

# Reducing Points In a Handwritten Curve (Improvement in a Note-taking Tool)

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**Abstract:** *To improve the performance of a handwriting input system for note takers, we attempt to reduce the number of points system contains. We construct an evaluation function for a "reasonable line segment" in a handwritten curve to create an adequate polygonal line for an input curve. We acquire training data using our experimental system, and we assign an evaluating value using "error sort model", that will be defined in the paper. Then we develop a method for reducing the number of points in the handwriting system. This method reflects the intentions of human being.*

## 1. Introduction

Students with impaired hearing need note-taking services during a class to aid oral communication. Note takers, i.e.; staff appointed to provide note-taking services, write down what teachers talk during a class. There are various software tools for "Electronic note-taking"[1]. Users can store and print out the contents, and some of them are online (sometimes it is wireless) system, and keyboard input, handwriting input are available. These tools have the following benefits: (1) one note taker can provide information to several students at a time, (2) written documents can be easily stored as electric data, and (3) two or more support stuffs do not have to change their seats when they are supporting students in rotation.

In a scientific class, we use some formulae and figures even in oral communication. In this case, since we can hardly express these contents by a text-only interface. In a conventional hand-writing input system, one inputs a curve as a family of points and the system performance depends on the number of points, since the note-taking task is performed very quickly. We attempt to find the smallest possible number of points and their optimal positions. One difficult problem is to decide whether or not the input curves are reasonable. It must be smooth if the drawing curve is a part of a circle, and at the vertex of a triangle, it must be sharply-peaked. It may be preferable to remove small bends if the curve is a straight line. In this study we will develop an evaluation function for reducing the points of the curves. This function will quantify the unnaturalness or unreasonableness. We attempt to understand the reason for reducing the points of the curves. Assume that a drawing line is given as a set of points and that the system selects some of these points to create an approximating polygonal line. The argument of the estimation function is each line segment in this polygonal line. The geometric features of this polygonal line will be valid for the approximation of a raw line. However, the raw line sometimes differs from intended line, causing the roughly skipped curve to be more reasonable. We developed a system to obtain data of approximating line segments for a drawn curve. Using this system, one can draw a curve using a mouse or an ultrasonic pen, select some dividing points manually, create an approximating polygonal line. Some test subjects obtained the data of line segments using this system. From the

collected data, it was found that some irregular line segments existed, and we selected some of them. The test subject checked these line segments and sorted them in descending order of unnaturalness.

Using the sorted data, we will construct the estimation function. Let us assume that there is a correct estimation function and that every test subject obtains these values with errors. He sorts the data on the basis of these values. The sorting order differs from person to person, however, for any two target line segments, there is only one correct sorting order based on the supposed estimation function. The ratio of the appearance of the reverse order depends on the difference in the values of the estimation function. Therefore, we can approximate the values of evaluation function for sample data using the method of maximum likelihood. Subsequently, the general evaluation function is expressed by the least squares method using generalized Choquet integral with respect to two-additive measure[2].

## 2. Sketch of Total System

We have arranged a handwriting note-taking tool for a scientific class [1]. This tool is a stand alone Windows application. We assume that a note taker inputs and edit contents, and a person with impaired hearing reads them. A client area consists of two pages and a note taker inputs the contents with ultrasonic pen in the right hand side page(Fig 2.1).

Several clipboard systems are available for reusing the inputted contents.

1. We have the standard cut, copy and paste functions, and the target contents are stored in the Windows Clipboard.
2. They display 5 icons representing additional memory areas (Fig 2.1).
3. The contents stored in the Windows Clipboard are character strings, and we can use any text editor as a temporal memory area.

Each task must be performed very quickly, and if the text-based expression is space-consuming, then the number of points must be small. Moreover, we want best possible reasonable shape with the least possible number of points, especially for a symbol that we use repeatedly.

