility of real and virtual environments which are both, meaningful to them and their community, and which naturally motivate a greater use of mathematical language in its different forms. As conventional assessment would contradict this position, especially as regards emphasizing informal mathematics, the article also discusses how to get out of the current deadlock situation.

Introduction

We would like to begin with a discussion why we have chosen the "pit stop" metaphor from car race. The first grounding comes from our experiences as teacher educators since more than 30 years. Within the institutional mathematics teaching, namely, students very often show lack of appropriate problem solving competence even when solving simple problems. We pick up such an amazing example from Haapasalo (2007, p. 3). The question is about a well-tailored learning program to find the idea of the gradient of a straight line. The program proceeds step-by-step beginning from the simplest case of just one line segment in a rectangular coordinate system with its movable end points at its axis, respectively. The student can manipulate the visual slope (based on spontaneous procedural knowledge[1]) of the segment to see how the ratio (i.e. more or less

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[1] We adopt the following characterizations of Haapasalo and Kadijevich (2000):

- *Procedural knowledge* denotes dynamic and successful use of specific rules, algorithms or procedures within relevant representational forms. This usually requires not only knowledge of the objects being used, but also knowledge of the format and syntax required for the representational system(s) expressing them.
- *Conceptual knowledge* denotes knowledge of particular networks and a skilful "drive" along them. The elements of these networks can be concepts, rules (algorithms, procedures, etc.), and even problems (a solved problem may introduce a new concept or rule) given in various representational forms.

The interaction between conceptual and procedural knowledge, especially the so-called *simultaneous activation principle* (SA) is discussed in Haapasalo (2007), and the software related to our example is freely downloadable at http://wanda.uef.fi/lenni/programs.html abstract conceptual knowledge) alongside is related to the situation. Everything seemed to go fine with these one or two components, which could be manipulated but something strange happened at the last stage when there are many chunks (Figure 1, left). When asking to move the end points of the lines with the mouse to see how k_1 and k_2 are changing, hundreds of students, student teachers and even mathematics teachers since 1995 have changed all possible problem components at the same time (illustrated in the middle), just to get a data overflow. So as to surprise us even more, students kept on acting like this in spite of several trials and when working in teams! However, those who would have basic skills in behaving in a problem situation would change just one component at a time to see the relation between the proportions and the line position (Fig. 1, right).

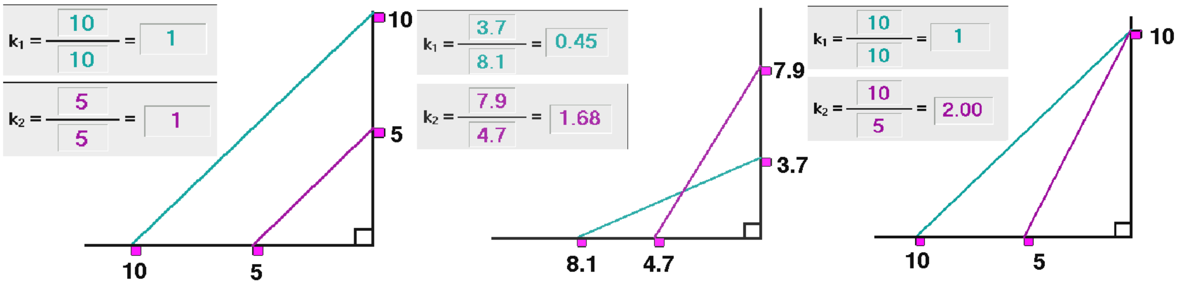


Figure 1. Utilizing the SA method in technology-based learning environment.

Luckily, the situation is not that bad outside the classroom. Eronen and Haapasalo (2006) noticed that even a quite mediocre student at 8th grade showed excellent competence in a much more complicated situation when playing with a ClassPad calculator (see http://www.classpad.org) voluntarily during her summer holiday (see Figure 2). Because there were only few days before they would go to their holiday, the administrators really had to act like a pit stop team in car race to equip students with "new tires and gasoline" for their race: with a leaflet of problems related to the above-mentioned "gradient". Both the drag-and drop technology and the *Geometry Link* –operation of the tool allow, namely, the student to manipulate mathematical objects between two windows, illustrating two different forms of mathematical representation. The portfolio sample below Figure 2 shows that the above-mentioned student showed a good problem solving competence by utilizing ClassPad properties in a sophisticated way without any tutoring from teacher's side.

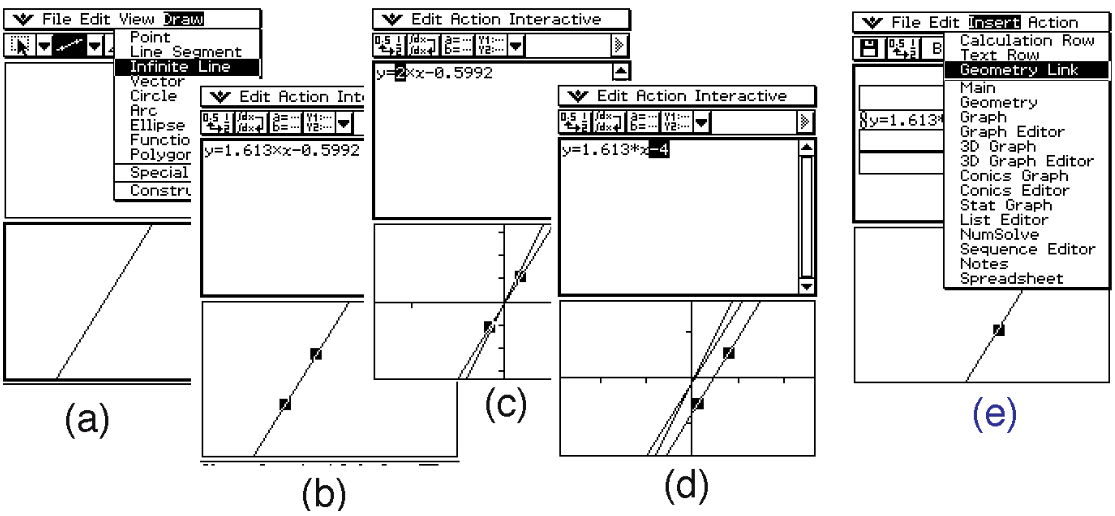


Figure 2. Utilizing SA-method through drag-and-drop technology (a-d) or Geometric Link (e).

Example of a student's 6th session on July 15th, 2005, at 00:27.

