Optimal control for Linear Singular Fuzzy System Using Simulink

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Abstract: *In this paper, optimal control for linear singular fuzzy system is obtained using Simulink. To obtain the optimal control, the solution of MRDE is computed using Simulink approach. The Simulink solution is equivalent or very close to the exact solution of the problem. An illustrative numerical example is presented for the proposed method.*

1. Introduction

A fuzzy system consists of linguistic IF-THEN rules that have fuzzy antecedent and consequent parts. It is a static nonlinear mapping from the input space to the output space. The inputs and outputs are crisp real numbers and not fuzzy sets. The fuzzification block converts the crisp inputs to fuzzy sets and then the inference mechanism uses the fuzzy rules in the rule-base to produce fuzzy conclusions or fuzzy aggregations and finally the defuzzification block converts these fuzzy conclusions into the crisp outputs. The fuzzy system with singleton fuzzifier, product inference engine, center average defuzzifier and Gaussian membership functions is called as standard fuzzy system [14].

Two main advantages of fuzzy systems for the control and modeling applications are (i) fuzzy systems are useful for uncertain or approximate reasoning, especially for the system with a mathematical model that is difficult to derive and (ii) fuzzy logic allows decision making with the estimated values under incomplete or uncertain information [17]. Fuzzy controllers are rule-based nonlinear controllers, therefore their main application should be the control of nonlinear systems. However, since linear systems are good approximations of nonlinear systems around the operating points, it is of interest to study fuzzy control of linear systems. Additionally, fuzzy controllers due to their nonlinear nature may be more robust than linear controllers even if the plant is linear. Furthermore, fuzzy controllers designed for linear systems may be used as initial controllers for nonlinear adaptive fuzzy control systems where on-line turning is employed to improve the controller performance. Therefore, a systematic fuzzy controllers for linear systems is of theoretical and practical interest. Stability and optimality are the most important requirements in any control system. Stable fuzzy control of linear systems has been studied by a number of researchers. It is well-known that nowadays that fuzzy controllers are universal nonlinear controllers. All these studies are preliminary in nature and deeper studies can be done. For optimality, it seems that the field of optimal fuzzy control is totally open.

Singular systems contain a mixture of algebraic and differential equations. In that sense, the algebraic equations represent the constraints to the solution of the differential part. Singular system is also called as differential algebraic system. The complex nature of singular system causes many difficulties in the analytical and numerical treatment of such systems, particularly when there is a need for their control. The system arises naturally as a linear approximation of system models or linear system models in many applications such as electrical networks, aircraft dynamics, neutral delay systems, chemical, thermal and diffusion processes, large scale systems, robotics, biology, etc., (see [2,3,4,8]).

As the theory of optimal control of linear systems with quadratic performance criteria is well developed, the results are most complete and close to use in many practical designing problems. The theory of the quadratic cost control problem has been treated as a more interesting problem and the optimal feedback with minimum cost control has been characterized by the solution of a Riccati equation. Da Prato and Ichikawa [5] showed that the optimal feedback control and the minimum cost are characterized by the solution of a Riccati equation. Solving the Matrix Riccati Differential Equation (MRDE) is a central issue in optimal control theory. The needs for solving such equations often arise in analysis and synthesis such as linear-quadratic optimal control systems, robust control systems with H_2 and H_∞ control [18] performance criteria, stochastic filtering and control systems, model reduction, differential games etc. One of the most intensely studied nonlinear matrix equations arising in Mathematics and Engineering is the Riccati equation. This equation, in one form or another, has an important role in optimal control problems, multivariable and large scale systems, scattering theory, estimation, detection, transportation and radiative transfer [6]. The solution of this equation is difficult to obtain from two points of view. One is nonlinear, and the other is in matrix form. Most general methods to solve MRDE with a terminal boundary condition are obtained on transforming MRDE into an equivalent linear differential Hamiltonian system. By using this approach, the solution of MRDE is obtained by partitioning the transition matrix of the associated Hamiltonian system [13]. Another class of methods is based on transforming MRDE into a linear matrix differential equation and then solving MRDE analytically or computationally [9,11,12]. However, the method in [10] is restricted for cases when certain coefficients of MRDE are non-singular. In [7], an analytic procedure of solving the MRDE of the linear quadratic control problem for homing missile systems is presented. The solution K(t) of MRDE is obtained by using K(t)=p(t)/f(t), where f(t) and p(t) are solutions of certain first order ordinary linear differential equations. However, the given technique is restricted to single input.

