Scientific Inquiry in Mathematics: A Case of Implementing Scientific Simulations for Analyzing Problems on Motion

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Abstract: *This study examines the current methods on solving problems on motion presented in some mathematics textbooks currently used in the US. We concluded that a wide diversity in the methods as well as the methods' lack of coherence with the scientific inquiry process results in students' weak understanding of solving these problems. This conclusion is supported by findings of a pre-test conducted on a sample of 80 pre-calculus students enrolled in one of the high schools in Texas. It is hypothesized that if the interpretation of these problems is supported by physics methods as well as by extensive graphical representations, the strategies of solving motion problems will be better understood by students. The proposed method addresses one of the most fundamental mathematics objectives stated in the Texas Education Agency's newly developed Texas End-of-Course Algebra Assessments, which requires students to analyze situations involving linear relationships and formulate linear functions. The suggested strategy is also aligned with the calculus interpretation of kinematics quantities. A significant percent gain of students' scores on post-test instruction proves that the hypothesis of the research is correct. Consequently, it is implied that students learn science application problems in mathematics classes successfully if presented methods that integrate science content and if they are parallel with students' prior knowledge.*

Background

The methodologies of mathematics and science are closely related. Mathematical concepts are utilized to model data generated from scientific experiments and formulate them in a concise and meaningful language of functions and formulas. A scientific law or scientific principle defined as a mathematical statement of a relation that expresses a fundamental principle of science [1] provides an opportunity for hypothetical thinking, a process seldom encountered by students in mathematics classes. The theoretical framework used to organize our study is based on a constructivist perspective in which learning is viewed as a process of experiencing dissonance and working to resolve these dissonances by building workable explanations (mathematical functions) [2]. There is a wide range of research pertaining to students' comprehension of graphing motion [3]. The findings show that students have difficulties reading and interpreting graphs of motion. For instance, several researchers have proven the tendency for students to interpret the position-time graph as a picture of the actual path of objects' motion [4]. In our proposal, we will also address this issue and will try to correct this misconception. Other research studies have shown that students also have difficulty understanding derived graphs, for example, a velocity-time graph derived from a position-time graph or an acceleration-time graph derived from a velocity graph. Researchers [5] proved that this difficulty is related to students' weak prior understanding of the concept of rate. It is notable that mathematical functions involving rates are usually not constructed due to gathered direct data; thus, their intuitive understanding might be difficult for students.

As teaching real-world application problems is difficult in mathematics classes [6], it seems that developing skills of constructing mathematical functions through the support of scientific inquiry and inductive reasoning from observations and calculations might help with this process. Furthermore, such created environment of mathematical modeling might help students not only in mathematics but also in science, where applying the process of inquiry is the primary method of knowledge acquisition. Research shows that even when the problems and methods encountered in class are similar to the real world, students tend to dissociate their real-world thinking from their problem-solving techniques [7]. It seems that integrating scientific inquiry with mathematical graphical representations of the problems might be a bridge solidifying the process of creating mathematical models and solving them.

Diversity of Representations of Motion Problems

Problem solving in introductory mathematics and science courses (at the college level) is often algorithmic or procedural; students are assigned lots of problems, which they solve by mimicking the way similar problems are solved by the teacher and the textbook [8]. The context of motion is typical for problem solving in high school mathematics textbooks. In most of these textbooks, problems on motion are included in the introductory sections called *Modeling with Equations.* Although no clear differentiation in difficulty level (e.g., by grade level) has been noted, motion problems can be grouped into three main categories, depending on the number of objects and variables:

1. Motion of one object with (usually) a piece-wise representation
2. Motion of two objects whose initial velocities and positions vary
3. Motion of one or two objects whose velocities are affected by a medium (wind or water stream)

Is there coherence in the methods used to present the solutions to these problems? Are these problems supported by a unified approach consistent with science methods?

After analyzing selected mathematics textbooks [9, 10, 11, 12, 13, 14, 15], we concluded that there is a range of methods that students learn while transferring from one mathematics level to another. These methods often contradict themselves, “forcing students to memorize solutions patterns to stereotyped versions of these problems” [16, p. 248]. We agree here with [8] that most of the assigned problems are strictly quantitative and have solutions that can be obtained by selecting and manipulating one or two appropriate equations. This type of problem solving does not result in a deep understanding of concepts nor in building robust problem-solving skills. While transitioning from pre-algebra to calculus class, students do not relay upon previously acquired skills and methods, but instead they learn new strategies from the beginning. Employed methodologies do not highlight, for instance, an establishment of frame of reference that is essential in determining objects’ positions or direction of motion. The difficulty level is also not considered when assigning these problems to students. For instance, category 3 problems (as classified above), usually analyzed in an advanced physics class [17] with intense vector representations, were found in an Algebra 1 textbook [9].