- I draw a line (cf. Fig. 2., (a)). When drag-dropping, the equation of the line is y= **1.613**x-0.5992 (b).
- By changing the equation to y=**2**x-0.5992 the angle between the line and y-axis is getting smaller (c).
- By changing the equation to y=**1**x-0.5992, the angle between the line and y-axis is getting larger.
- I change the equation to y= 1.613x-**0.4**. I don't see any changes (in the graphic window).
- I change the equation to y= 1.613x-**4**, the line moves to the same direction away from origin (d).
- When changing the equation to y= 1.613x+**4**, the line moves in the same way, but to another direction on the x-axis with equal distance from the origin.
- I will continue in the morning. Time is now 1:42 a.m.. I worked 1 h 15 min.

When analysing the profiles of that student within the eight sustainable activities from the history of mathematics (see Figure 3), Eronen and Haapasalo found that working with ClassPad had, even during a short period of time outside the classroom, extended the so-called mathematical profile, self-confidence profile and techno-profile of the student. This finding was strongly supported by the interview, as well: *"In May I could not even think to play with ClassPad in summer holiday. However, I noticed, that it was very capable for playing with mathematics.*" The student also noticed the versatility of the calculator, which decreased the relative amount of the calculating belief: *"ClassPad is suitable for calculating, but if you want to learn how to calculate, you have to do something by hand".*

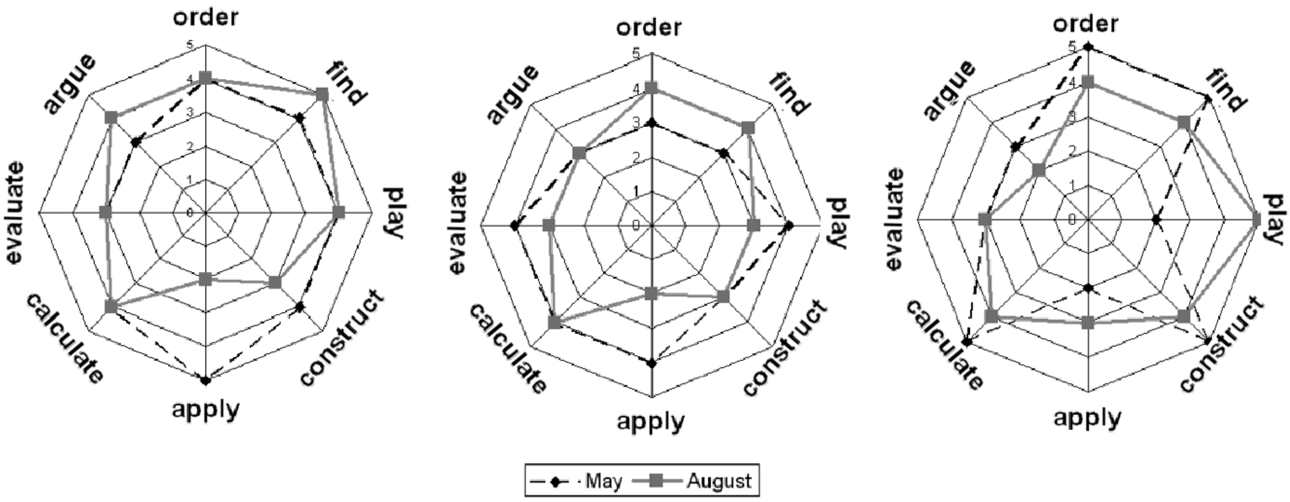


Figure 3. Student's view on mathematics before and after her ClassPad work. 'What is mathematics all about?' (on the left), 'How good I am in making mathematics?' (in the middle), and 'What kinds of mathematics can be made by using computer?' (on the right).

The quite amazing findings above triggered the so-called *ClassPad project*. Eronen and Haapasalo carried this kind of sponatenous playing with CladdPad into the classroom and reduced the instruction according to the so-called *Minimalist Instruction* principle (see Haapasalo 2007, and later on this article) to allow students learn – actually *make* - the mathematics of 9th grade totally by using ClassPad without any conventional textbooks or homework. After this kind of learning period, students scored in all test items significantly better than in the pre-test. A postponed test, after 5 months revealed that this scoring level remained consistent, and for many students it even improved. Students liked the feeling that they had reached action potential, which is one of the main aspects in assessment within minimalism. They also liked the learning without any pre-set goals or tutoring from teacher's side.

As second grounding we would like to mention the experiences gained in Switzerland (see http://www.spiegel.de/schulspiegel/0,1518,359073,00.html, accessed 20.06.2011): As a school had no money to hire a teacher, it had to shift the responsibility of the learning to students, using school as a "pit stop" to support students if they got difficulties. The learning results exceeded expectations, being not worse than in normal teaching.

Even though these kinds of suggestive groundings could be given more, we would like to stop by mentioning the huge amount of Internet forums, where people, even small children, participate in problem solving and discussion even on complicated questions. Because today almost every student owns not only a mobile phone but also many other personal devices, it would be important to get these devices integrated into mathematics learning. Many studies show that enjoyment and possibilities to be creative are seen as the key to affective motivation leading to an identity and attitude change towards mathematics (c.f. Loveless, 2002; de Freitas & Oliver, 2006; Harlen & Deakin Crick, 2003). An integrated computer algebra system and dynamic geometry software, for example, might be one of the most fascinating combinations to trigger an environment for the construction of mathematical knowledge. This instrumental genesis has already changed our views on making and teaching mathematics - and probably will change it even more radically. It might be quite evident that most part of students' instrumentation (i.e. technology is used to do mathematical actions) instrumentalization (i.e. technology is shaping also the mathematical objects under consideration) often happens on his or her free time. This situation implies that educators should shift their focus from well-prepared classroom lessons to minimalist instruction. Instead of acting like a pace car in a race, institutions should be types of pit stops to scaffold students' "race" outside the classroom.

Finally, we would also emphasize that governments and schools probably never have enough money to equip schools with the newest technology. Most students in well-fare countries will always have a little bit more sophisticated technology at home – perhaps even in their pocket as hand-held technology. Therefore, by looking at the relationship between technology and mathematics education from five perspectives, Haapasalo (2007) suggests that instead of speaking about 'implementing modern technology into classroom' it might be more appropriate to speak about 'adapting mathematics teaching to the needs of information technology in modern society'. This means emphasizing more the making of informal than formal mathematics within the framework of the above-mentioned eight main activities and motives, which have proved to be sustainable in the history of human thinking processes and making of mathematics (see Fig. 3; Zimmermann 2003).

