Geometric Ornaments in Istanbul

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Abstract: *Islamic ornaments are one of the greatest achievements of ancient geometers, artisans and craftsmen in the Middle East, Turkey, India, Spain, and North Africa. These ornaments are frequently used to decorate secular and civil buildings, books, and furniture. Istanbul, in particular, is a place where such ornaments were, and still are, frequently used.*

In this paper we will construct selected geometric ornaments from Istanbul. All these ornaments were created using very precise geometric constructions and multiple grids. We will analyze the structure of these ornaments and show how grids used to draw them were constructed. We will use a computer program, Geometer's Sketchpad, to construct these grids as well as complete ornaments. Examples presented in this paper have a significant didactical value. While creating them students can learn a number of important topics in constructive geometry, e.g. constructions of regular polygons, constructions of figures circumscribed or inscribed in a circle, division of angles and segments into a given number of parts, transformations of figures, and many other topics.

Introduction

Islamic geometric ornaments are essential element of my life. I see them every day and everywhere around me in the place where I live for last ten years. It is quite natural for me as a mathematician educated in geometry, and a person with some experience in computer graphics, to look at these ornaments as at geometric constructions. In fact each Islamic geometric ornament is a very precise geometric construction. There is evidence, see for example [5], that these ornaments were constructed by the middle century Islamic mathematicians and then created by, quite often, illiterate craftsmen.

In this paper I will show some of the techniques used to construct these ornaments and some of the basic components of these constructions. The paper is based on five different examples. Each of these examples was taken from a concrete place in Istanbul, and each of them demonstrates different features of constructions of Islamic geometric ornaments.

Islamic geometric ornaments are usually constructed on a base of grids and multiple subgrids. Therefore in the first example I show the role of grids and subgrids in these constructions. Grids form an overall structure of the ornament, and if we know the pattern filling the cells of the grid we can easily reconstruct the whole ornament. However, grids with pentagonal or even hexagonal cells are not the most convenient means to reconstruct the ornament. There is strong evidence that people who created these ornaments frequently used a square or a rectangular fragment of the ornament to cover a larger area, e.g. a wall or space on the minbar, cover of a paper, etc. Such rectangle usually is described as a repeat unit. Not all ornaments can be created using a rectangular repeat unit. Many of them require more complex techniques. In example 1 I show why a hexagonal repeat unit is not the most convenient tool to use. All remaining examples in this paper are created using a rectangular repeat unit.

Many of the Islamic geometric ornaments contain a star or multiple stars. Usually these stars are inscribed into regular polygons. Therefore, where it is possible I show the role of regular polygons and stars in constructions of Islamic geometric ornaments. There are many different types of stars used in Islamic geometric ornaments. In this paper I show only very few of them.

In example 2 I show one of the simplest methods used to create Islamic geometric ornaments with regular hexagons and six-fold regular stars.

In example 3 I discuss a geometric ornament based on multiple circular subgrids. In this example subgrids are used only to establish points where the lines of the ornament meet or cross. Lines of the ornament do not follow lines of subgrids. In this example the repeat unit has an easy to indentify square shape.

Example 4 is again based on circular subgrids. However, here the role of the subgrids is totally different than in the previous example. In this example the lines of the ornament follow tightly lines of the circular subgrids. This example can be a starting point to many popular Islamic geometric ornaments.

Finally example 5 shows a very popular type of construction where the repeat unit is a rectangle and there are two stars with centers in two opposite vertices of the repeat unit. According to my observations this type of construction is the most frequently used in designing Islamic geometric ornaments. A very interesting is the group of constructions where each star has different type of symmetry, e.g. one of them with six-fold symmetry and another one with ten-fold symmetry.

This paper may not be entirely consistent. It was created from fragments of my book *Islamic Geometric Ornaments in Istanbul*. All photographs, as well as all presented here constructions, are my sole work. All presented here geometric constructions were developed with the help of The Geometer's Sketchpad.

Example 1 – grids, subgrids and stars

In a small book on sacred geometry, see [4], Miranda Lundy wrote a very short chapter entitled "*Islamic designs – stars are born from subgrids*." In this chapter we can find two very meaningful sentences "*Many beautiful ornaments are sitting in very simple subgrid, just waiting to be pulled out"* and later "*The subgrids themselves are rarely shown in traditional art. They are considered part of the underlying structure of reality, with the cosmos* (*'cosmos' means 'adornment'*) *overlaid*."

1.1 Grids and subgrids

In this section we will investigate how these grids and subgrids look and how they are used to create Islamic geometric ornaments. We will have to learn how to find these grids in geometric ornaments and then how to construct them? They are there, and if we are able to identify them then we can decipher the miracle of the ornament design. Without knowing how to find grids in Islamic geometric ornaments, and how to use them, our understanding of geometry behind the ornament will be incomplete. So, let us examine some examples of grids and see how they work in a design of an Islamic geometric ornament. Let us look at a very simple example. The next figure shows a geometric ornament that is very popular in Istanbul. We can see it almost everywhere – on fences, window grills, in mosques, on covers of some books, etc. The first look at the whole ornament reveals that we are dealing here with a regular hexagon and some detailed pattern inside the hexagon. Then the whole hexagon is replicated along some invisible lines, using reflections about these lines. We can draw these lines easily in the space between two neighboring hexagons. This way we will create a grid built out of hexagons. The hexagon can be considered as the repeat unit. Then we can look at the pattern inside the hexagon. A grid, or grids, used to create this internal pattern we will call subgrids. In fact, the main grid is usually very simple, the subgrids to create the repeat unit are quite often very complicated.

Fig. 1 A typical Islamic geometric ornament in Istanbul

The repeat unit here is a hexagon with invisible sides. In the next figure we show how these lines look.

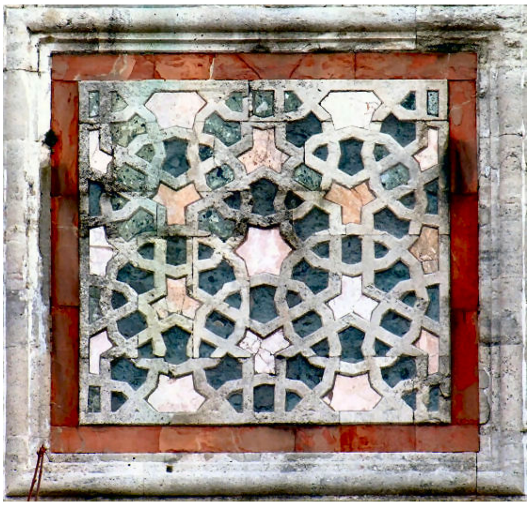
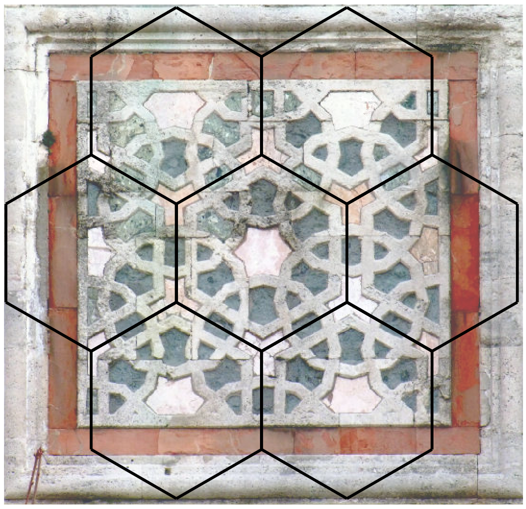


Fig. 2 The same ornament with the grid drawn on top of the photograph

The main grid is built out of regular hexagons. Each hexagon with its content can be considered as a repeat unit.



Our next goal will be to create the subgrids of the ornament and based on them construct the repeat unit. We will start by creating the outline of the repeat unit, i.e. the regular hexagon. Then we will try to reconstruct one of the round figures that wrap each vertex of the hexagon in the photograph.

Fig. 3 First stage of reconstructing the ornament from the figure 1

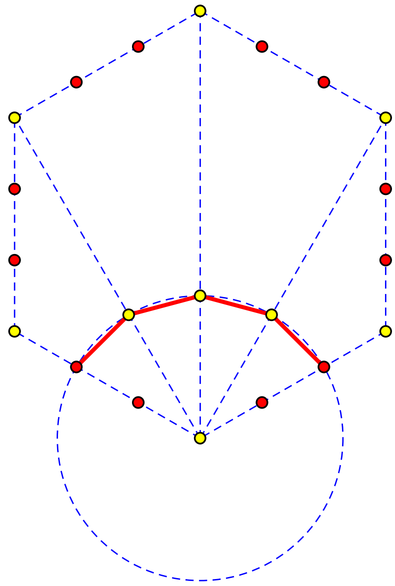
STEP 1: Boundary of the repeat unit and the first subgrid

Develop a regular hexagon. This will be the boundary of the repeat unit. Divide each side of the hexagon it into three equal parts. We know how to divide a segment into any number of equal parts.