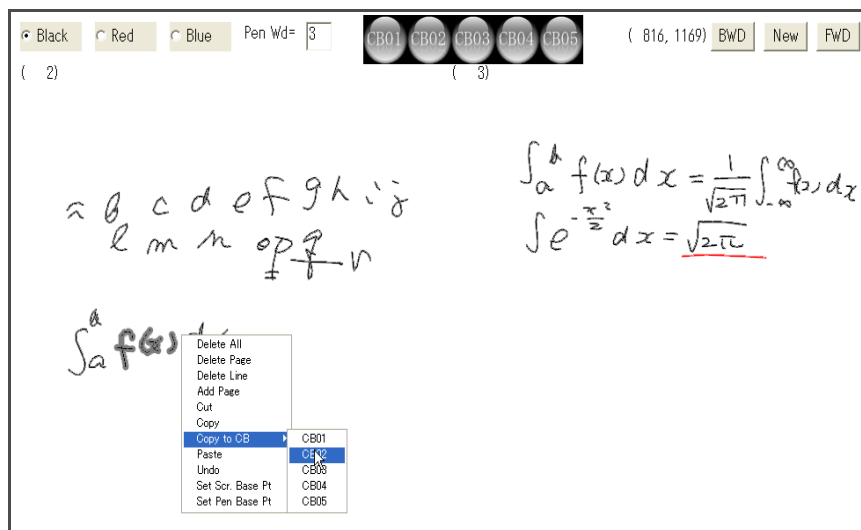


Figure 2.1 System Layout

### 3. A System for Polygonal Line Data

We developed a system to create polygonal line data. Using this system, test subjects draw curves according to the model BMP images, and they select dividing points. Then, we can obtain data of reasonable shapes of polygonal lines. In this section we will explain the their details.

#### 3.1 Guide BMP Pictures

We prepare some model shapes for the curve data. If the data include all the typical aspects of the curves and a sufficient number of test subjects draw them, we would only need to develop a function fitting these data. However, it is very difficult to assemble sufficient data owing to the presence of shapes and some of them rarely appear. Then, the model images with various curve types can be valid to assemble curve data. A model picture is read into the system by typing the file name.

#### 3.2 Drawing a Line

Before drawing a line, we have to fit its position. There are 4 cross marks at the 4 corners of every BMP image. These cross marks must be clicked with the left button of mouse to fit the position of mouse(Fig. 3.1).

This system comprises an ultrasonic pen, which is a ball point pen with ultrasonic generator (Pegasus Technologies), we can input handwriting lines using the pen. We use a printed paper of the model BMP image for this input. The cross mark at the 4 corners of the image in the paper must be clicked with the ultra sonic pen, to fit the position. Then, test subjects input handwritten curve according to the model image .

#### 3.3 Select Dividing Points

An input curve is a set of points and equally spaced dividing points are given when a polygonal line is drawn. The default number of dividing points is displayed in a text box and the dividing points are initialized when this number is changed. Each dividing point is marked with a small circle. These small circles can be moved by dragging the mouse over them, and a test subject arranges these points to create a reasonable polygonal line (Fig.3.2). After setting the polygonal line, the total number of all points, the total number and the indices of dividing points, all coordinates of the points and the time data are stored in a text file.

#### 3.4 Check the Data

To estimate the polygonal lines, a test subject compare several line segments. Among the several line segments in a polygonal line, one line segment is displayed in a different color and the corresponding index of the point is displayed in a text box. A test subject changes the polygonal line and the line segment to compare their smoothness or reasonability.

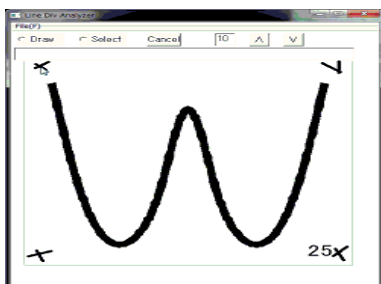


Figure 3.1 Model Image

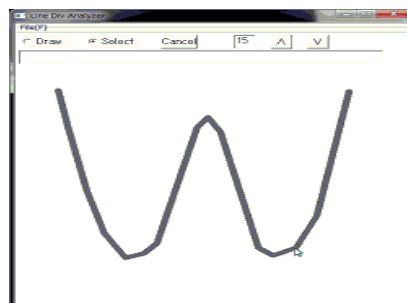


Figure 3.2 Line Segments

## 4. Training Data

We will develop an evaluation function for an interval as a part of a reasonable polygonal line. First, we obtain some reasonable and unreasonable polygonal lines drawn by test subjects. Next, we select several line segments from among. These line segments are sorted by the test subjects, and using the error sort model (our new method), we obtain the evaluating values for these line segments.