Concerning the educational policy, the problem of "math dropouts" has increased now that "mathematics for all" has come into fashion as a slogan. 'Mathematics' is normally presented as a meaningless collection of knowledge - unrelated to the experience of the students and totally uninteresting. Sterile "logical connections" seldom lead to understanding or appreciation. This has given rise to a flourishing enterprise - empirical research - which studies and characterizes the symptoms without producing a cure. We need a new approach to the teaching of mathematics but there is little hope it will emanate from this psychological perspective. Epistemological perspectives and historical sources offer much more hope. Besides, they must not be forgotten when planning curriculum or constructivist learning environments for students´ productive activity.

The fact that ordinary people can realize outstanding examples of simple and powerful ideas from the history of mathematics implies that also organizing the content of the curriculum should be made in a meaningful way instead of treating the same idea in several disguised forms under the name of "spiral curriculum". In this contribution we restrict us in giving examples in the spirit of *modelling*, being understood in wide sense. At the end we will discuss the problematic of assessment.

Remarks on modelling through technology

The term ‚modelling' seems to be used usually to mean representing real-life situations through mathematics, as can be seen in the cavalcade of researchers' views in Greefrath & Siller (2008). Gjone (2008), for example, emphasizes "transforming a problem into mathematical form, solving it and assessing the validity." We, however, would like to stress that *even more important than to solve a given problem is to promote students to find and pose new own problems related to a situation which is meaningful for them*. Instrumentation releases the student from cognitive overload whereas instrumentalization scaffolds him/her to make own mental models (which might not always be viable ones when considering the objectivity of mathematical models).

Some researchers like Treffers (1987) speak about 'horizontal modelling' when a real-world situation is interpreted through mathematical structures, whereas 'vertical modelling' refers to manipulating structures inside mathematics. The often-used term 'mathematization' would mean rather the first one. We would like, however, to avoid using those complicated terms, which might be irrelevant from pedagogical point of view. Very seldom, namely, a real world situation is directly interpretable through mathematical structures before making a simplification – sometimes even a very progressive one. A virtual model on a computer screen, for example, can very often offer to the learner, even for a child, a more appropriate *investigation space* (Erkundungsraum) than a conventional "real-world situation". Hence, from constructivist viewpoint we prefer to make the following generalization, seeing 'modelling' almost synonym to the term 'mentally modelling': *Modelling* means mathematical interpretations made by the student when interpreting a situation, which is psychologically meaningful for him or her.

From a constructivist viewpoint learning and applying mathematics are basically triggered by the same kinds of modelling processes, general aspects of problem solving in the sense of Pólya playing a central role within the respective activities. The interpretations can appear in different forms of representation (*V*erbal, *S*ymbolic, *G*raphic), and on different cognitive levels (***I***dentification, ***P***roduction). Drag-and-Drop activity with *ClassPad*, for example, allows ***P***roduction from ***G***raphic to ***S***ymbolic form (***PGS***), showing how opportunities to develop and apply mathematical ideas appear in modern society through *instrumentation* and *instrumentalization*. Figure 4 below illustrates our view that technology cannot only be an interface for a one-way street from "meaningful situations" to mathematics but it could also enhance and empower the interpretations in different kinds of modelling processes as can be identified in the student portfolio above.

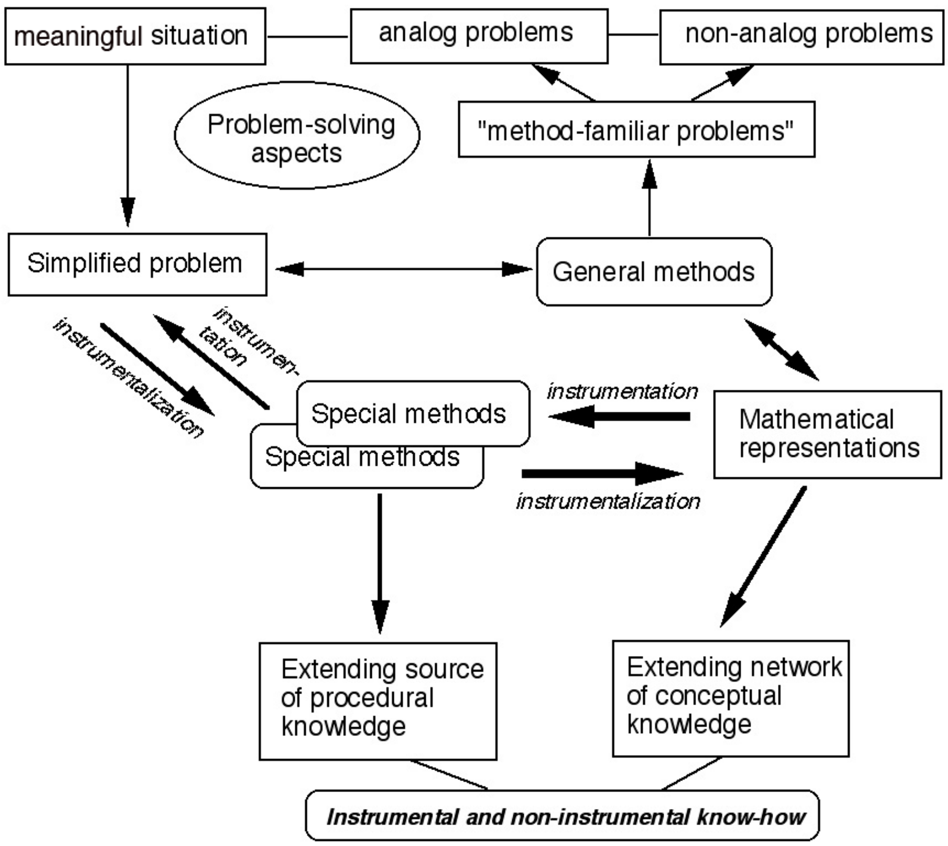


Figure 4. The role of technology in different types of modelling processes.

Examples from the history of mathematics

Let us consider the *cycloid*, which was one of the most investigated objects in 17_{th} century by Galilei, Torricelli, Roberval, Descartes, and Huygens, for example. However, the numerous websites (see e.g. http://mathworld.wolfram.com/Cycloid.html) to illustrate it as an animation are based on parametric representation without even trying to unravel how to construct such an animation without using any symbolic mathematics in coordinate system. Not only the basic construction of cycloid but also many other features of it can be made without any symbolic mathematics just by using a Dynamic Geometry program. Figures 5-9 illustrate some of spectacular features, animated by Stowasser. This, as many other examples of revitalizing geometric ideas from the history of mathematics can be downloaded at http://users.jyu.fi/~laurikah/TUBerlin/home.html, the website referring to material production within the first author's Joint European project (see Haapasalo & Stowasser 1994, and http://wanda.uef.fi/lenni/modem.html). Those visualizations can be utilized in many ways almost on any level of mathematics teaching. The first level of modelling could be just to watch the beautiful simulation and try to explain in own words what happens on the screen. The highest level of modelling would be to make an own computer-based model, which makes the analogous simulation or perhaps improves it.