Draw a circle with its center in one of the vertices of the hexagon and radius equal to 2/3 of length of the side of hexagon.

Finally divide the arc inside the hexagon into four equal parts by drawing diagonals from the center of the circle to the three opposite vertices of hexagon. The lines are part of the subgrid that we are looking for.

Using points of intersection of the circle with the subgrid you can create the first part of the repeat unit.



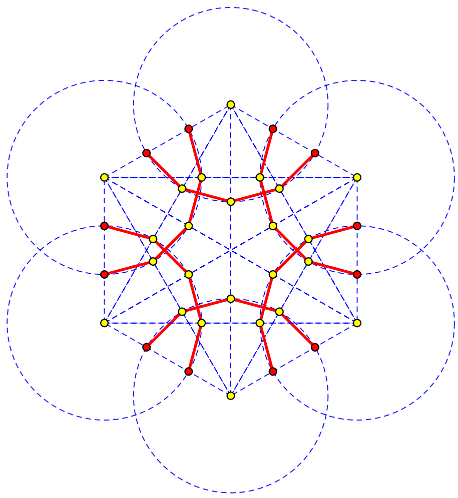
After repeating this task for each vertex of the hexagon we will obtain a large part of our ornament and a subgrid containing three sets of lines perpendicular to the edges of the hexagon.

Fig. 4 Part of the ornament and subgrids used to create the ornament

Repeat the described above tasks to each vertex of the hexagon. You will obtain a set of lines parallel or perpendicular to a side of the hexagon. This is the first subgrid of the ornament.

The set of circles can be considered as a second subgrid.

NOTE – the hexagon shape in our picture is the boundary of the repeat unit. This hexagon is invisible in the original ornament. However, there is also a visible regular hexagon that we will have to construct in the next step.



STEP 2: Constructing the visible regular hexagon

The last task of the construction will be to create the hexagon shape that we see in the photograph. Vertices of the this hexagon can be created by adding short segments connecting intersection points of each circle with sides of the repeat unit (here for example points P and Q) and finding their points of intersection with the first subgrid.

By joining these points we get our regular hexagon (solid, medium line).

Now hide the subgrids, all or some points of intersection, and replicate the repeat unit by reflecting it about each side of the grid hexagon.

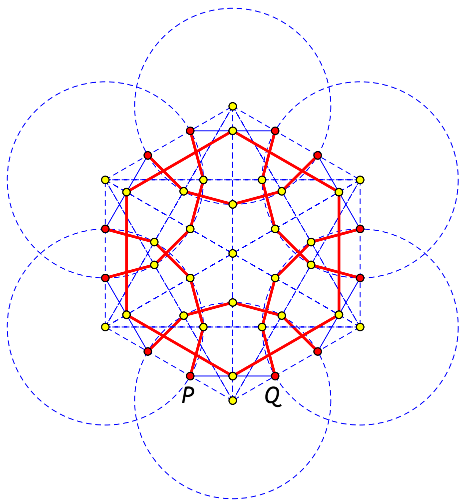
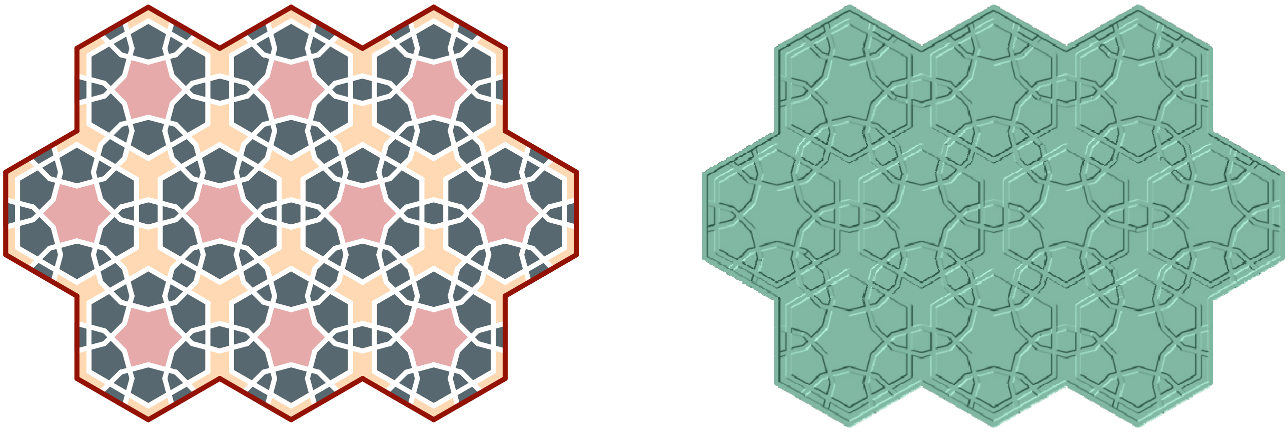


Fig. 5 Two different versions of the ornament from the last construction



This example shows why a non-rectangular repeat unit is not the handiest instrument to develop geometric ornaments. We always get some empty spaces near the edge of the area which we wish, or have, to fill with the ornament.

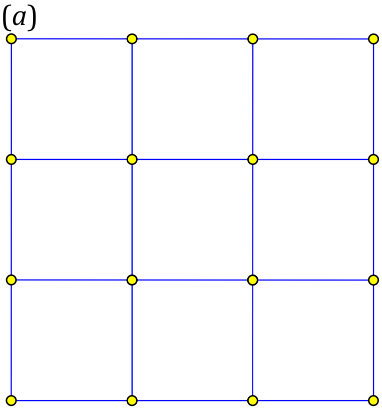
While reading publications on Islamic ornaments we can notice that the grid and subgrid story is a bit complicated and I would say quite “misty”. There is no precise definition what should be called a grid or a subgrid. In this text we did distinguish the main grid from subgrids. This is all. However, we did not attempt to classify subgrids and we did not say exactly how we look at them. We just used the term subgrid for the bunch of lines and circles that were used to create the ornament. However, there are many possible ways of looking at these lines. One can consider each set of parallel lines as a separate subgrid – this way we could talk about three subgrids; or we could consider the set of lines going out of each vertex of hexagon as a separate subgrid – this way we could have six subgrids. The next question would arise – what to do with these six circles? Are these circles parts of a separate grid? If we agree that a subgrid is a set of lines, or circles sharing the same property, then we could say that in our construction we have three subgrids: the set of lines joining vertices, the set of circles, and the set of short lines connecting points of intersection of circles with edges of the hexagon.

Let us concentrate for a while on grids of Islamic geometric ornaments. The one that we used in the last example was a network of hexagons. However, we can imagine grids built out of other regular and not necessary regular polygons. Figure 6 shows selected examples of grids that are used to create geometric ornaments.

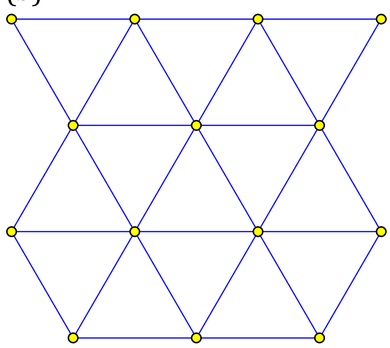
Fig. 6 Examples of grids used in Islamic geometric ornaments:

(a) squares,
(b) equilateral triangles,
(c) octagon and square,
(d) hexagon, square and triangle,
(e) octagon, hexagon and pentagon (not a regular pentagon)

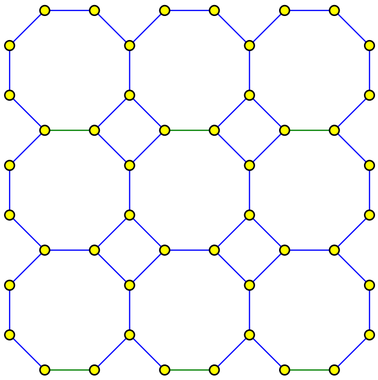
(a)



(b)



(c)



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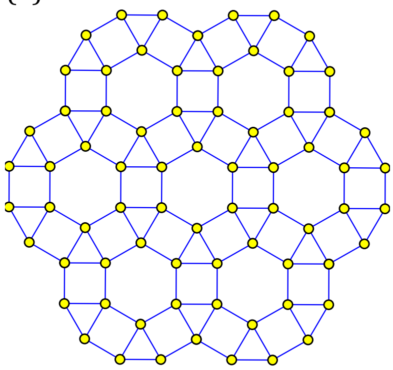
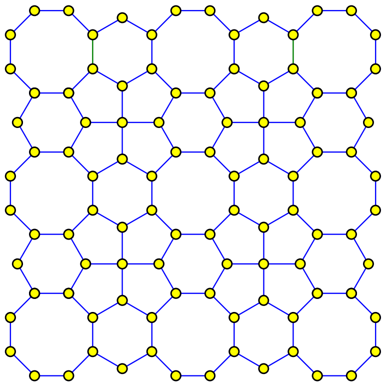


(e)



The collection of subgrids used in this example was fairly simple. In many sometimes even very simple ornaments the set of subgrids can be quite complex.