Simulink is a MATLAB add-on package that many professional engineers use to model dynamical processes in control systems. Simulink allows to create a block diagram representation of a system and run simulations very easily. Simulink is really translating block diagram into a system of ordinary differential equations. Simulink is the tool of choice for control system design, digital signal processing (DSP) design, communication system design and other simulation applications [1]. This paper focuses upon the implementation of Simulink approach to compute optimal control for linear singular fuzzy system.

Although parallel algorithms can compute the solutions faster than sequential algorithms, there have been no report on Simulink solutions for MRDE. This paper focuses upon the implementation of Simulink approach for solving MRDE in order to get the optimal solution.

This paper is organized as follows. In section 2, the statement of the problem is given. In section 3, solution of the MRDE is presented. In section 4, numerical example is discussed. The final conclusion section demonstrates the efficiency of the method.

2. Statement of the Problem

Consider the linear singular fuzzy system [15,16] that can be expressed in the form:

R^i : If x_j is $T_{ji}(m_{ji},\sigma_{ji})$, i = 1,...,r and j = 1,...,n, then

$$E_i\, x'(t)=A_i x(t)+B_i u(t), \quad x(0)=x_0, \tag{2.1}$$

where the matrix E_i is singular, $x(t) \in \Re^n$ is a generalized state space vector and $u(t) \in \Re^m$ is a control variable. $A_i \in \Re^{nxn}$ and $B_i \in \Re^{nxm}$ are known as coefficient matrices associated with x(t) and u(t) respectively, x_0 is given initial state vector and $m \le n$.

If all state variables are measurable, then a linear state feedback control law

$$u(t)= - R^{-1} B_i^T \lambda(t) \tag{2.2}$$

can be obtained to the system described by equation (2.1), where

$$\lambda(t)= K_i(t)E_i x(t), \tag{2.3}$$

$K_i(t) \in \Re^{nxn}$ matrix such that $K_i(t_f)= E_i^T S B_i$.

In order to minimize both state and control signals of the feedback control system, a quadratic performance index is usually minimized:

$$J= ½\ x^T(t_f)E_i^T S E_i x(t_f)+ ½ \int_0^{t_f}[x^T(t)Qx(t)+u^T(t)Ru(t)]dt,$$

where the superscript T denotes the transpose operator, $S \in \Re^{nxn}$ and $Q \in \Re^{nxn}$ are symmetric and positive definite (or semidefinite) weighting matrices for x(t), $R \in \Re^{mxm}$ is a symmetric and positive definite weighting matrix for u(t). It will be assumed that $|sE_i - A_i| \ne 0$ for some s. This assumption guarantees that any input u(t) will generate one and only one state trajectory x(t).

It is well known in the control literature that to minimize J is equivalent to minimize the Hamiltonian equation

$$H(x(t),u(t), \lambda(t))= ½\ [x^T(t)Qx(t)+u^T(t)Ru(t)]+ \lambda^T(t)[A_i x(t)+B_i u(t)]$$