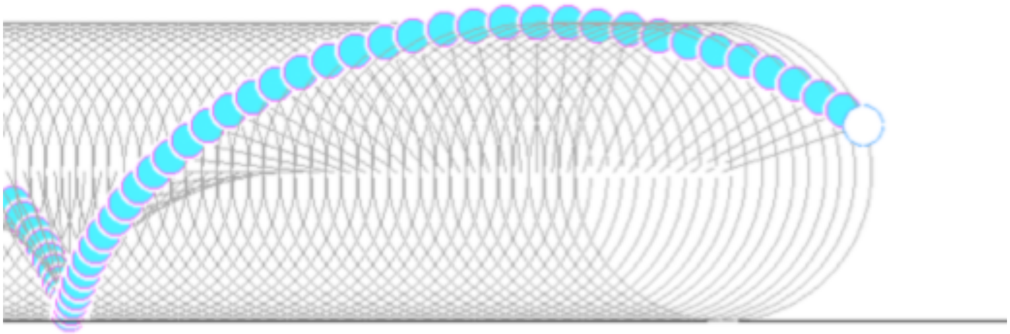


Figure 5. The basic construction of the cycloid (Galilei).

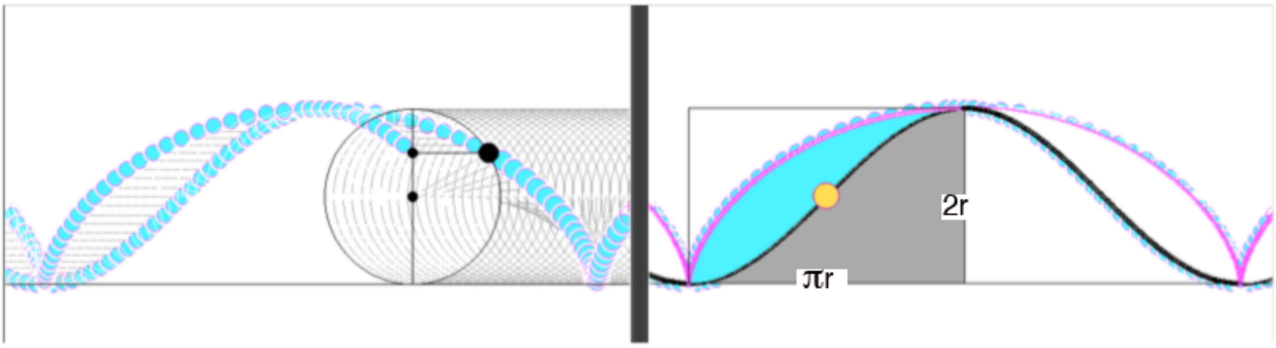


Figure 6. Getting the idea to find out the cycloid area (Roberval).

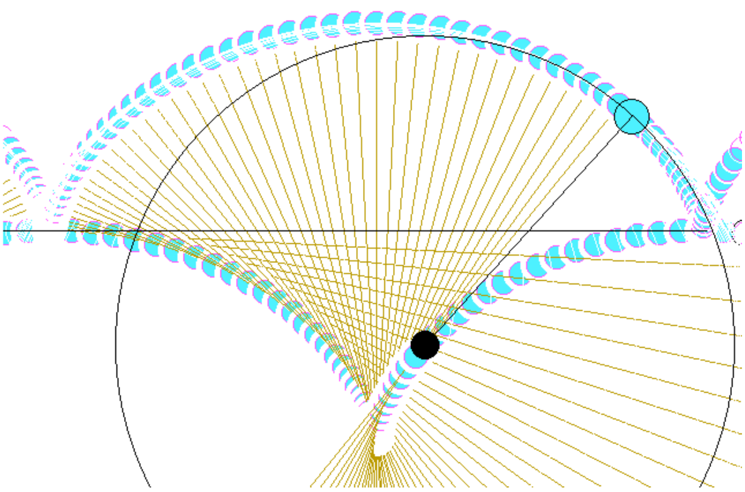
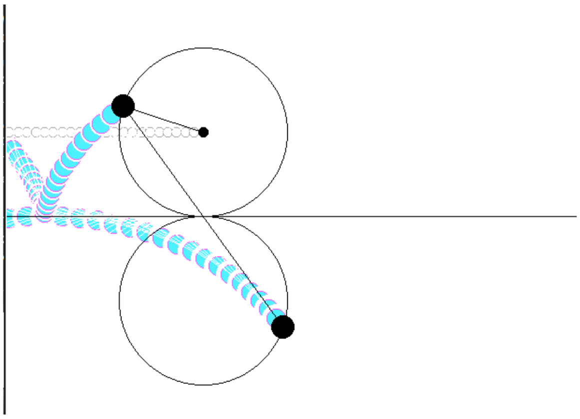


Figure 7. Cycloid constructed through momentanoues rotations (Descartes).



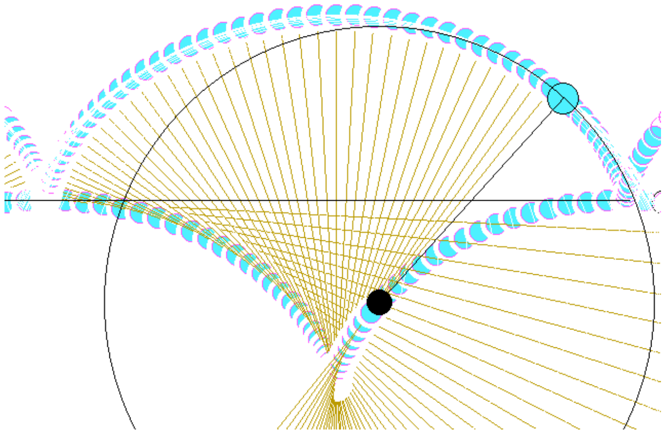


Figure 8. Cycloid with evolute (Huygens). Figure 9. Cycloid constructed as curvature.

The screen shot in Figure 10 represents a GeoGebra construction of a cycloid, for which the student only needs to understand the basic idea of angle measurement: *angle = arch / radius, i.e.* the arch length = radius * angle (in radians). By making a simple physical model by using a CD and a marker pen, for example, the student can test and find out how the movements of the middle point of the circle (FreePoint) and the current arch point (ConstructedPoint) relate to each other.