1.2 Construction of a double star ornament

Let us talk a moment about stars. Islamic art is famous for the sophisticated star shapes used there. In this paper we will have a few opportunities to construct such stars. Therefore, here I will examine a very simple and popular example of a star ornament and show how it can be used to fill a grid.

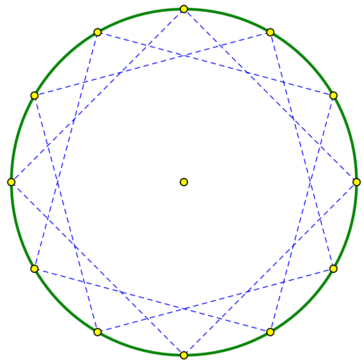
In our example we will use a regular dodecagon, the 12-sided polygon, inscribed in a circle. Precisely speaking we will need only vertices of the polygon. We will not use its sides. Based on the vertices of the dodecagon we will create three squares rotated 30° about the center of the dodecagon. This will give us the first subgrid and a way to construct a large star. Later another subgrid will be created inside the star and this way we will be able to create another internal star. Finally we will fill parts of the ornament and replicate it by using reflections.

Fig. 7 A subgrid created using dodecagon vertices

A dodecagon inscribed in a circle was constructed. Its sides were removed.

STEP 1: Construction of the first subgrid

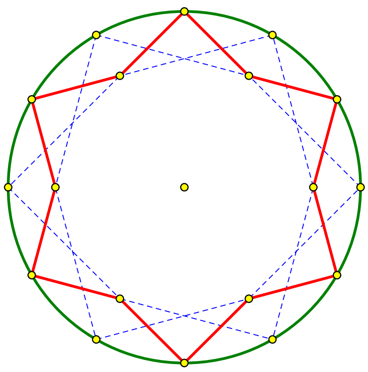
By connecting every fourth vertex of the dodecagon we obtained three squares. This is our first subgrid. This subgrid can be a starting point for many interesting constructions.



STEP 2: Construction of the external star

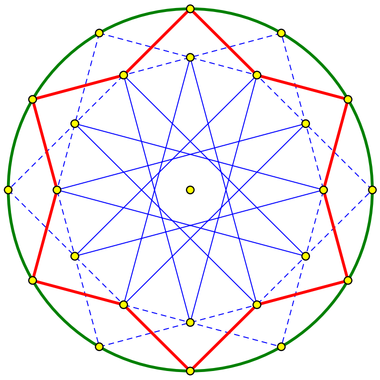
By connecting vertices (every second) with intersections of subgrid lines we obtain a large star. From this point the construction can go many ways. For example we could use all internal intersection points and create another subgrid and this way we could add an additional, identical star inside the current one.

We can also notice that some of the edges of the obtained star are perpendicular to each other and have the same length. Therefore we can think about constructing a collection of small squares using the existing edges of the star.



STEP 3: Construction of the second subgrid

Here we show one of the possible additional subgrids that can be added to our construction. The solid, thin lines form the second subgrid. Now, we can easily see that each two perpendicular sides of the star are part of a small square.



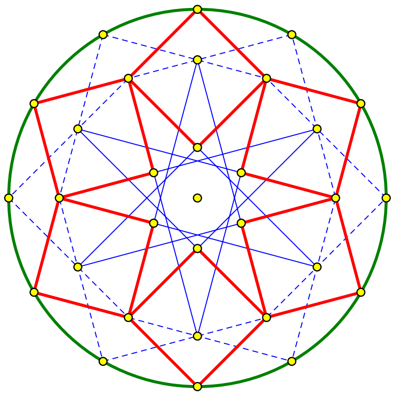
STEP 4: Construction of the internal star

Use the second subgrid to draw the missing edges of squares.

We obtained a double star ornament – one external and another one inside the first one.

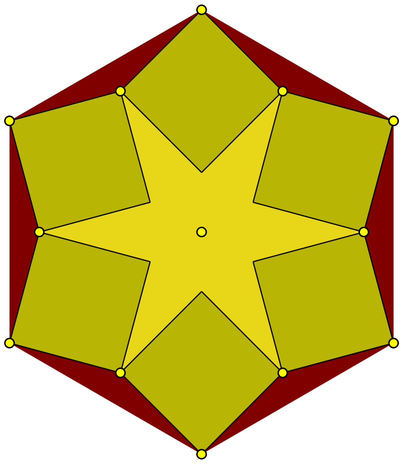
We can finish our construction at this very moment, remove all unnecessary elements and fill the empty spaces.

If you do not wish to stop here, you still can use the second subgrid and create another smaller star-like shape in the empty space in the center of the second star.



The image to the right shows how the repeat unit of the double star ornament may look. One of the peculiarities of this construction is that we started with a dodecagon, i.e. a regular polygon with 12 sides and we finished with a shape that is bound by a hexagon. This way we can fit this ornament into a hexagonal grid.

One can also think about squeezing the double star shape into a square or even a pentagon.



The next figure shows an ornament created using the double star ornament. It is one of the very popular ornaments used for wedding invitations, decoration of greeting cards, or even books.

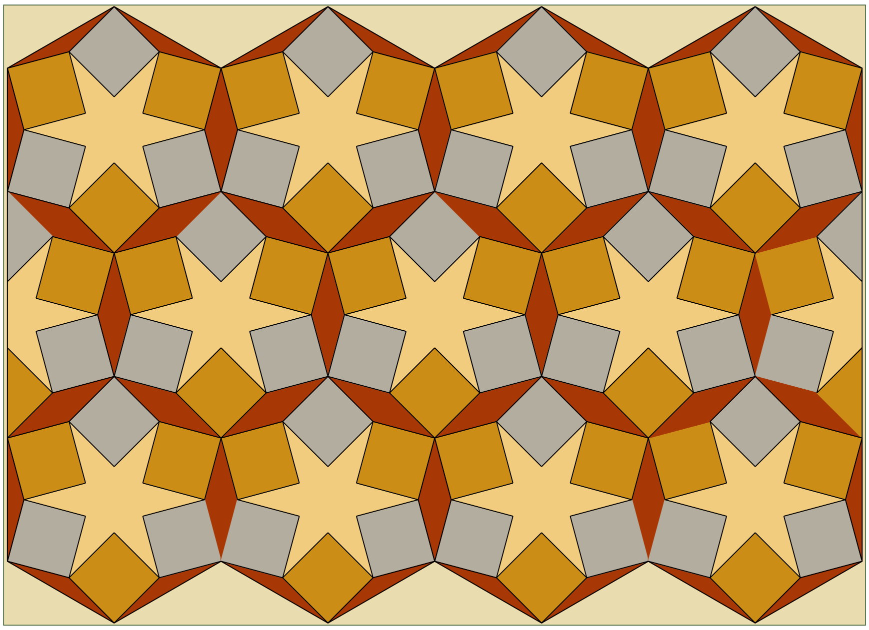
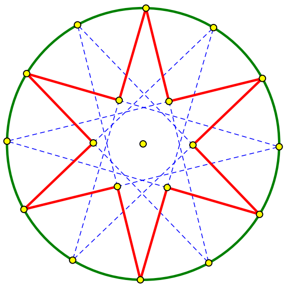


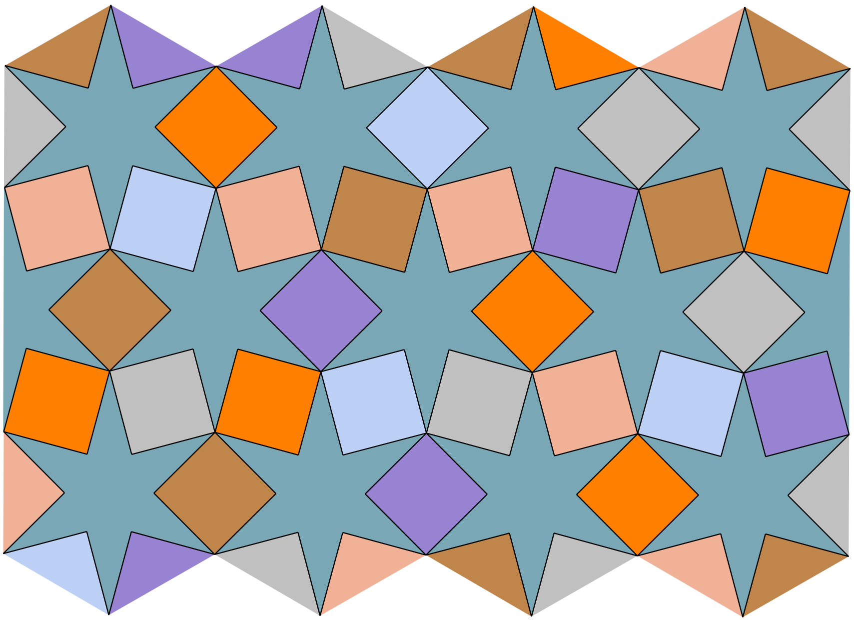
Fig. 8 An ornament created using the double star ornament

A slight modification of the first subgrid, where we draw from each vertex a line to the fifth vertex to the right and left, gives us a completely different ornament.