A Closer Look

Physics Interpretation of Direction of Motion

In their physics classes, students learn that an object's motion has a definite direction and position determined by an established frame of reference. By convention, forward motion is associated with a positive velocity and backward motion is associated with a negative velocity [18]. Respectively, if an object moves along a straight line in an eastward or northward direction, its velocity is considered positive, as opposed to an object's velocity moving in a westward or southward direction, which is considered negative. This distinction is fundamental to correctly constructing position and velocity functions of one or two moving objects. (It is understood that while studying *related rates* in calculus, other conditions determine the signs of an object's velocities. It is usually dictated by an increase or decrease of an object's distance as measured with reference to established axes.)

Lack of Consistency in Labeling the Direction of Motion in Mathematics Textbooks

In the pre-algebra textbook *PreAlgebra* [12], in example 1 on page 158, students are given the speed of an object moving *forward* at 25 feet per second and the distance it traveled. The suggested function describing the object position is $d = 500 - 25t$. Although the author restricts the interpretation of d as the distance left from the object's destination, the negative sign for the speed contradicts with the vector nature of velocity. A similarly misleading interpretation can be found in the textbook *Algebra 2* [10] on page 53 (example 3), where again forward motion is represented by a negative velocity: $d(t) = 380 - 156t$. In another textbook, *Health Algebra 1* [13], the velocities of two objects moving toward each other are both considered with positive values. A similar approach can be found in *Precalculus* [14] on page 65, where forward and backward motion are both considered with positive velocities. A consistent interpretation of velocity was found in the textbook *College Algebra Enhanced with Graphing Utilities* [15], where on page 144 forward motion is given a positive value.

Lack of Consistencies in Applied Methods

Problems on motion are usually not supported by functional analysis, although they generate great content for exercising this fundamental mathematical tool. The applied strategy often refers to equating objects' distances or time intervals without establishing position-time graphs for the motions. Accompanying graphics depict scenarios, not mathematical representations of these problems.

In [15], on page 144 in example 5, students construct two equations to analyze the motion of two objects applying *rate, time, and distance*. This approach suggests using applications of properties *of proportions* to solve the problem, rather than properties of linear functions, which are a much broader tool in algebra. In [14], on page 66, the process of finding the speed of an object with varying forward and backward velocities is supported by the construction of a complex rational equation of the form:

$\dfrac{4200}{s} + \dfrac{4200}{s+100} = 13$ (where s represents unknown speed).

Although a rationale can be provided for this equation, its complex form does not shed any light on the importance of rate of change of an object's motion utilized in the process of constructing the equation.

Furthermore, the properties of linear functions and the utilization of graphing technology that could seemingly enhance the process of visualization of these problems are neglected. There is another argument against teaching these structures; these methods of solving problems on motion are completely abounded in calculus, where the analysis is supported by the process of differentiation or integration of position, velocity, or acceleration functions [19]. Since motion problems are the most common type of problems on the free response sections of AP-calculus exams since 1956 [20], enhancing the process of functional analysis and scientific inquiry, especially in pre-calculus, seems to be beneficial for the students.

Suggested Ways of Improving

After the analysis, we concluded that there was a need for establishing a consistency in the methodology of teaching motion problems across high school mathematics curriculum, not only in considering the vast applications of these problems in mathematics but also in considering coherence of these methods with physics. We propose that if physics simulations that have been created by the Physics Educational Team (PhET) and whose effectiveness is proven by extensive research [21] are introduced to mathematics classes, students will better understand the mechanics of constructing graphs and functions describing motion. Research also shows that if students use interactive computer simulations, they are much more engaged with the concept [22] and are, in fact, doing *simulated research activities* [23].

Research Question

Will implementing scientific inquiry to support graphical analysis, frame of reference, and respective position-time graphs improve students' strategies of solving motion problems?

Research Hypothesis

Just as [24] concluded that "interdisciplinary curriculum fosters intellectual development and students' capacities for critical thinking" (p. 25), we hypothesize that enriching the concepts of motion by physical representations and sound functional analysis will improve their strategies to solve these problems.