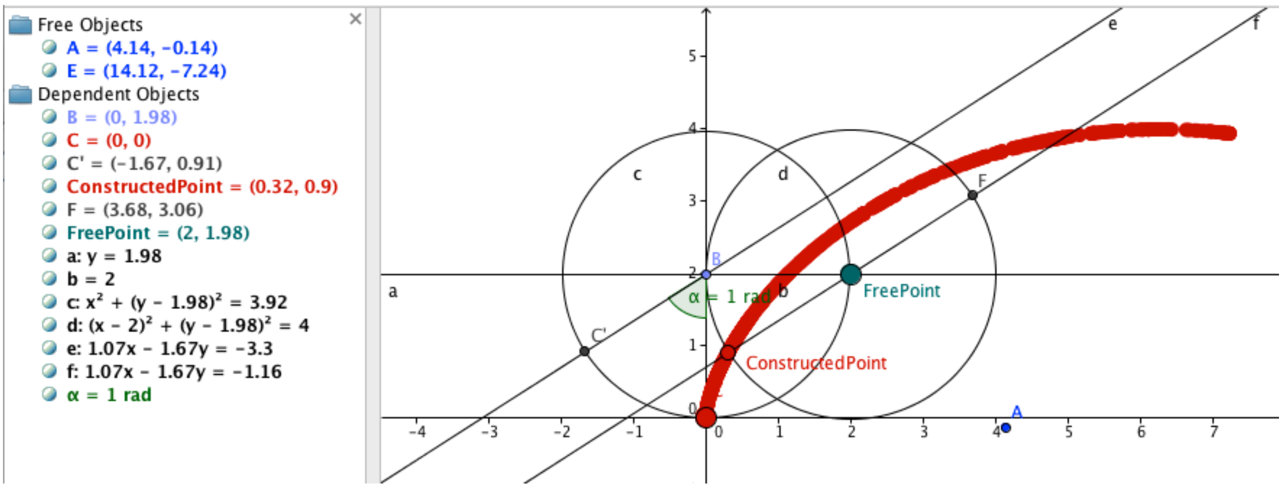


Figure 10. Simple cycloid construction with GeoGebra avoiding symbolic mathematics.

After being successful with the basic construction of a cycloid, the student might try to understand the idea of other constructions above, or perhaps go for epicycloid animations at http://mathworld.wolfram.com/Epicycloid.html, for example. In fact, the construction of a epicycloid is not more complicated than that of a cycloid. Figure 11 represents a GeoGebra construction (in this case Ranuncoloid), where the user can use the slider to choose the radii of the circles (in this case 1 and 5, respectively).

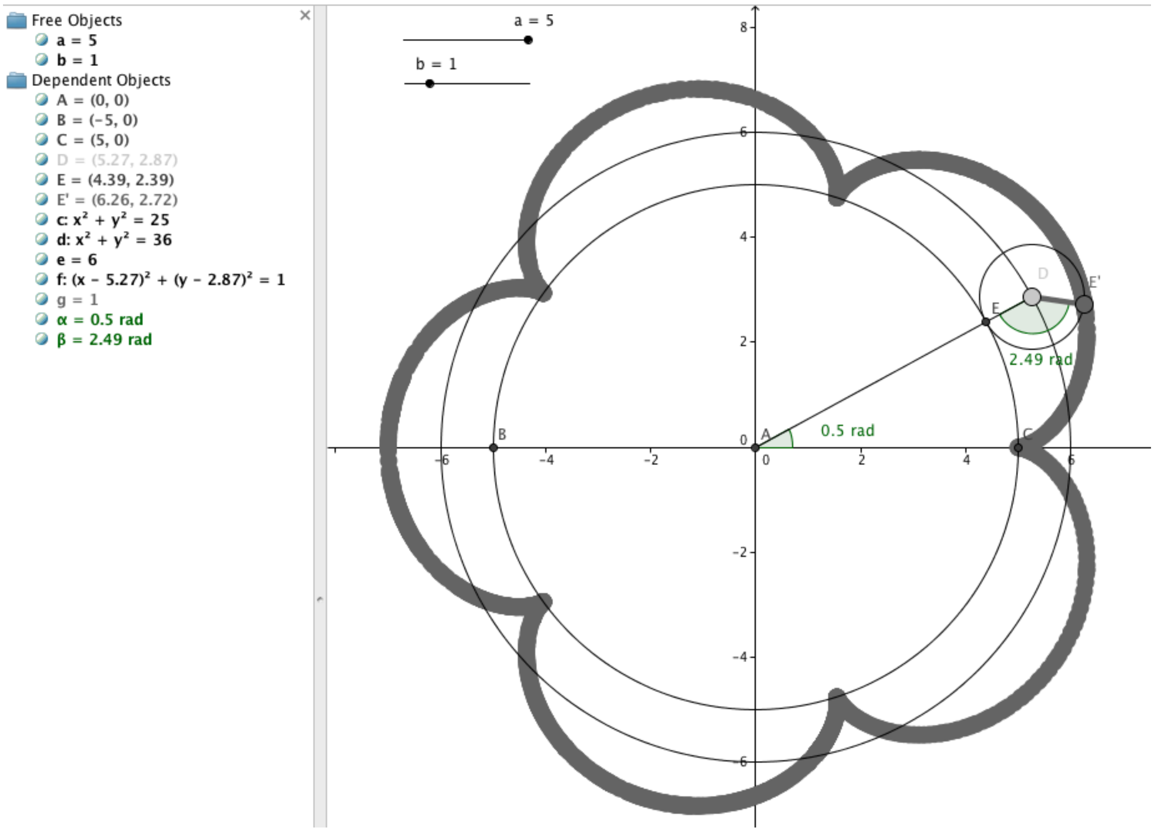


Figure 11. Simple epicycloid construction with GeoGebra avoiding symbolic mathematics.

Examples from educational robotics

Samuels and Haapasalo (2011) give an overview of available small educational robot environments. In their other recent article (Haapasalo and Samuels 2011) they represent a simple robot, which has the capacity to run GeoGebra and the LEGO Mindstorms NXT software. They discuss the interaction between the physical environment of robotics and the virtual environment of GeoGebra. The confidence and interest in using mathematical language is encouraged via the interplay between these two environments and social interaction can be created by collaborative problem solving. They show that a relatively simply formulated problem can yield a rich challenge in geometric and algebraic problem solving.

The screenshots in Figure 12 are taken from that animation where the symbolic representation have been purposely hidden. It is simple to play with the sliders to try to steer the pivot wheel of the robot through the given points. The position of the pivot wheel has been animated and its trace has been represented as a sequence of dots using the point tracing functionality. The values of the variables on the sliders can then be entered directly into the LEGO NXT program to make the robot move through the single point physically.