Fig. 9 An ornament obtained from the previous example by modification of the first subgrid. There was no need for the second subgrid.

In this example colors are completely random.





A number of Islamic geometric ornaments can be created using the regular dodecagon and very simple subgrids based on it.

Example 2 – the ornament from the entrance to the Sultan Ahmed Mosque

Let us examine a simple ornament that can be seen in Istanbul in many places, in particular in hotels or mosques. We show it in the next figure.

Fig. 10 A simple star ornament that can be seen on the floor in one of the entrances to the Sultan Ahmed Mosque in Istanbul

There are many methods of reconstructing this ornament. We will choose the simplest one based on a rectangular grid described by Hankin in1925 (see [3]).

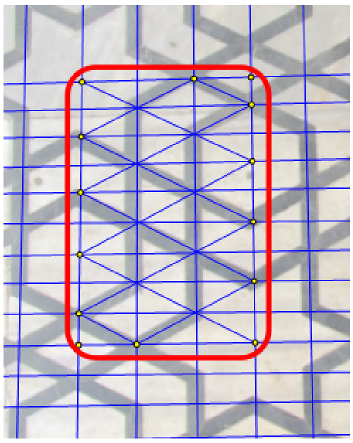
Let us take a fragment of the above photograph and draw on it a rectangular grid. In fact we do not need to cover the whole photograph. We need only a small portion of it, like the one shown in the next figure. After constructing the enclosed in a rounded rectangle fragment of the ornament we can use it to create a larger part of the ornament. We will have to use reflections about its edges, and this way cover a bigger area of the plane.

As we said in the beginning of this paper – the part of the ornament that is enough to create the whole ornament will be called a *repeat unit* or a *fundamental motif*. Many of the Islamic geometric ornaments can be created from a repeat unit using geometric transformations: reflections about edges of the repeat unit, rotations about a point, translations about a vector, and compositions of these transformations. Depending on our point of view the concept of repeat unit can be slightly different. An orthodox mathematician would probably ask about the smallest region that can be used to create the whole ornament using the mentioned transformations. An artist or a craftsman would probably choose the region that is the most convenient to control – a rectangle or another figure that will be easy to use. The same geometric ornament can be created using different its parts, that means by different repeat units.

Fig. 11One of the possible grids that can be used to reproduce the ornament from the entrance to the Sultan Ahmed Mosque.

The repeat unit (here enclosed in the rectangle with rounded corners), contains 3x9 rectangular cells. The angles between diagonal lines are 60°. Each cell is a rectangle with the short side equal to a *, and long side equal to* $b = a\sqrt{3}$ *. The same is valid for the repeat unit, if* a *is the length of the short side and* b *is the length of the long side then* $a : b = 1 : \sqrt{3}$ *.*

Note – the proportion $a : b = 1 : \sqrt{3}$ *is one of the most frequent proportions of the repeat unit for Islamic geometric ornaments.*



Let us start by constructing a rectangular grid similar to the one in the figure 11. Construction of a single cell is shown in figure 12.

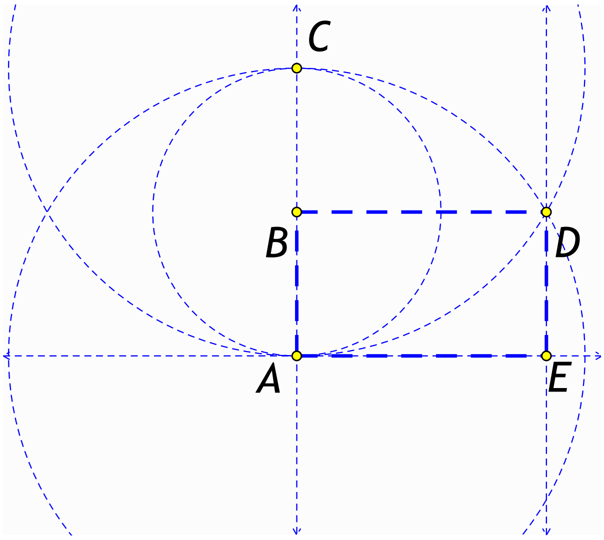
Fig. 12 Construction of a cell for the grid that we will use to create the ornament from the floor in Sultan Ahmed Mosque.

Start by drawing a vertical segment AB as well as a line passing through points A and B. Draw a circle with radius AB and center in B. You will get a new point C.

Now draw two large circles with centers in A and C, and radii equal to AC. One of the points of intersections of these two circles we mark as D.

Finally construct a line passing through D and parallel to the segment AB, and another line through A and parallel to BD.

By connecting points A, B, D and E we will obtain a rectangle with required proportions.



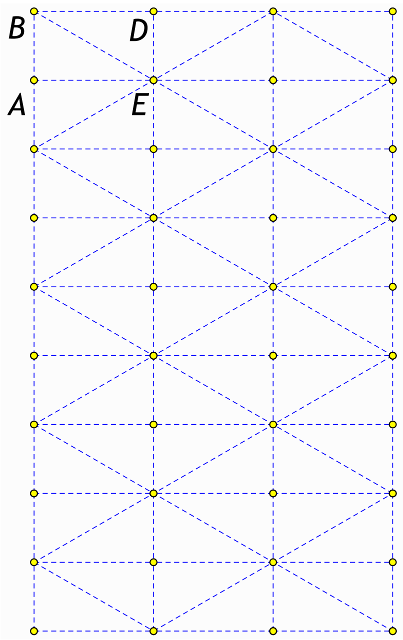
Before proceeding to the next step we will have to hide all unnecessary lines and circles leaving only the rectangle and its vertices. The rectangle obtained this way can be replicated to the right and down using reflections about its edges. The result of this task is shown in the following figure.

Fig. 13 The grid for the repeat unit of the ornament.

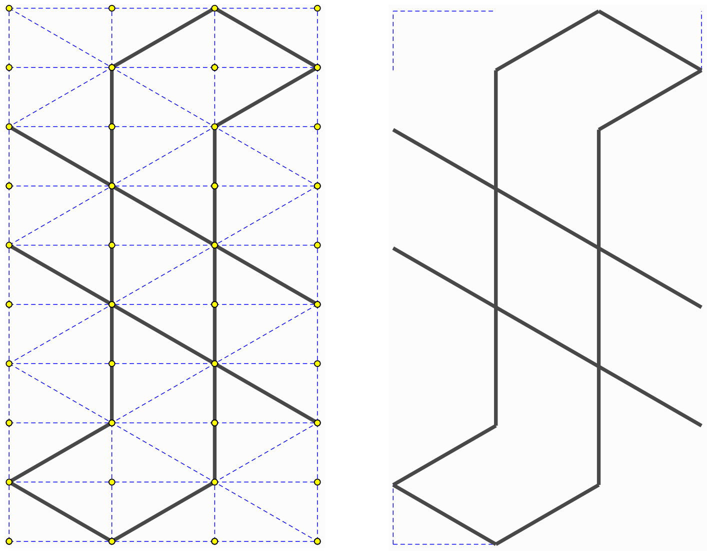
Rectangle ABDE was replicated two times to the right using mirror reflection about its right edge. Then the whole triplet of rectangles was replicated the same way down 8 times.

Then the diagonal segments were created starting from the point B.

Comment – in reality we do not need all diagonal segments in this grid. In order to create our ornament we need only selected points of intersection of the horizontal and vertical segments. We created all diagonal segments in order to emphasize the whole structure of the ornament. A rectangular grid with horizontal and vertical lines can be enough to recreate many Islamic ornaments as well as Chinese lattices. The only difference is that Chinese lattices are mostly based on a square grid, while Islamic ornaments use rectangular square root of 2, square root of 3, etc. grids.



In the next few steps we will create the repeat unit using the rectangular grid we created just a moment ago. Then we will hide all gridlines leaving only selected segments on the edge of the repeat unit in order to mark mirrors for further reflections. Finally, we will have to find a good marble texture, put it under the whole ornament and hide all unnecessary elements. Now, let's have a look at the sequence of images that we have to create on the way from the rectangular grid to the final ornament.