number of person	24
age	22- 24
etat	student
department	computer science and system engineering

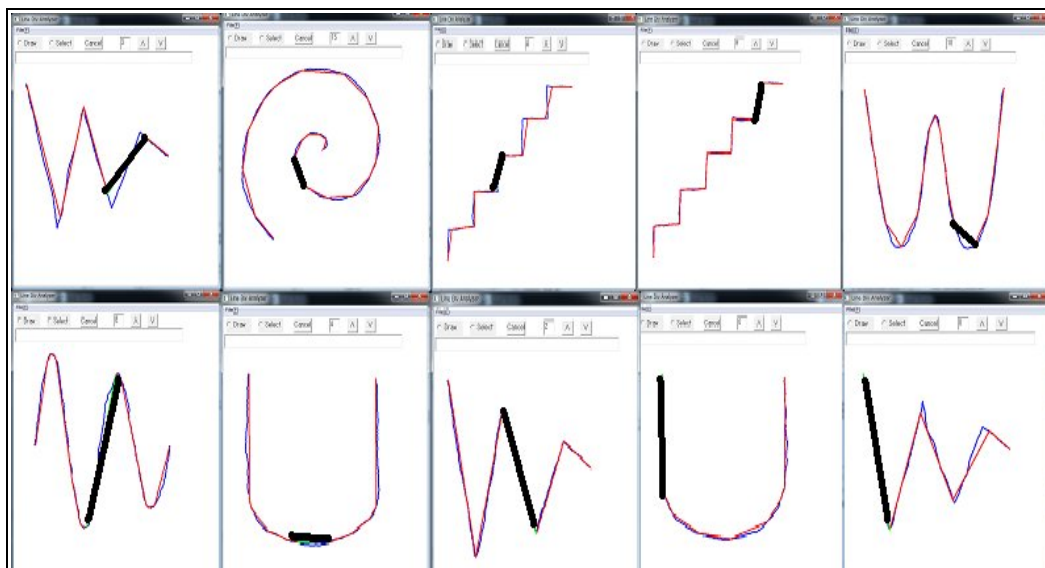
**Table 4.1** Test Subjects

### 4.1 Polygonal Line Data

Using the system for obtaining polygonal line data, several test subjects set polygonal lines according to guide BMP images. For the variation of the data, each one makes perfect lines and imperfect lines. The following table is their details for test subjects.

### 4.2 Sorting Line Segments

Among all the polygonal line data, we select 32 line segments. Some of them are unreasonable because of the miss choice of dividing points. For this set of line segments, each test subject sort them according to his own judgment. No rules or criteria are provided.



**Figure 4.1** Target Line Segments (Sample)

### 4.3 Error Sort Model and Evaluation Values

Let us assume that all the line segments have their correct evaluation values and that the test subjects observe them with errors. We attempt to approximate the line segments using the sorting data obtained by the test subjects.

In general, let  $\vec{x}=(x_j)_{j \leq n}$  be a set of evaluation values. Let  $\{\vec{\varepsilon}_k\}_{k < K} = \{(\varepsilon_k^{(j)})_{j \leq n}\}_{k < K}$  be a sequence of independent random vectors whose distribution is  $N(0, I)$  ( $0$  is the  $n$ -dimensional zero vector and  $I$  is an  $n \times n$  unit matrix). We consider  $n$  targets for the evaluation and that  $x_j + \varepsilon_k^{(j)}$  is the evaluation value for the  $j$ -th target of  $k$ -th test subject. Moreover, we assume that  $\{x_j\}_{j \leq n}$  is an increasing sequence and  $a_{i,j}$  ( $i < j$ ) is defined as the number of  $k$  such that  $x_i + \varepsilon_k^{(i)} > x_j + \varepsilon_k^{(j)}$ . Then, the smaller the difference between  $x_i$  and  $x_j$  is, the larger the reverse order number  $a_{i,j}$  may be. Our problem is to approximate the values of  $(x_j)$  using the sorted data of  $\{x_j + \varepsilon_k^{(j)}\}$ . Then using maximum likelihood method we can approximate  $\{x_j\}_{j \leq n}$ .