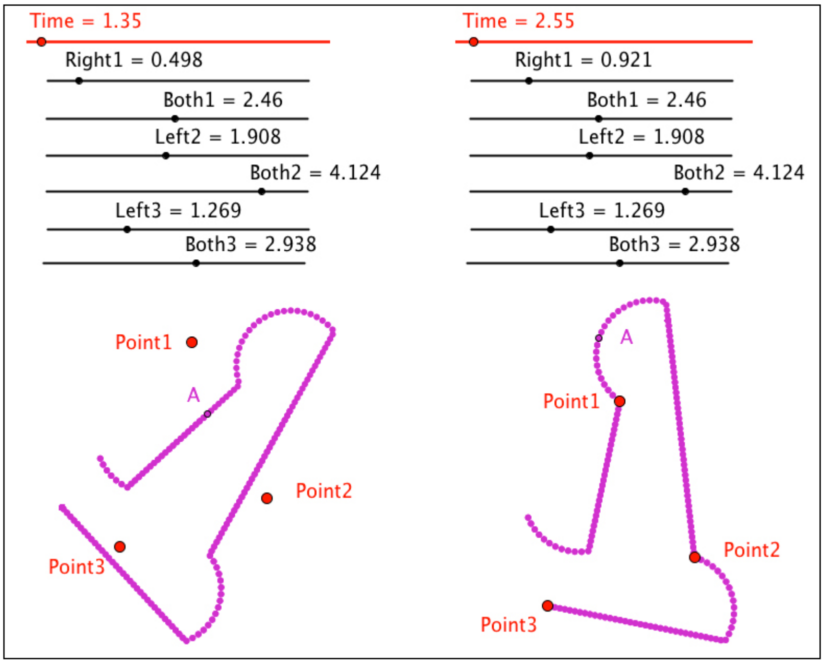


Figure 12. Playing with GeoGebra animation to find parameters for the robot when asking it to go through three points on a plane.

The same animation can be illustrated, of course, with numerical and algebraic representations in the algebra window on the left-hand side of the GeoGebra screen. The authors discuss the pedagogical value of this kind of problem-solving environment in detail, coming to the conclusion that the seven challenges of instrumental orchestration, summarized in the next chapter, can be responded appropriately.

Evaluating the examples

We will now summarize why our examples could be used to respond to the seven challenges of Haapasalo (2008).

(i) The promotion of collaborative social constructions

Posing problems, which are psychologically meaningful for the learners provokes them (or teams) to consider the situation from subjective viewpoints and to use multi-discipline criteria to find solutions. Both, the physical models and the virtual animation provide a sense of enjoyment or satisfaction when they work correctly. The students can change the problem formulation according to their own interest, provided that the way they try to change it is still within the scope of their ability and the amount of time available. They can design their own product (as a robot, for example), or share duties between the teams who are going to solve a particular sub-problem. At higher level, students might be given some indication of how to formulate the problem symbolically in order to solve it exactly (e.g. the cycloid animations in Figures 5-9). Thus the problem can be interpreted in several different interconnected areas simultaneously, leading to the possibility of delegation and teamwork. The student activity even through Internet forums can be therefore collaborative, perhaps facilitated by "pit stop instructors", sharpened by competition and directed towards assessment criteria, which focus on encouraging teamwork, the learning of deeper mathematical concepts and the interaction between different mathematical ideas. Archetypical examples of this kind of working culture are Internet forums to solve problems with hardware and software in general. Robotic forums as (http://mindboards.sourceforge.net), and GeoGebra forums as (http://www.geogebra.org/forum), can be utilized in the same way. A virtual robot arm movement designed with GeoGebra by a team from Nanyang Polytechnic in Singapore is represented at http://concordrobotics.com/arm, and the real robot arm is demonstrated with a video clip at http://www.youtube.com/watch?v=cYP4pshmW3k. These are examples of "education for cyber-generation", one of the most often mentioned aspects is collaborativity, as in the philosophy of Schneiderman (1998): *relating* work in collaborative teams, *creating* ambitious projects, and *donating* meaningful results for someone outside the classroom.

(ii) Linking of conceptual and procedural knowledge

We know from basics cognitive psychology that our world is a world of meanings, not a world of stimuli. This implies the need to apply the *developmental approach* (see footnote #1) in instructional design: students should have opportunities to use their more or less spontaneous procedural knowledge. On the other hand, perhaps the most important educational goal in a modern society is – especially if we trust on mathematics' power to trigger general educational goals – to scaffold citizens' abilities to identify and construct links within complicated multi-causal and multi-disciplined knowledge networks. This means investing in conceptual knowledge, even in such a way, that students also learn appropriate procedural skills. This so-called *educational approach* causes the following conflict: *Does a student have to understand before being able to do, or vice versa*?

Recalling Figures 1, 2 and 12, the student can freely manipulate components, which he or she understands based on his or her more or less spontaneous knowledge – very often procedural one. The immediate updating between the two windows on the screen basically gives the student the opportunity to link this representation to an abstract one – very often more or less conceptual one. The first author's MODEM –project[2], for example, would give empirically tested models to scaffold an interplay between conceptual and procedural knowledge, utilising especially the process of *Identification* during the mathematical concept building (see Haapasalo 2007 and Eronen & Haapasalo 2009). The same kind of identification process happens when the student is trying to explain what happens in an animation, for example.

By utilizing educational robotics, Petre and Price (2004) noticed that many of the children had come to terms with topics (such as programming, gearing, and mathematical representations) which they had previously found difficult, in order to make the robot work. They also noticed that students' learning is concrete, associated with phenomena they create, observe and interact with, and so the abstractions they derive (or apply later) are grounded and relevant. Events drove the child to discover concepts and principles that are often considered difficult.

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[2] See http://wanda.uef.fi/lenni/modemempe.html.

(iii) Solving the dilemma between systematization and minimalism

The term *Minimalist Instruction*, introduced by Carroll (1990), is crucial not only for teachers but also for those who write software tutorials. We pick up the following characteristics (see Haapasalo 2007 or Haapasalo & Samules 2011): (1) Instead of specific content and outcomes, only a core knowledge domain may be determined stressing doing and exploring, (2) Learning is modelled and coached with unscripted teacher responses, (3) Errors are not avoided but used for instruction, (4) Learners construct multiple perspectives or solutions through discussion and collaboration; and (5) Criterion for success is the transfer of learning and a change in students' action potential.