(a) (b)

Fig. 14 Last two steps in creating the repeat unit:

(a) the grid and the ornament of the repeat unit,

(b) the repeat unit – all unnecessary elements are hidden. The dashed segments were left in order to mark mirrors of reflection while developing the final ornament. You should hide these segments at the end of the construction of the whole design.

NOTE – in this example it was not necessary to distinguish between subgrids and the main grid, their roles were quite evident.

Now we have everything ready to finish our ornament. The repeat unit can be reflected as many times as we wish to the right and below in order to create a larger fragment of the ornament. The final result is shown in figure 15.

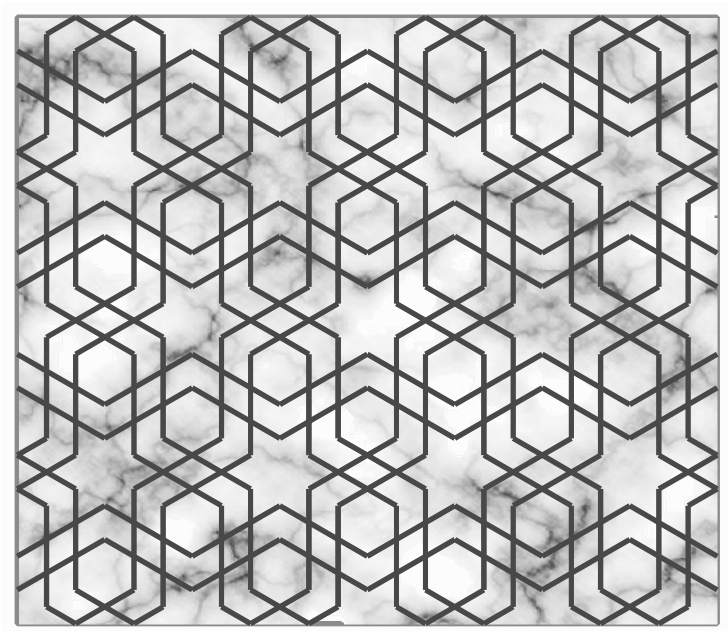


Fig. 15The final construction of the ornament from the floor in one of the entrances to the Sultan Ahmed Mosque in Istanbul.

The marble texture was created using a computer generated image of gray marble.

Finally one can think how this ornament can be further modified. For example in some places in Istanbul a different color of the marble was used to create an identical or a similar ornament. In some other places stars in the ornament were filled with a different color of stone and sometimes a metal. This ornament was quite often developed using wood or ceramic tiles. In some other places the hexagon shape was additionally filled with a more detailed geometric ornament. I have also seen the same ornament where all polygons were filled with a beautiful floral arabesque.

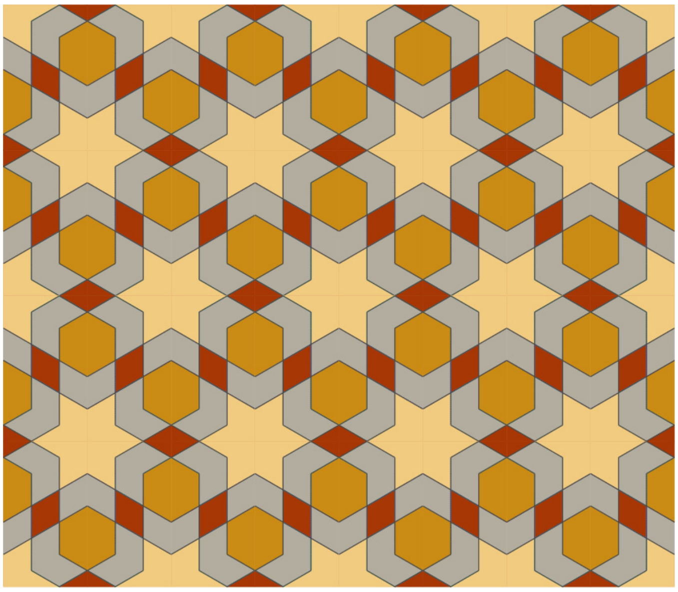


Fig. 16 The ornament from the entrance to the Sultan Ahmed Mosque in Istanbul – this time the image was created using tiles with colors that mimic one of the large tiled panels on the wall of Hagia Irene (near the Topkapi Palace). RGB values of colors used here are as follows:

Dark red: 167, 56, 5
Dusty orange: 203, 140, 21
Dusty yellow: 241, 203, 127
Gray: 179, 173, 159.

Example 3 – the Minbar from the Sultan Ahmed Mosque

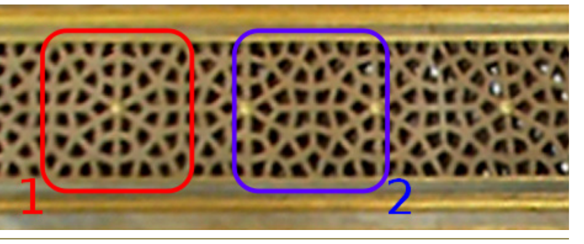
The Sultan Ahmed Mosque is one of the treasures of Istanbul. It is probably the most frequently visited mosque in Turkey. It is also my favorite place. I can spend many hours in this Mosque and every time I am there I discover something new.

Fig. 17 The Sultan Ahmed mosque is one of the masterpieces of Islamic architecture

Now, let us enter the Blue Mosque. The first thing that we see inside is an incredible space with huge columns, walls covered with blue tiles and sophisticated arabesques. Just opposite the main entrance we can see the minbar. In most of the mosques in Istanbul a minbar is the place where we can find some interesting geometric ornaments. This one is not an exception. On both sides of the minbar there is long belt of a geometric ornament along the balustrade. Let us have a closer look at it, and try to construct it?

Fig. 18 The minbar in the Sultan Ahmed mosque contains a lot of arabesque decorations and quite an intriguing geometric ornament along its balustrade.

Fig. 19 A detail of the geometric ornament on the minbar. The two rounded squares show two different square tiles that can be used as a repeat unit to create the whole ornament.



As we can see, the repeat unit of the ornament can be created in a few ways. One of them could be to create the repeat unit as a square with the circled star ornament in the middle (rectangle 1). Another way could be to create a square determined by the vertical bars going through the whole ornament (rectangle 2). This way on the sides of the square we will have centers of four circles and inside the square there will be four halves of a circle with half of a star in each of them. Without any special reason we will choose the second method.

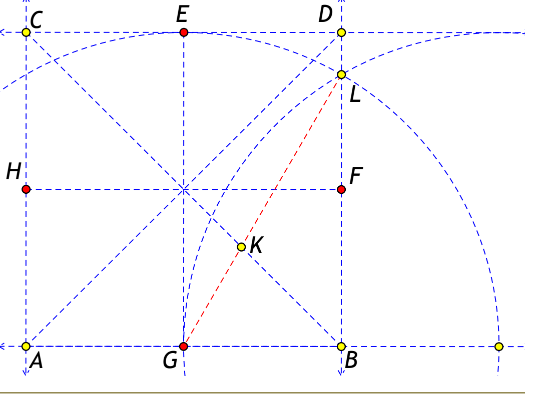
The following images show steps of our construction. We start with a preparation step, and before going to the actual construction we will hide all unnecessary elements. This way we will have much clearer understanding of what we are trying to do.

Fig. 20 Construction of the ornament on the minbar in the Sultan Ahmed Mosque

STEP 1: Construction of the fist subgrid

Create a square, mark the center of each side. Draw diagonals of the square as well as lines connecting centers of opposite sides. This will be our first subgrid.

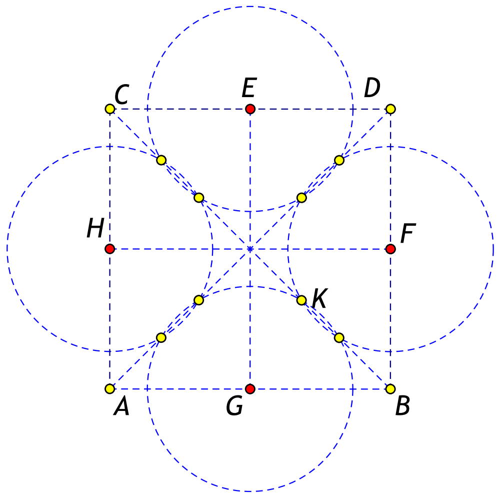
Draw a circle with center in G and radius equal to GE (length of the side of the square), find intersection point of it with the right side of the square (point L). Draw line GL and mark point K. Note: ∠LGB=60°.