Design

Logistics of the Research

The research, which included a diagnostic pre-test, an instructional unit, and a post-test, was conducted in a South Texas high school's college algebra classes consisting of 80 students. The group of students was concurrently taking a dual credit college algebra course through one of the local colleges. A pre-test containing three main problems with three parallel sub-problems was given to the students to assess their current strategies for solving motion problems. The pre-test targeted the following objectives:

- Graphical representation of the solution process
- Creation of system of linear functions expressed algebraically
- Final answer that included interpretation due to physical constrains of the problems

Although all three problems targeted the properties of motion of two objects, each had a unique character. In the first problem, students were to realize that if one object starts later, a horizontal transformation can be applied to create the position function. In the second problem, students were to realize that motion in the southward direction is represented by a negative velocity. In the third problem, velocity of one of the objects was supposed to be expressed in terms of the velocity of the other object.

Although the analysis of the results did not target these details (students were asked to *show their work*), the purpose of the variation was to create an environment where students would critically analyze the contents and then select the appropriate solution strategy. The results of the pre-test were not shared, nor were they discussed with the students before the research was finished.
The post-test was taken after about a 10-day interval and followed a lesson entitled “Motion of Two Objects,” which was designed to enhance graphical representation of category 1 and 2 problems.

Outline of the Lesson “Motion of Two Objects”

The lesson’s main objectives were as follows:

- Students will apply mathematical modeling to transfer physical conditions such as an object’s initial position and velocity in mathematical representations.
- Students will learn to identify intersections as the mathematical solution of the system of equation and interpret it due to physical constraints.

During the lesson, the significance of frame of reference as well as the interpretation of negative and positive velocity was especially emphasized. The main teaching aide of the lesson was a physics simulation called “The Moving Man,” which we created and uploaded at http://phet .colorado.edu/simulations/sims.php?sim=The_Moving_Man. This simulation was selected because of its rich, *naturalistic* scientific context and high variability of the physical quantities.

For independent practice, we created a student-centered lab utilizing the simulation and focused on having the student create a mathematical model as defined by [25]: *mathematical model* is a representation of the essential aspects of an existing system (or a system to be constructed) that presents knowledge of that system in usable form (e.g., mathematical formula). We chose inductive reasoning to support the process of mathematization of scientific representation. Reasoning is a thought process that involves judging, inferring, generalizing, and comparing. Among many types of reasoning, inductive reasoning is one of the most commonly utilized across all subjects and grade levels [26]. Inductive reasoning or induction leads to a general law (statement) derived from specific cases.This type of reasoning also has many applications in engineering. Expanding its applications in high school not only enhances the goals of high school curriculum but also encourages and prepares students to enter the field of engineering.

Instruction units involving inductive teaching usually contain four stages [26] beginning with *focus*. In order to parallel this process to its scientific counterpart, we included a *problem statement* stage in the process. The stages we used are as follows:

1. *Problem statement*: the question that students are going to answer while working on a given task.

2. *Focus*: building (collecting) the data set, studying the attributes of the data set, and formulating the hypothesis.
3. *Conceptual control* (analysis): classifying the facts and identifying patterns of regularity.
4. *Inference*: making a generalization (formulation of a pattern or law) about the relations between the collected facts that leads to acquiring a general (mathematical) function.
5. *Confirmation*: verifying the derived model in new circumstances conducted through testing inference and further observations.

The process began with giving students a real-world problem, for instance, *starting from a position 10m east, a man walks in the westward direction at 1.5 m/s for 12 seconds*. In the process of *focus* and *conceptual control*, students identified a constant rate of change (man's velocity), as well as dependent (his position/location) and independent (time) variables. In the process of *inference*, students selected a linear function of the general form $y = mx + b$. By translating given scientific quantities into a mathematical representation, they obtained $x = -1.5t + 10$. In the process of *interpretation*, they sketched the position function and discussed its limitations (domain). They further *tested* the derived model by answering additional questions. In the processes of *confirmation*, they verified their model by the physics simulation and a graphing technology. Figure 1 below illustrates selected print screens from one of the motion problems that students worked on.