The necessary conditions for optimality is

$$\partial H/\partial u =0$$

implies that

$$Ru(t)+B_i^T\lambda(t)=0 \text{ and } \partial H/\partial x= E_i^T\lambda'(t)$$

$$\Rightarrow E_i^T\lambda'(t) = -Qx(t) - A_i^T\lambda(t) \tag{2.4}$$

$$\partial H / \partial\lambda = E_i\, x'(t)$$

$$\Rightarrow E_i\, x'(t) = A_ix(t)+B_iu(t)]$$

and from (2.2), we have

$$E_i\, x'(t) = A_ix(t) - B_iR^{-1}B_i^T\lambda(t). \tag{2.5}$$

Equations (2.4) and (2.5) can be written in a matrix form as follows :

$$\begin{pmatrix} E_i & 0 \\ 0 & E_i^T \end{pmatrix} \begin{pmatrix} x'(t) \\ \lambda'(t) \end{pmatrix} = \begin{pmatrix} A_i & -B_iR^{-1}B_i^T \\ -Q & -A_i^T \end{pmatrix} \begin{pmatrix} x(t) \\ \lambda(t) \end{pmatrix}$$

Assuming that $|R| \neq 0$, from (2.3) we have

$$\lambda'(t)= K_i'(t)\, E_i\, x(t)+ K_i(t)\, E_i\, x'(t)$$

and

$$E_i^T\lambda'(t) = E_i^TK_i'(t)E_i\, x\,(t) + E_i^TK_i(t)E_i\, x'(t) \tag{2.6}$$

Using the equations (2.3) – (2.5) in (2.6), we obtain

$$[E_i^TK_i'(t)E_i + E_i^TK_i(t)A_i + A_i^TK_i(t)E_i +Q - E_i^TK_i(t)\, B_iR^{-1}B_i^TK_i(t)E_i\,]\, x(t) = 0 \tag{2.7}$$

Since equation (2.7) holds for all non-zero x(t), the term pre-multiplying x(t) must be zero. Therefore, we obtain the following MRDE for the linear singular system (2.1)

$$E_i^TK_i'(t)E_i + E_i^TK_i(t)A_i + A_i^TK_i(t)E_i +Q - E_i^TK_i(t)\, B_iR^{-1}B_i^TK_i(t)E_i = 0 \tag{2.8}$$

In the following section, the MRDE (2.8) is going to be solved for $K_i(t)$ in order to get the optimal control of the singular system.

3. Solution of MRDE by Simulink

Simulink is an interactive tool for modelling, simulating and analyzing dynamic systems. It enables engineers to build graphical block diagrams, evaluate system performance and refine their designs. Simulink integrates seamlessly with MATLAB and is tightly integrated with state flow for modelling event driven behavior. Simulink is built on top of MATLAB. A Simulink model for the given problem can be constructed using building blocks from the Simulink library. The solution curves can be obtained from the model without writing any codes.

A Simulink model is constructed for the following system of two differential equations as shown in Figure 1.