## 5. Features for Evaluation

The evaluation function for a reasonable polygonal line quantifies its reasonability for each line segment in a polygonal line. An original curve is a set of points, and a line segment is a selection of two points among them. The evaluating value is determined by the information regarding the curve between these two points. We also available the corresponding time information.

### 5.1 Basic Features.

Let  $L$  be the line segment,  $n$  be the number of points between two edge points of  $L$ ,  $\{\vec{p}_k\}_{k \leq n}$  be a set of position vector, and  $\{t_k\}_{k \leq n}$  be the corresponding time records. Then, we use the following feature values.

1. Maximum distance:  $\max_k \text{dist}(L, \vec{p}_k)$ .

2. Maximum angle difference:  $\max_k |\text{angle}(\vec{p}_n - \vec{p}_0) - \text{angle}(\vec{p}_{k+1} - \vec{p}_{k-1})|$ ,

where  $\text{angle}(\vec{v}) = \tan^{-1}(\frac{v_x}{v_y})$  ( $\vec{v} = (v_x, v_y)$ ) and the subtractions of angles are generalized considering circular identification.

3. Maximum drawing speed:  $\max_k \frac{|\vec{p}_{k+1} - \vec{p}_{k-1}|}{t_{k+1} - t_{k-1}}$ .

4. Maximum acceleration:  $\max_k \frac{\frac{|\vec{p}_{k+1} - \vec{p}_k|}{t_{k+1} - t_k} - \frac{|\vec{p}_k - \vec{p}_{k-1}|}{t_k - t_{k-1}}}{(t_{k+1} - t_{k-1})/2}$ .

5. Maximum angle change:  $\max_{j < k \leq n} |\text{angle}(\vec{p}_{k+1} - \vec{p}_{k-1}) - \text{angle}(\vec{p}_{j+1} - \vec{p}_{j-1})|$ .

6. Maximum angular speed:  $\max_{k \leq n} \frac{|\text{angle}(\vec{p}_{k+1} - \vec{p}_k) - \text{angle}(\vec{p}_k - \vec{p}_{k-1})|}{t_{k+1} - t_{k-1}}$

7. Drawing time:  $t_n - t_0$

## 5.2 Normalization

We use a generalized Choquet integral with respect to a two-additive measure [2] for an evaluation function. Then, an integrand function must take value in  $[0,1]$ . We set truncated linear transform that takes values in  $[0,1]$ , using the percent points (99%) of normal distribution, and all feature values are converted into  $[0,1]$ . Then we obtain the normalized feature vectors from the training data. In the case of the actual use, the situation changes and same transformations are not valid. It is necessary to make some fine adjustments, for which we use experimental values.

## 6. Experimental Results

### 6.1 Outline To Obtain Training Data

As we explained above, the training data were collected through the following procedural steps:

1. Draw curves according to model curves and divide them into several line segments.
2. Select some irregular line segments and fix a family of line segments.
3. Sort these segments and estimate an evaluation value for each line segment using the error sort model.
4. Calculate feature values using the data of original lines.

The following table lists the information and data collected by the test subjects:

Model Line Types (BMP files)	6
Line Segments (Sort target)	32
Feature Values	7
Test Subjects for Curve Drawing	24
Test Subjects for Sorting	24

**Table 6.1** Test Subjects

### 6.2 Approximation of Evaluation Function

We use a generalized Choquet integral with respect to a two additive measure to express an evaluation function. Let  $A$  be a finite and  $\mu$  be a set function defined on  $2^A$  (the set of all subsets of  $A$ ). Assume that  $\mu$  is a two additive measure, that is, there are  $n+n(n-1)/2$  numbers  $\{\mu_x\}_{x \in A}, \{v_{x,y}\}_{x,y \in A}$  such that

$$\mu(B) = \sum_{x \in B} \mu_x + \sum_{\{x,y\} \subset B} v_{x,y}$$

Let  $f$  be a function from  $A$  to  $[0,1]$ , and the Choquet integral  $\int f d\mu$  with respect to t-Norm  $\otimes$  is defined by

$$\int f d\mu = \sum_{x \in A} f(x) \mu_x + \sum_{\{x,y\} \subset A} f(x) \otimes f(y) v_{x,y}.$$