To emphasize the genesis of heuristic processes and students' ability to develop intuition and mathematical ideas within a constructivist perspective, a quasi-systematic planning of the instrumental orchestration is needed to fulfil the role of a pit stop. In learning situations, however, students must have freedom to choose the problems they want to solve within continuous self-evaluation instead of relying on guidance by the teacher. This can mean modifying a GeoGebra environment, for example, as illustrated in Figure 12. Petre and Price (2004) observed that teams described learning from mistakes, but some teams captured attention to obstacles as part of their problem-solving strategies: "Disasters can produce insights" and "Problems can lead to improvements". They found that a drive to build a functioning robot had carried them into new and sometimes daunting territory. It had helped them to take step-by-step and systematic approaches to learning what they needed to know.

(iv) Learning by design

Our examples can be interpreted as open-ended processes, which promote students to develop and test their mathematical ideas within collaboration and own design. This happens in terms of the level of accuracy to which the problem is solved and demonstrated, both physically and virtually. The level of accuracy will also depend on the approach. A more formal approach requires a deeper understanding but it will eventually yield a better and more efficient solution. Students can learn not only about mathematical and scientific ideas, but also about the process of design itself (cf. Resnick et *al*. (1988). This is in accordance with Jonassen's (2000) view that "*those who learn more from the instructional materials are their developers, not users.*

(v) Respecting the sustainable heuristics from history of mathematics

Zimmermann's (2003) long-term study of the history of mathematics reveals eight main motives and activities, which proved to lead very often to new mathematical results at different times and in different cultures for more than 5000 years. It seems, as suggested in Haapasalo (2007 and 2008), appropriate to take this network of activities as an element in a theoretical framework when structuring learning environments and for analyzing student's cognitive and affective variables. Especially the 'find' and 'play' corners represent heuristic activities appearing very often in most natural way in spontaneous activities without any demand to learn. Table 1 is a modification from Haapasalo and Samuels (2011) to show how those activities appear within some possible modelling processes within our examples.

Table 1. Summary of Zimmerman's activities in real and virtual modelling.

Activity	Sub-activity supported by modelling with virtual and real modelling
Find	Finding a way to describe the real or virtual phenomena in spoken language. Looking for links between mathematical concepts and procedures to find mathematical methods for description.
Apply	Applying mathematical concepts and procedures to represent the phenomena. Applying Dynamic Geometry software to make own animations and modifying those animations to the physical environment.
Construct	Constructing geometric orbits and auxiliary geometric objects to illustrate the virtual movements, combination of virtual motions, and constraints relating to real motions.
Order	Ordering the sequence of movements and the algorithms and commands for those motions.
Calculate	Calibrating the parameters for single movements and rotations. Using calculators and software for numerical methods (including interpolation, extrapolation, successive approximation), Using algebra to solve the simultaneous equations and comparing the exact solution provided by GeoGebra with an algebraic solution.
Play	Playing with graphic calculators and software. Playing with the physical model when it is ready. Using the sliders in the animation to compare this with the motions. Trying out different values and observing their effect – in both the computer program and the virtual animation.
Evaluate	Evaluating approximations, the accuracy of the model, the accuracy of the physical motion, and the appropriateness of each task. Refining the model of the movement. Evaluating beliefs of mathematics, including own self-confidence in working with mathematics.
Argue	Arguing inside own team and between the other teams when solving problems and discussing physical and virtual motions. Proving and testing of ideas and methods directly and indirectly.

(vi) Applying business principles to shift the bad social reputation of mathematics

Perhaps the following two managerial points of view, suggested by Hvorecky (2007), would be relevant to be taken into account in the teaching and educational research: "(1) Each market segment has its own expectations. Thus, we should set up relevant priorities for different groups of pupils/students; (2) As the customer is always right, we should make mathematics more edible and digestible for each segment i.e. closer to their environment and cultural values." Concerning those customers, Prensky (2001) coined the term 'Digital Natives' to emphasize how progressive technology is an integral part of the lives of contemporary learners born after about 1980 and how their interaction with technology go far further and deeper than most educators suspect or realize. Deploying a Learning-by-Design-approach to robotics and virtual animations via integrated mathematics software within the curriculum at the secondary level – tertiary level transition would therefore seem to be a potential way to engage digital natives, motivate mathematical learning and change the image of mathematics. Johnson (2003) argues that robotics offers special educational leverage because of its multi-disciplinary nature and the way it involves a synthesis of many technical topics, including algebra and trigonometry, design and innovation, electronics and programming, forces and laws of motion, and materials and physical processes. Nagchaudhuri et *al.* (2002) report of improved creativity in mathematics, physics and engineering design courses resulting from robotics projects in pre-university courses. diSessa (1986) describes science learning as a re-experiencing process, whereby children must experience and re-experience the very specific concept in different contexts. Through these experiences, children gradually reorganize their intuitions into more complete models.

The first author has tried to get student teachers out from sterile task posing to emphasize that mathematics should appear connected to meaningful situations. Such a narrative example related to Figure 11 is the following task in the Basic Course of Mathematics Pedagogy:

> Patrick, who was fallen in love with his classmate Julia, wanted to impress her by planning a personal screen saver for her birthday. He remembered from the biology class that Julia likes ranunculus (flowers). So, he typed the term into Google, and found in few minutes interesting animation sites http://mathworld.wolfram.com/Ranunculoid.html http://mathworld.wolfram.com/Epicycloid.html. He was excited but on the other hand the complicated mathematics killed his interest very soon. So, he gave up and went to sleep. Suddenly he got a crazy idea: he searched his coin collection and chose two of them, having seemingly the ratio of their radii 1:2. When he started to play with them he remembered something happening in the animation, he noticed that the smaller coin rotates with double velocity around the bigger one. He took a marker pen and found out a method to describe the motion without any complicated mathematics. He started GeoGebra and made the animation. He was happy when going back to sleep because he knew that making "real happening" on the screen would be a peace of cake tomorrow. What kinds of problems do you think Patrick recognized, formulated, and solved? What mathematical concepts and procedures do you think he utilized in his animation? Would you like to try to make an own animation with your team?