$$x'(t) = -x(t) + 1, \quad x(0) = -1$$
$$y'(t) = -y(t) + 1, \quad y(0) = 1.$$

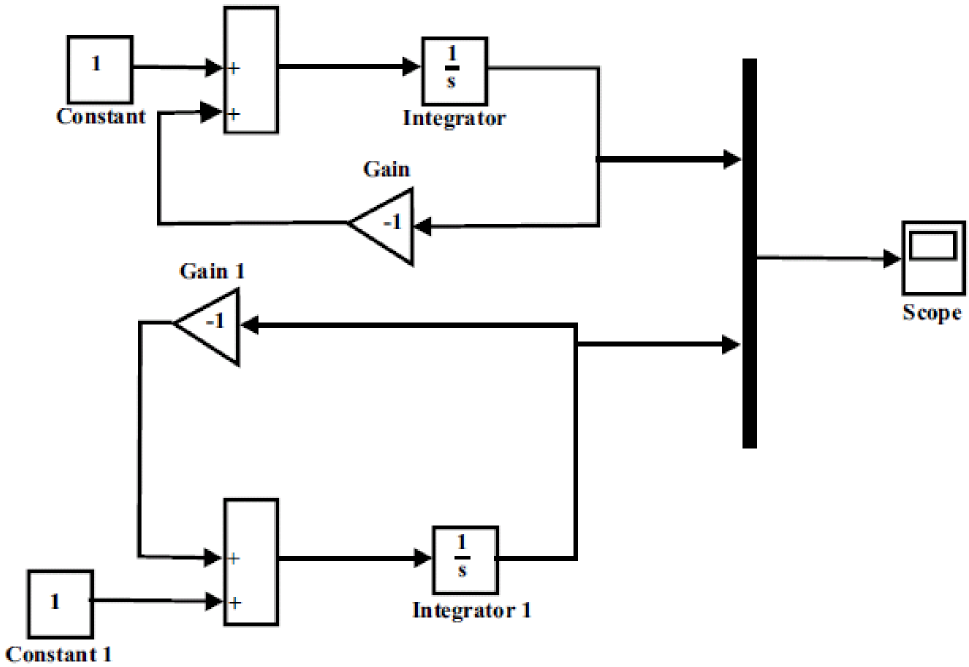

Figure 1: Simulink Model

As soon as the model is constructed, the Simulink parameters can be changed according to the problem. The solution of the system of differential equation can be obtained in the display block by running the model.

3.1 Procedure for Simulink Solution

Step 1. Select the required number of blocks from the Simulink Library.

Step 2. Connect the appropriate blocks.

Step 3. Make the required changes in the simulation parameters.

Step 4. Run the Simulink model to obtain the solution.

4. Numerical Example

Consider the optimal control problem:
Minimize

$$J = \tfrac{1}{2}\, x^T(t_f)E_i^T S E_i x(t_f) + \tfrac{1}{2}\int_0^{t_f} [x^T(t)Qx(t) + u^T(t)Ru(t)]dt,$$

subject to the linear singular fuzzy system R^i : If x_j is $T_{ji}(m_{ji},\sigma_{ji})$, i = 1, 2 and j = 1, 2,3, then

$$E_i\, x'(t) = A_i x(t) + B_i u(t), \quad x(0) = x_0 ,$$

where

$$S = \begin{pmatrix} 1.1517 & 0.1517 \\ 0.1517 & 1 \end{pmatrix},\ E_i = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},\ A_1 = \begin{pmatrix} -1 & 1 \\ 0 & -2 \end{pmatrix},\ A_2 = \begin{pmatrix} -2 & 2 \\ 0 & -4 \end{pmatrix},\ B_i = \begin{pmatrix} 0 \\ 1 \end{pmatrix},\ R=1,\ Q = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

The numerical implementation could be adapted by taking t_f =2 for solving the related MRDE of the above linear singular fuzzy system with the matrix A_1 . The appropriate matrices are substituted in MRDE. The MRDE is transformed into differentia algebraic equation (DAE) in k_{11} and k_{12}. The DAE can be changed into a system of differential equations by differentiating the algebraic equation. In this problem, the value of k_{22} of the symmetric matrix K(t) is free and let k_{22}=0. Then the optimal control of the system can be found out by the solution of MRDE.

4.1 Solution Obtained Using Simulink

The Simulink model is constructed for MRDE. The Simulink model is shown in Figure 2. The numerical solution of MRDE is calculated by Simulink and displayed in Table 1. The numerical solution curve of MRDE by Simulink is illustrated in Figure 3.