In the case where  $\otimes$  is "minimum", this is a standard Choquet integral. The training data are several sets of 7 feature values  $\{f_k(x): x \in A\}$ , whose values are adjusted to  $[0,1]$ , and evaluation values  $y_k$  ( $k \leq K$ ). Then using extended feature vectors:

$$\{f_k(x), f_k(x) \otimes f_k(y): x, y \in A\} \quad (n+n(n-1)/2 \text{ -dimensional vectors})$$

and the evaluation value  $y_k$ , we obtain an optimal two-additive measure by the least squares method (Table 6.2)

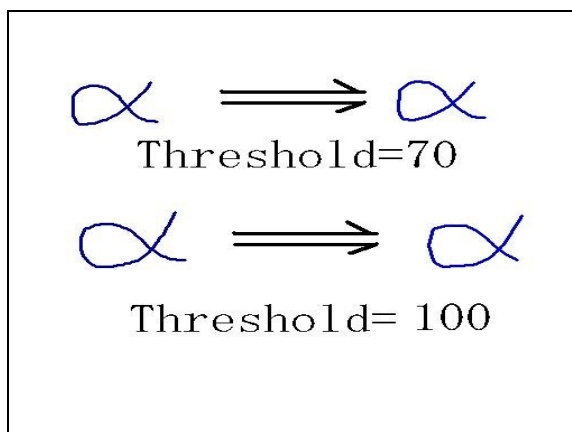
point mass		main adjustment terms		parameter =3.08 least square error =3.732
1	2.59E+02	1,2	1.60E+03	
2	4.39E+02	2,3	1.40E+03	
3	2.45E+02	1,5	-1.99E+03	
4	9.65E+01	3,5	-1.36E+03	
5	9.07E+01	4,5	-1.86E+03	
6	-1.82E+02	2,6	-1.11E+03	
7	1.42E+02	5,6	1.03E+03	

**Table 6.2** Coefficients

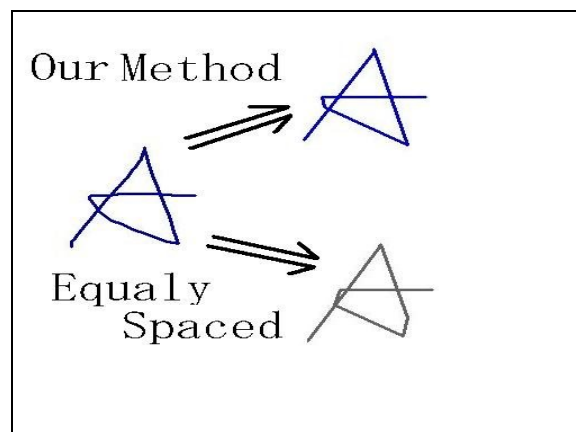
### 6.3 Point Reduction for Drawn Curves

We reduce the number of drawn curves using the evaluation function defined in previous section. According to the increase of drawn points, the value of the evaluation function changes. When the value of the evaluation function exceeds the threshold value, the last point is the end point of current line segment and the start point of new line segments. Figure 6.1 is an example of the point reduction. If we ignore some key points under the reduction of curve points, the reduced curves sometimes are very unnatural. Figure 6.2 is an example of unnatural point reduction. The upper one is a reduced curve by our rule, and the other one is equally-divided.

Two curves in Figure 6.2 consist of ten points. In the equally-divided curve, some peak points disappear because there are no dividing points near the corresponding peak points. Then the curve is unnatural comparing with original curve (the left hand side curve). If there are no rules to reduce the curve points, we need more dividing points to obtain natural curve.



**Figure 6.1** Point Reduction



**Figure 6.2** 10 points Curves

## **7. Conclusion**

We constructed an evaluation function for a line segment as a part of a reasonable polygonal curve. Their training values of evaluation values are based on ranking and will reflect human sensibility. We only need to obtain adequate threshold value to divide the drawn curves into reasonable sets of line segments.

## **References**

- [1] Comparison of notetaking software, Web Page,  
[http://en.wikipedia.org/wiki/Comparison\\_of\\_notetaking\\_software](http://en.wikipedia.org/wiki/Comparison_of_notetaking_software)
- [2] Ryoji Fukuda, Koji Nii (2009), Handwriting Tool For Note Takers in Mathematical Classes, Proceedings of the 14th Asian Technology Conference in Mathematics.