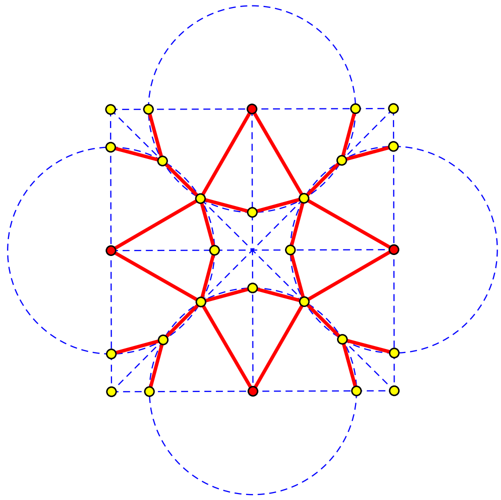
STEP 2: Construction of the second subgrid (circular)

Before starting this step hide all unnecessary elements leaving only first subgrid, all points including and point K.

Draw the first circle with center in G and passing through K. Draw another circle with center in F passing through K also. The two remaining circles with centers E and H we draw using the points of intersection of the diagonal lines with the two existing circles.



STEP 3: Use the existing points and the points of intersection of the four circles with diagonals of the square to draw a part of the ornament.



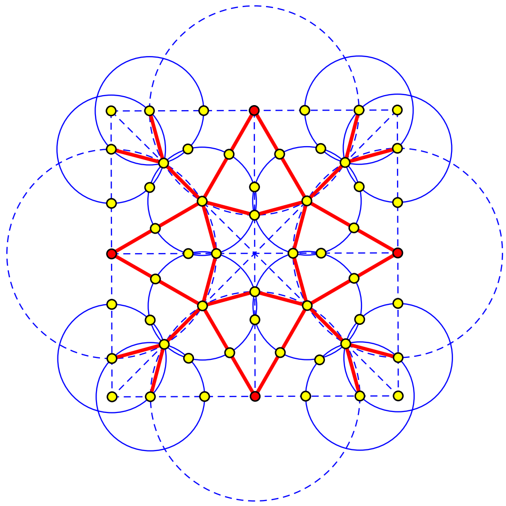
STEP 4: Construction of the third subgrid (small circles)

In order to create the remaining elements of the repeat unit we need another subgrid of circles. Create 12 small circles – 4 in the middle and 8 close to the edge of the square.

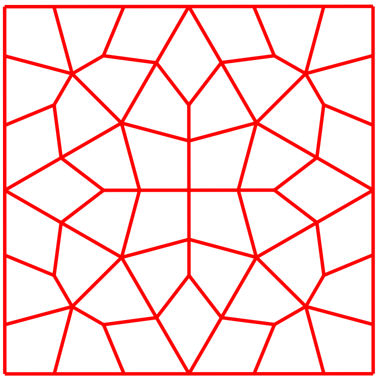
Circles of the new subgrid are marked here using a thin solid line.

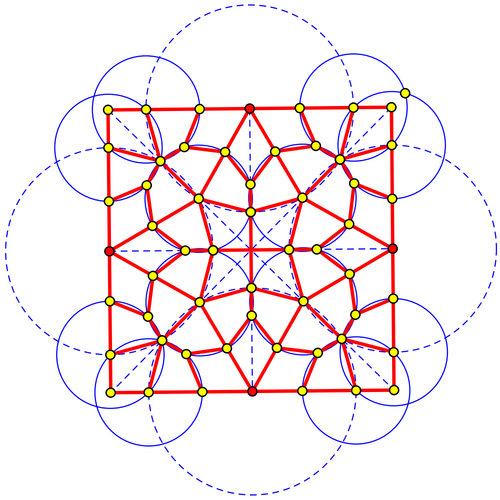
Mark all points of intersection of the new subgrid of circles with the existing subgrids and part of the ornament created in the previous step.

Now we are ready to finish the repeat unit.



STEP 4: Draw the missing lines of the ornament, i.e. the lines shown inside small circles. Hide all points and the three subgrids: diagonals of the square, large circles and small circles. The repeat unit is ready (below).

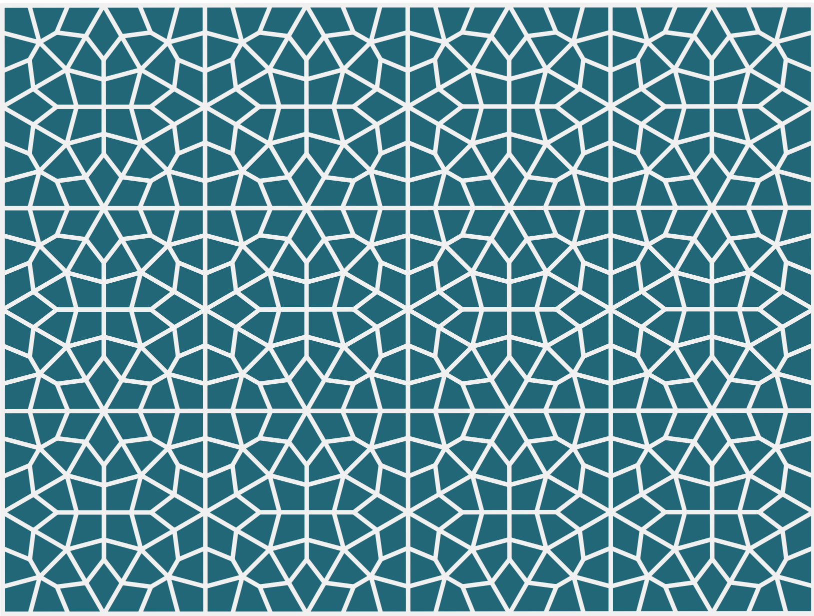




The next figure shows a larger ornament created using this repeat unit.

Fig. 21 A geometric ornament created using the repeat unit constructed above. This ornament is very specific to Istanbul. We find it in many older mosques here, as well as in the Topkapi palace.

The same ornament can be found also in many other mosques in Turkey.



Example 4 – the ornament from the courtyard in the Şehzade Mosque

The Şehzade Mosque has its own sad history. In 1543, the eldest son of Suleyman, the prince (Şehzade) Mehmet died at the age of 21. Suleyman mourned three days beside the coffin before allowing to be buried his son. Then he ordered his famous architect Mimar Sinan to design and build a mosque to commemorate his son.

Fig. 22 The Şehzade Mosque (photo from the back of the garden).

Fig. 23 The courtyard of the Şehzade Mosque with some geometric ornaments

Now let us look for geometric ornaments in this mosque. We can find many of them, but due to the limited space in this paper we will concentrate on one of them only. We will examine one of the ornaments that we find in the courtyard.

The ornaments in the courtyard were frequently copied and used in many other mosques. We find them in many places in Istanbul. The next photograph shows one of these ornaments. It was constructed using circles, and it can be created using a simple hexagonal grid. However, this ornament can also be created using a rectangular grid. This way the repeat unit will have the form of a rectangle and it will be very easy to spread such an ornament over a larger area. Finally, one can think about creating the repeat unit in the form of an equilateral triangle.

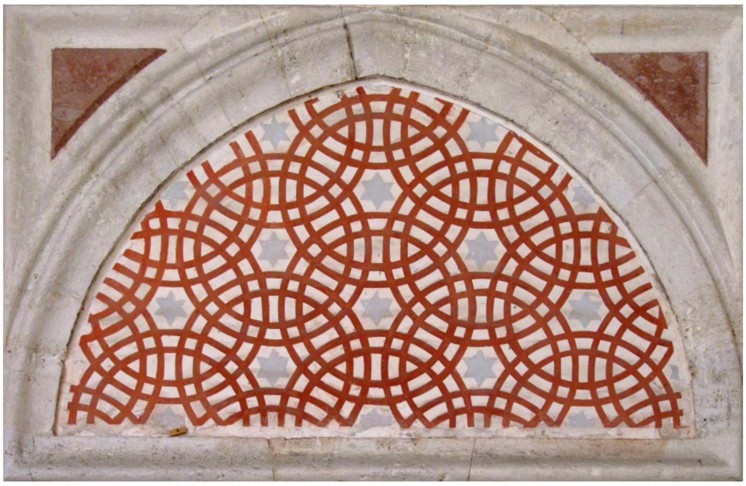


Fig. 24 A simple geometric ornament in the courtyard of the Şehzade Mosque. The ornament was constructed using multiple circles.

Let us start developing the ornament shown in the photograph. For this example we will create a rectangular repeat unit. It will be a rectangle with given length of its longer side (horizontal). We can imagine that ancient masters usually got the size of the area they had to cover and then they had to fit an ornament in this space. Therefore, starting with the repeat unit of a given size makes a lot of sense.