Stage A. *Stage B.*

Figure 1: *Screens of Applied Physics Simulation*

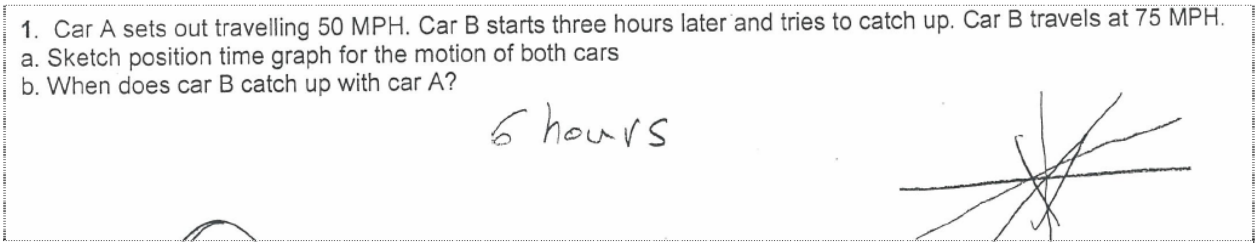
Source: PhET Interactive Simulations, University of Colorado, http://phet.colorado.edu.

The man's initial position and his velocity were assigned due to given values (Stage A). These conditions were entered into the simulation. After every student was finished with constructing and sketching the position function, Stage B of the simulation was played. Students' attention was focused on the interpretation of the negative slope of the graph, its horizontal intercept, and the position below the time axis representing the man's position on the west to the reference point. Motion with a negative velocity is seldom analyzed in algebra classes; it is, however, one of the essential quantities in calculus. Students note that the man moves along a horizontal line; however, the generated position-time graph does not represent the same geometrical object. This stage clearly distinguishes between path of motion and position-time graph.

After students mastered the process of formulating and sketching position functions of one object, the motion of two objects was introduced. In these scenarios, students' attention was focused on the intersection of constructed position functions that represented the time and location where one object overpassed the other. The critical information to extract from the coordinates of intersections was not only their numerical value but more importantly their physical interpretations and necessary modifications that reflected the physical conditions. For example, in one of the problems, students were to modify the time instances the objects met due to a given *clock* time. In another, students were to convert a negative *y-coordinate* of the intersection to a position in the southward direction. The lesson was concluded with a homework assignment that correlated with the objectives of the lesson.

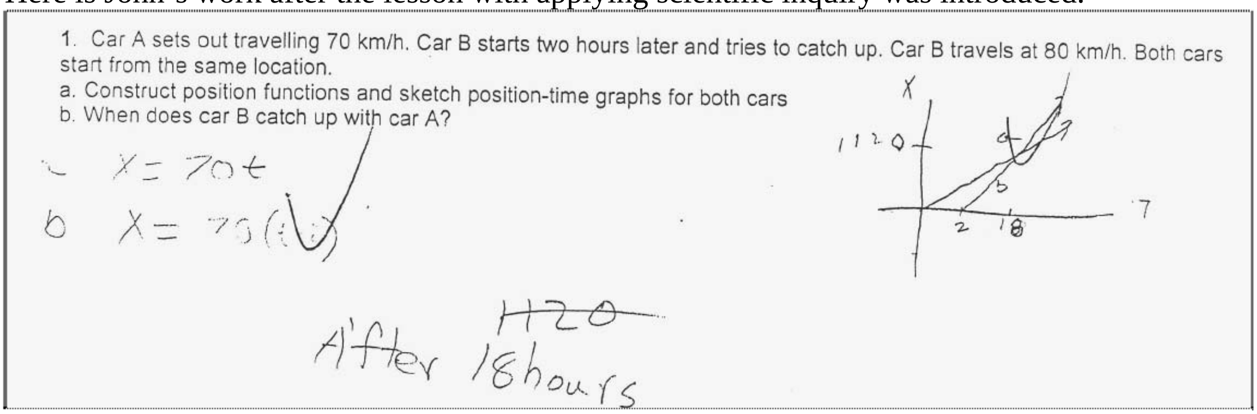
Discussion of Students' Answers

Below are sample solutions by two students—John and Mark (names are changed)—from a pre-test and post-test. The purpose of problem #1 was to verify if students could identify scientific quantities and convert them into mathematical representations (position functions). In order to focus on conceptual understanding, a graphing calculator was available on both tests.


1. Car A sets out travelling 50 MPH. Car B starts three hours later and tries to catch up. Car B travels at 75 MPH.
a. Sketch position time graph for the motion of both cars
b. When does car B catch up with car A?