(vii) Relating instructional design and assessment to instrumental genesis

The fact that several kinds of instrumentations and instrumentalizations very often happen in students' free time in a natural way, implies that educators should shift their focus from well-prepared classroom lessons on minimalism. Instead of acting like a pace car in a race, institutions should be types of pit stops to scaffold students' "race" outside the classroom. By looking at the relationship between technology and mathematics education from five perspectives, Haapasalo (2007) suggests that 'instead of speaking about 'implementing modern technology into the classroom' it might be more appropriate to speak about 'adapting mathematics teaching to the needs of information technology in modern society'. This means emphasizing more the making of informal than formal mathematics to promote sustainable activities (cf. Table 1). This implies that we should give students a variety of ways to show their action potential. Overcoming the gap between traditional methodologies targeting "perfect manual performance of algebraic manipulations and geometric constructions" and new ones emphasising the "understanding of mathematical concepts and their role in solving real-life problems" is difficult but challenging. Modern approaches emphasise technology not only because it reduces the amount of tedious exercises. ICT, if applied properly, demonstrates the role of mathematics in our everyday life better than many traditional approaches do.

Still, educational policy makers in many countries are against allowing the use of modern technology in examinations. This implies that teachers are even less voluntary to learn to develop their own instrumental genesis, and even less ready to make the same concerning instrumental orchestration. On the other hand, allowing the use of technology in teaching and examination does not necessarily improve students' understanding and motivation when staying with conventional tasks. An extra problem might be that technological tools are made for those who apply mathematics, and not for learning purpose. Manuals, as for ClassPad for example, consist often several hundreds of pages containing huge amount of conceptual mathematical knowledge. This causes a contradiction between the versatility of the tool and minimizing the continuous tutoring from teacher's side.

Haapasalo & Hvorecky (2010) express their worry about obstacles, which seem to prevent using technology in mathematics education, especially in assessment. They triggered discussion about how to shift assessment tasks by providing a matrix illustrating potential task types, which could be considered as "task generators" and serve as control mechanisms allowing establishing proportionality between categories of tasks based on different principles. This matrix consists of the quadruple *Starting point – Conceptual knowledge – Procedural knowledge – End point.* Each of those components can be given or not given, defining a new type of problem in which remaining components are present. This produces basically 2*2*2*2 =16 different categories, and if allowing for each given component the sub-dichotomy right vs. wrong, the amount of categories is 3*3*3*3=81. Table 2 represents five of those alternatives, demonstrating the current situation of assessment in mathematics. Those examples show, how minor changes in problem posing can shift a task from one category to another, and such can cause big impact on the solving process.

This matrix can be used as a framework when trying to get out from a certain deadlock situation as regards allowing the usage of technology. Let us begin with the most traditional task type when everything is given except the result (first row of Table 2). As regards technology, we basically have the following three options when solving the task:

(i) it *is not allowed* to use technology
(ii) technology *can* be used freely
(iii) technology *has to* be applied

The first alternative represents the situation in many countries, and in most of the sixteen 'Bundesländer' of Germany, for example. Because very often a sophisticated tool as *ClassPad* can be used to get the result by pressing one button or several ones, we must be honest to ask why all those mechanical paper-and-pencil tricks are still taught at school. On the other hand, the second alternative causes a serious conflict with the *Equity Standard*, emphasized by NCTM (1993), for example. Those who have opportunity to buy sophisticated tools, namely, profit from this kind of assessment policy. The third alternative would not only make the situation even worse, but devaluate the appreciation of sophisticated mathematical methods being applicable without technology. This would probably even accelerate "cookbook-teaching".

We notice that the task type on the first row of the matrix in Table 2 is not only almost complementing authentic problems occurring in real life but causes a deadlock when used for assessment purpose. Still, it seems to be the most common one even in those countries, which allow the usage of technology. All other types of task posing (cf. examples in Table 2) would allow the use of technology in much more sophisticated and appropriate way. Because there are 15 (or 80, if posing with wrong alternatives) different ways to get out of this conflict, it would be appropriate to shift the focus on task types, which would be independent from the usage of technology. This would mean a thorough shift in mathematical assessment in general and therefore "might be problematic". If the teacher is aware of the richness to come up with different kind of problem posing, the student has almost unlimited amount of opportunities to use the tool. We would like to quote Haapasalo & Hvorecky (2010): "Still, institutions emphasizing mathematics as goal aim to explicit knowledge even though economists, physicians, engineers, for example, do not expect that mathematicians could co-operate in solving their problems. Accepting mathematics as a tool means a balance between soft skills and hard skills". We repeat their very progressive problem posing, fitting our pit stop philosophy and discussion on assessment. The reader might notice that our narrative Ranunculoid example above fits this kind of problem posing:

Peter noticed that *ClassPad* was able to solve simple equations as 2x-7=0. He continued testing if the tool was able to solve equations like $x^2+x-6=0$. Yes, it worked! Now Peter got excited about what all *ClassPad* could actually do for x^2+x-6. He tapped menus for factorizing, simplifying, for example, and noticed there is an interesting connection between the roots 2 and -3 with (x-2)(x+3). He continued playing with the tool and made a conjecture. Which kind of conjecture you think he made? Can you make own ones and perhaps see some mathematical concepts and procedures associated with the situation?

Table 2. Classification of task types in assessment of mathematical knowledge

Starting point	**Concep-tual know-ledge**	**Proce-dural know-ledge**	**End point**	**Frequ-ency in the assess-ment**	**Example**
1	1	1	0	Very tradi-tional	A quadratic polynomial P can be written in the form $P(x)= ax^2 + bx + c$, where x is the variable and a, b and c are given real constants. The values of x, where P(x) = 0 are called *roots* of the polynomial. Those roots can be found by the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Find the roots of $x^2+x-6=0$.
1	0	1	1	Very rare	The values of x, where a polynomial P=P(x) gets the value 0 can be found by the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Which mathematical concepts have been used to get this formula and what can be said about the polynomial P?
1	1	0	1	Rare, except of uni-versity mathe-matics	A quadratic polynomial P can be written in the form $P(x)= ax^2 + bx + c$, where x is the variable and a, b and c are given real constants. The values of x, where P(x) = 0 are called *roots* of the polynomial and can be found by the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Which method has been used to get this formula?
0	1	1	1	Very rare	The values of x (i.e. roots), where a quadratic polynomial $Q=Q(x)= ax^2 + bx + c$ gets the value zero, are 2 and -3. If we know that any quadratic polynomial Q(x) can be written as $Q(x) = (x - x_1)(x - x_2)$, where x_1 and x_2 are the roots of Q(x)=0, find at least three such of polynomials Q.
1	0	0	1	Very rare	Explain in as many ways as you can why the equation $x^2 + x -6 = 0$ has the roots 2 and -3. What can be said about the polynomial $Q(x) = x^2 + x -6$?

Coda

To avoid a possible misinterpretation that our pit stop metaphor would mean devaluating the importance of teacher and his or her competence, we would emphasize that even the most spectacular driver would never become world champion without a very professional pit stop team. Neither would do a soccer team if just the trainer would run all over the field trying to make the scores on behalf of the team.

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