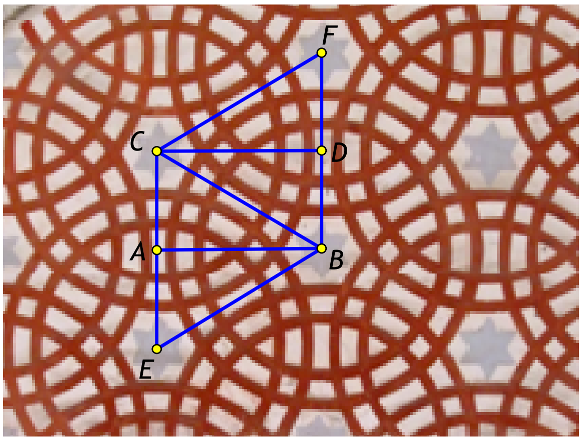


Fig. 25 A rectangular repeat unit ABDC for the geometric ornament from the Şehzade Mosque.

Note the overall design is based on a hexagonal grid. Therefore we can easily draw two equilateral triangles on our photograph, each of them with one side vertical. Their heights are equal to the longer side of the repeat unit.

Note also the other side of the repeat unit is equal to the half of the triangle side.

We can easily notice that if we start from a length of the repeat unit, here the segment AB, then we will have to construct an equilateral triangle with a given height. This is a very easy construction and can be done by a high school student.

Fig. 26 Construction of the rectangular repeat unit

STEP 1: Construction of the outline of the repeat unit

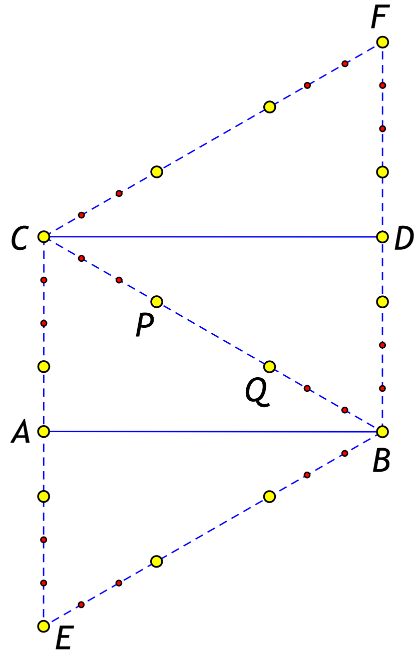
Draw segment AB and construct an equilateral triangle CEB with AB as its height.

Use points C and B to construct another equilateral triangle CBF. You should obtain two equilateral triangles with a common side and parallel heights.

The rectangle ABDC will be our repeat unit, and now we have to fill it with the pattern.

Divide each side of the two triangles into 3 equal parts (large points) and then each external 1/3 of the side divide again into three equal parts (small points).

We will use these points to create a circular subgrid.



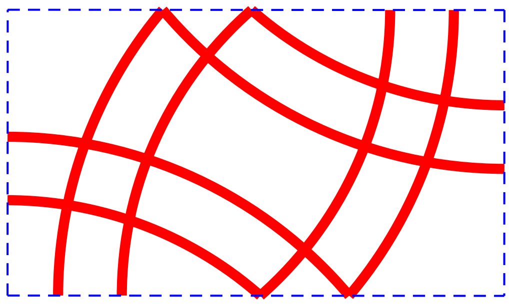
STEP 2: Construction of the pattern for the repeat unit

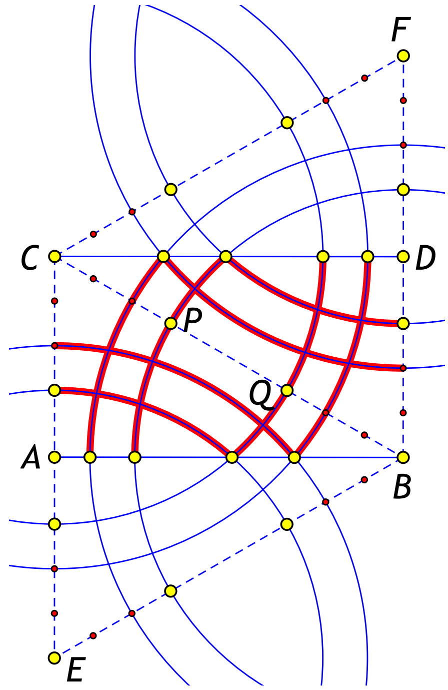
From each vertex of both triangles draw two circles – one passing through the large point obtained from division of the side of a triangle into three equal parts, and another one passing through small red point obtained by dividing external parts of sides into three smaller parts.

Finally use these circles as a subgrid to draw only this part of the ornament that is inside the rectangle.

The repeat unit is almost ready. Now you have to hide all unnecessary elements.

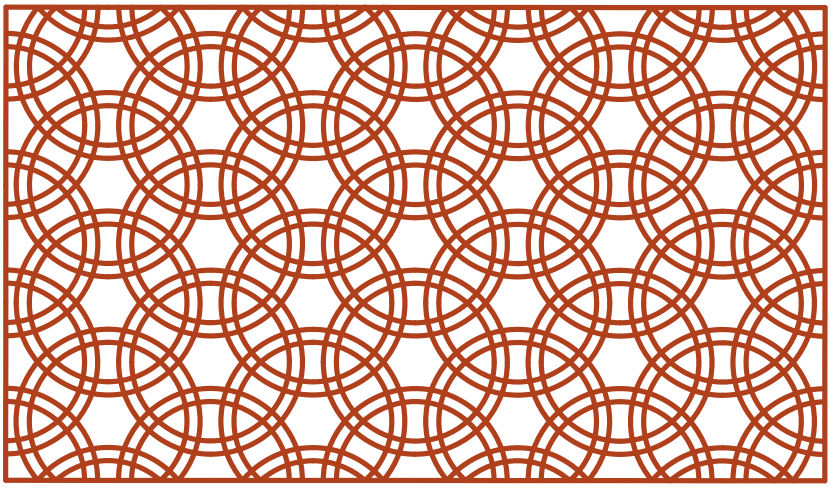
Below I show the final shape of the repeat unit.





The next figure shows an ornament obtained by multiple reflections of the repeat unit created a while ago.

Fig. 27 A geometric ornament from the courtyard of the Şehzade Mosque



Example 5 – the tomb of Mahmut Paşa

Every time I visit the Mahmut Paşa mosque I see a neglected cemetery next to the mosque and an octagonal tomb of Mahmut Paşa with interesting Moorish style decorations with small tiles in blue, black, green, yellow and turquoise.

The Mahmut Paşa mosque was built in 1462, nine years after Istanbul was conquered by Ottomans. This was the first large mosque built inside the city walls. Its founder, Mahmut Paşa, was a Byzantine aristocrat of Greek origin, who after converting to Islam became the grand vizier under Mehmet the Conqueror. He was fortunate to finish his own tomb in time. In 1474 after a disastrous defeat in the Upper Euphrates region of Anatolia, he was executed on the order of sultan.

In this chapter we will examine one of the ornaments on the tomb of Mahmut Paşa. Here we can see two different geometric ornaments – one of them with a constellation of stars with eight and five arms, and another one with overlapping regular dodecagons. We will construct the ornament with stars.

Fig. 28 The tomb of Mahmut Paşa with Moorish style decorations

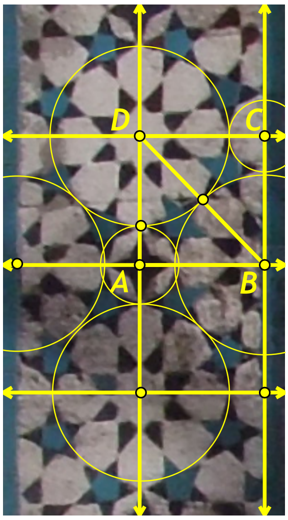
The figure below shows how these stars are arranged on a square grid.

Fig. 29 The starry ornament from the Mahmut Paşa tomb

We can easily see that the ornament can be outlined with the help of large touching circles, as well as smaller circles inscribed in the empty space between large circles. The large stars are constructed on the base of octagons inscribed in large circles. Finally centers of circles are located on vertices of a square grid.

The small circles are used to create small four corner stars with diamond shaped arms.

A square with corners located in centers of stars, here ABCD, can be the most convenient repeat unit for this construction.



Now, we know almost everything about this ornament. Let us start our construction.