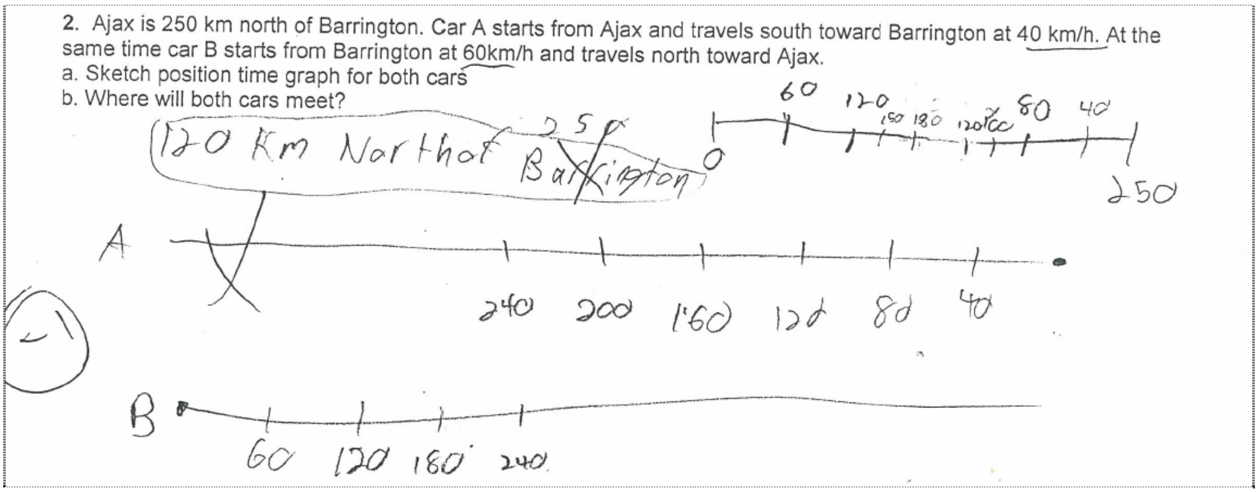
It is apparent that John could not handle the problem. He attempted to sketch graphs, but the graphs lacked many details. He missed the domain (he constructed a graph for negative time values), and he did not properly transfer the expression "three hours later" into mathematics. He also did not label the graphs.

Here is John's work after the lesson with applying scientific inquiry was introduced.

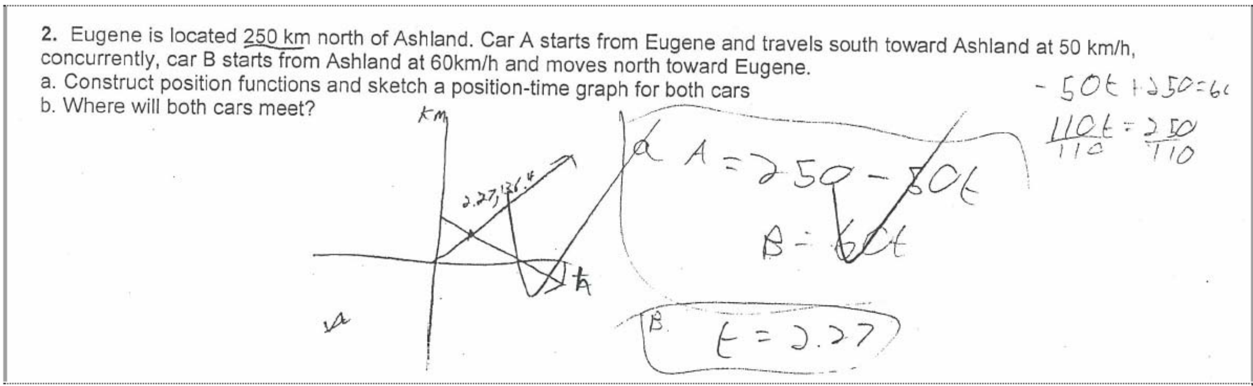

1. Car A sets out travelling 70 km/h. Car B starts two hours later and tries to catch up. Car B travels at 80 km/h. Both cars start from the same location.
a. Construct position functions and sketch position-time graphs for both cars
b. When does car B catch up with car A?

John not only constructed the functions (answer *b* reads *x=75(t-2)*), but he also sketched them correctly. He realized that both graphs will be located in the first quadrant. He also identified the domains of both graphs correctly.