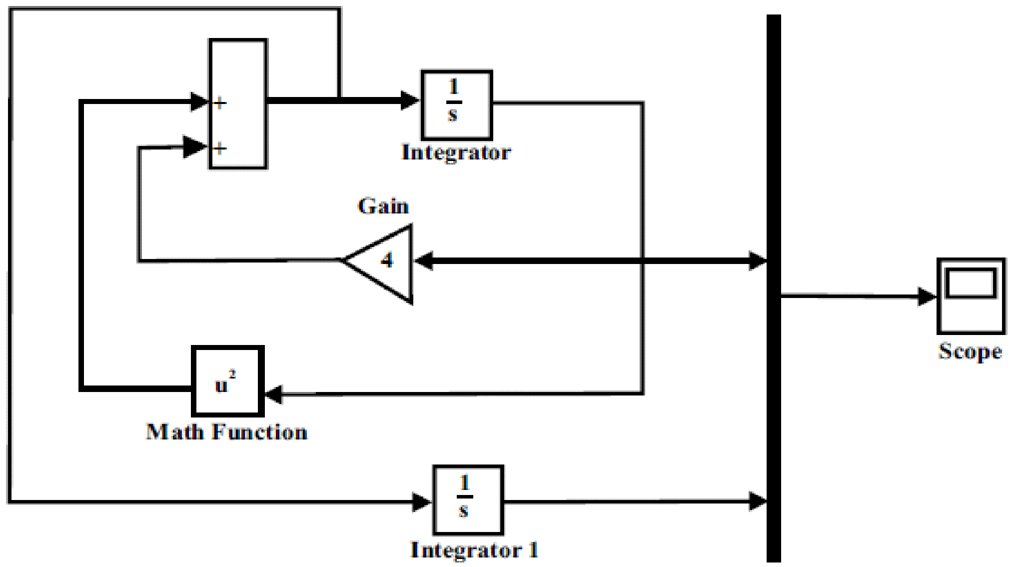

Figure 2 : Simulink Model for MRDE

t	k_{11}	k_{12}
0.0	0.0003	-0.9997
0.2	0.0007	-0.9993
0.4	0.0015	-0.9985
0.6	0.0033	-0.9967
0.8	0.0074	-0.9926
1.0	0.0164	-0.9836
1.2	0.0368	-0.9632
1.4	0.0828	-0.9172
1.6	0.1891	-0.8109
1.8	0.4467	-0.5533
2.0	1.1517	0.1517

Table 1: Simulink Solution of MRDE

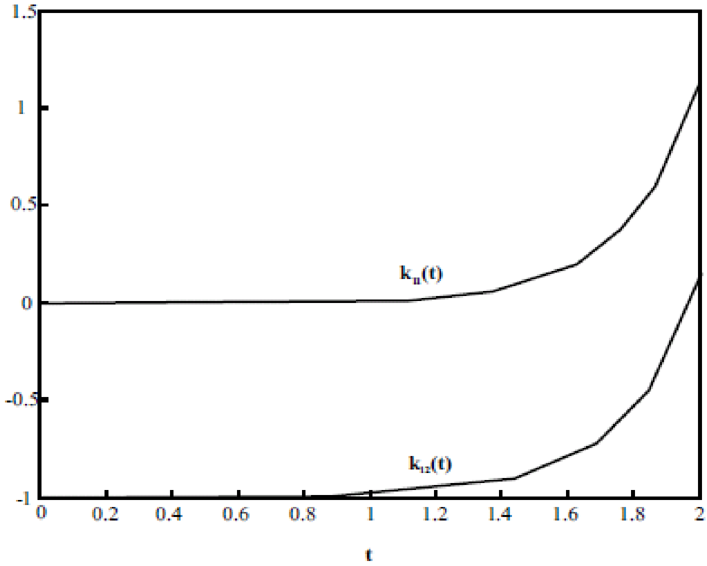

Figure 3: Simulink Curve for MRDE

Similarly the solution of the above system with the matrix A_2 can be found out using Simulink.

5. Conclusion

The optimal control for the linear singular fuzzy system can be found by Simulink approach. To obtain the optimal control, the solution of MRDE is computed by solving Differential algebraic equation (DAE) using Simulink. The Simulink solution is equivalent to the exact solution of the problem. Accuracy of the solution computed by Simulink approach to the problem is qualitatively better. A numerical example is given to illustrate the proposed method.

Acknowledgement: The funding of this work by the UMRG grant (Account No: RG099/10AFR) is gratefully acknowledged.

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