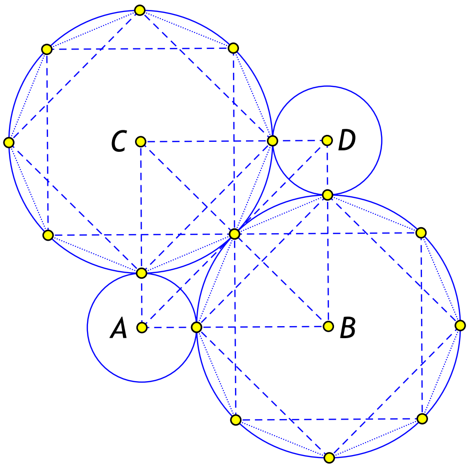
Fig. 30 Construction of the star ornament from the Mahmut Paşa tomb

STEP 1: Construction of the first subgrid

Draw a segment AB, and construct a square ABCD with AB as its base. Draw diagonals of the square and two circles with centers in C and B respectively and radius equal half of the diagonal CB.

Construct two regular octagons with centers in C and B respectively, inscribed in circles.

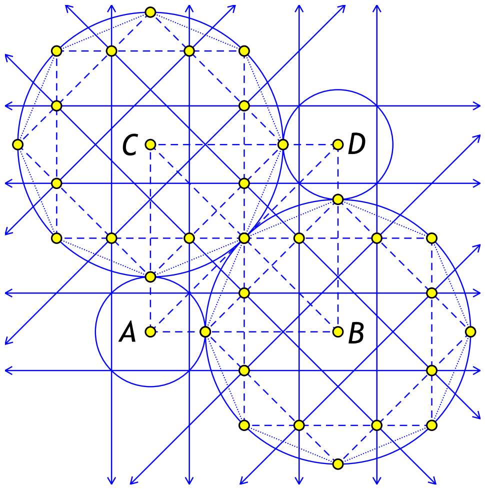
For each of the octagons create a subgrid of lines connecting every second vertex (dashed lines). For each circle you should get two squares inscribed in it. Finally, add the small circles with centers in A and C respectively, tangent to the large circles.



STEP 2: Construction of the second subgrid

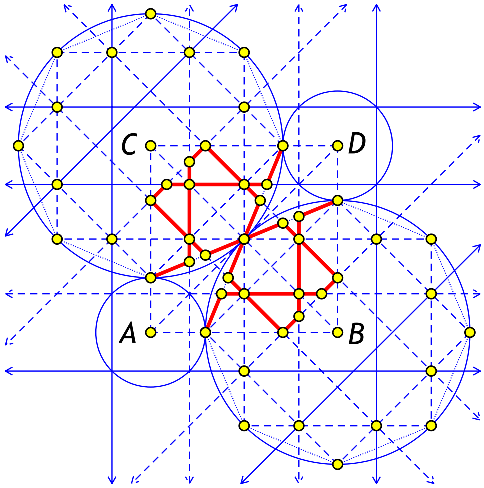
For each of the octagons draw a subgrid of pairs of parallel, vertical and horizontal lines (solid, thin lines). These lines are passing through points of intersection of the first subgrid and are parallel to the sides of one of the squares inscribed in each circle.

This is our second subgrid.



STEP 3: Construction of the pattern for the repeat unit

This part is quite simple. Draw only a quarter of each star – the part that falls inside the boundaries of the repeat unit. You do not need to draw anything outside of the repeat unit, even if the subgrids are tempting you to do this.

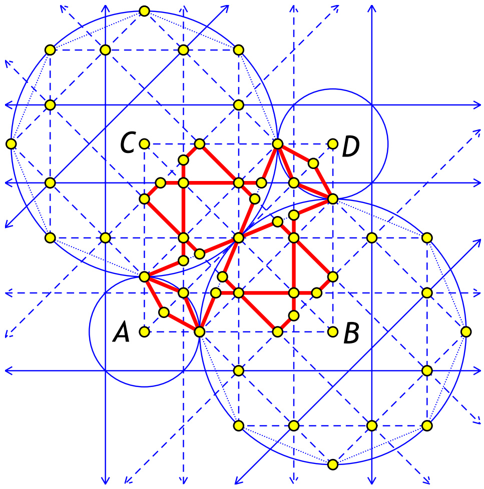


STEP 4: Filling the small circles

Note each small circle contains a short fragment of a diagonal of the square. Find the center of each of these fragments, and then use them to draw a diamond like shape inside each small circle.

This is all. Now clean the maze of subgrids and circles, and remove all labels if you have them.

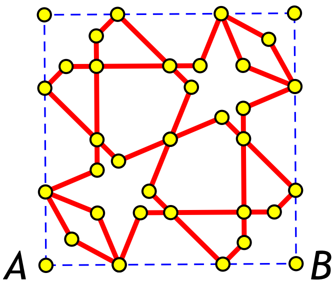
The repeat unit should look like the one in the pictures below.

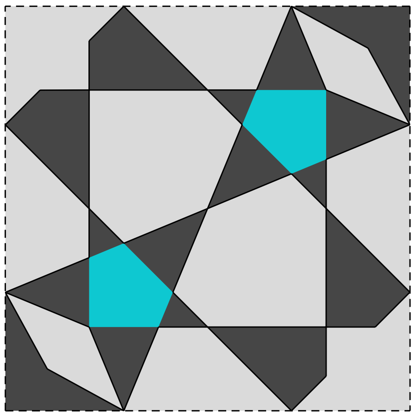


The repeat unit for the star ornament from the Mahmut Paşa tomb

Below – outlines of the pattern only

Right – a ready repeat unit with fills.

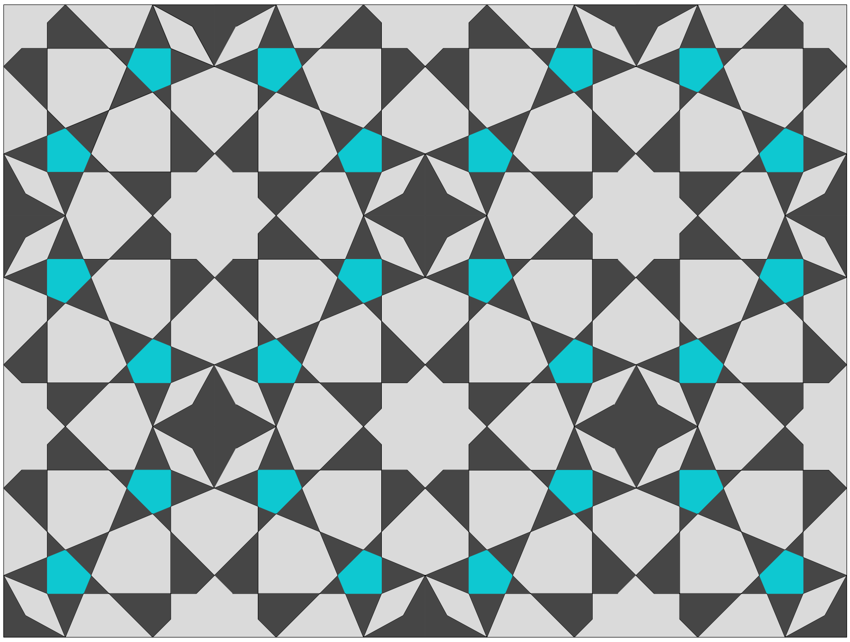




In the next figure I show one of the possible versions of this ornament. However, this ornament, although it is not very popular, it can be found in different color versions.

Fig. 31 The starry ornament from the Mahmut Paşa tomb

Although the original ornament on the Mahmut Paşa tomb is in a terrible state, this ornament is one of the most crystal clear ornaments I have ever seen.



Summary

In this paper I presented a few basic techniques used for constructing Islamic geometric ornaments as well as selected components of these ornaments: grids, subgrids, star patterns, the concept of repeat unit, ornaments using subgrid points only as well as ornaments with lines following tightly lines of subgrids. All examples of Islamic geometric ornaments presented in this paper are very precise geometric constructions. The construction techniques used in this paper are very similar to those used by medieval mathematicians in the Middle East, e.g. *House of Wisdom in Baghdad* (813-833AD) or *House of Knowledge in Egipt* (1004 AD). While constructing examples for this paper I used The Geometer's Sketchpad. However, each of these ornaments can be drawn by hand using only classical tools: straightedge and compass. Each of the presented here constructions can be used by mathematics teachers as an instrument for developing problem solving skills of students. Finally Islamic geometric ornaments as a very interesting combination of mathematics and art can be used in mathematics curriculum as well as in the curriculum of many arts related courses.

References

1. Clevenot D., *Ornament and Decoration in Islamic Architecture*, Thames & Hudson, Ltd., London, 2000.
2. Critchlow K., *Islamic Ornaments, An Analytical and Cosmological Approch*, Thames and Hudson, London, 1995
3. Hankin E. H., *The Drawing of Geometric Ornaments in Saracenic Art*, Memoires of the Archæological Survey of India, Vol. 15, Calcutta, India, 1925.
4. Lundy Miranda, *Sacred Geometry*, Wooden Books Ltd, 2006.

Internet Resources

5. Background Notes on Ornaments in Islamic Art, `http://www.ornamentinislamicart.com/`