Here is Mark's work on problem #2 on the pre-test.



Mark also faced difficulties with translating scientific quantities into mathematical representations. He constructed a strobe-like path of motion instead of functions. He did not consider direction of motion of the cars as a factor affecting the signs of functions' slopes and could not visualize the graphs. Mark's post-test solution on equivalent problem is inserted below:



Mark showed that the velocity of car A in the south direction must be considered negative. He also considered initial position of both cars as the y-intercepts of the respective functions. Although he did not answer *part b* of the question asking for the position (distance) where the cars meet, he found the time where car B overtakes car A.

General Analysis of Results

Analysis of Pre- and Post-Instruction

Problems on motion, according to mathematics curriculum in the selected district where the research was conducted, are taught in Algebra 1 and Algebra 2. Before taking the pre-test in pre-calculus class, students were not taught these concepts, with the exception of properties of linear functions. The test evaluated three categories: (1) ability to formulate function equations, (2) ability to sketch graphs while labeling the physical constraints, and (3) ability to arrive at the correct final answer. The pre-test evaluated students' understanding of these problems due to pre-instruction.
It is evident from the results of this research, as summarized in Table 1, that the current methods used to teach problems on motion produced low results. On the pre-test, only 20% of the linear functions were correctly formulated. Although 55% of the graphs were correct, only 33% were properly used to answer the main question of each problem.

Table 1: *Pre-Test and Post-Test Mean Scores with Respective Gains*

Objective Tested	Mean Pre-test Score	Mean Post-test Score	Mean Gain Score
Correct Function	20%	88%	85%
Correct Graphs	55%	93%	84%
Correct Final Answer	33%	86%	79%

Students' gain of scores following post-test instruction was calculated by using a method developed by Richard Hake from Indiana University [27]:

$$Gain = \frac{Average\ Post-test\ score\ \text{-}\ Average\ Pre-test\ Score}{100\% - Pre-test\ Score} .$$

The post-test, taken by students after they were introduced to the "Motion of Two Objects" unit, shows about an 83% average gain in each category. The most significant gain of 85% is observed in the process of constructing functions due to given initial conditions. With correct function equations, students correctly graphed the functions and correctly identified the intersection of each system, which led them to a final correct answer. A calculated z-test for all sets of scores proved their statistical significance with $p < 0.01$.

Analysis of the Survey

In order to assess how students felt about being exposed to the simulations, the following free response question, based on a 5-point Likert scale, was asked: *Did simulations help you understand applications of physics concepts in math classes?* The students' responses are summarized in Table 2 below.

Table 2: *Students' Responses to Survey Question*

Type of Response	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
Percent of Respondents	20%	55%	30%	0%	0%

The results of the survey correlate with students' gain of understanding of strategies used to solve motion problems. About 75% of the students either strongly agreed or agreed that the graphical representation generated by the simulation helped them understand the application of physics concepts in mathematics class. While answering students' questions and observing their reactions could also be incorporated as a qualitative factor, it was clear that the attractiveness of the simulation along with its clear scientific content created an engaging environment in which students could relate mathematics concepts to real-life situations.

Conclusions and Recommendations

In this paper, we made an attempt to transfer problem solving processes in mathematics into a coherent field with science by applying scientific inquiry. The findings support the hypothesis that students learn effectively if they are immersed in an interactive environment where the scientific aspects are clearly defined and applied. Although we suggested physics simulations as a visual aide to enhance the correlation with science, simple real scientific experiments, although more time-consuming to prepare, will also provide a content-rich learning environment for students.

As the application problems applied in mathematics textbooks also encompass other physics concepts as well as concepts from chemistry, biology, and engineering, perhaps verifying their coherence and adequate transferability would also benefit students.

There is also a need for expanding the population sample, which would allow for applying not only descriptive but also inferential statistical analysis of the data and increase the significance of the research. For example, the impact of these methods on students' progress in their science classes could also be analyzed and provide a valuable source for further study. We further envision applying qualitative methodology and zooming into the students' thinking processes while they construct the function. Students' doubts and obstacles in these regards would help educators construct teaching materials and organize the scientific inquiry process more efficiently.

It seems that obtaining access to students' AP-calculus exams and analyzing their performance with a focus on solving problems on motion would also serve as an evaluation tool of the current stage of teaching motion problems in high schools.

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