Constructing and Collaborating through ICT

Celia Hoyles O.B.E[1]
London Knowledge Lab
Institute of Education, University of London, U.K

Abstract
In this paper, I will describe research over several decades that has aimed to enhance mathematics learning and engagement through construction and collaboration making innovative use of digital technology. I will dwell in some detail on my most recent and ongoing project that is designing and building a computational system to support teachers in helping students to make the step to algebraic generalisation, with functionalities that encourage students to share and reflect upon their own constructions and rules as well as those of others.

Introduction

The paper raises issues concerning the ways that mathematical meanings are shaped by the symbolic tools in use, and the representational infrastructures that underpin them. It was Kaput who coined the term ‘representational infrastructure’ to refer to the kind of cultural tool epitomized by the Arabic numeral system (his work in this regard and its implications for mathematics learning is summarised in Hoyles & Noss, 2008). One characteristic of such a representational system is that it is taken-for-granted: this ubiquity and invisibility are critical facets of tool systems that become infrastructural. A key point here is that students of mathematics learning need to be aware not only of *how* mathematics is learned but also *what* is learned and the *language* in which this is expressed. Multiplication, like Newton’s laws, or elementary calculus, is *learnable,* precisely *because* we have Arabic numerals, the machinery of simple equations and Leibniz’s calculus notation respectively. What is to be learned depends on the representational forms with which it is expressed. We ignore the semiotic mediation of the tools we use at our peril! This is of course particularly apparent in the use of digital technology.

Turning to the issue of connectivity: it continues to change the landscape of human-human and human-computer interaction. To what extent is this shift reflected in the mathematical meanings learners develop? There is no lack of potential, see for example Roschelle, Penuel, & Abrahamson (2004), Hoyles & Lagrange, (2009). However given that access to computers is still an issue in many schools, there is rather limited research to identify in any systematic way the implications and potential of enhanced connectivity on mathematical learning and development.

Background

I distinguish two areas where I consider connectivity has considerable potential for enhancing the teaching and learning. First, for connectivity within and between classrooms, an individual’s communication can be changed into an object in a shared workspace, and thus become available for collective reflection and manipulation by the originator of the communication - but also by others. Second, the very need for remote communication of mathematical ideas – either synchronous or asynchronous – provides a motivation to produce explicit formal expression of

[1] I acknowledge the work of many colleagues in London who have contributed to the ideas in this paper. Most particularly I note the crucial input of Prof Richard Noss.

mathematical ideas including the processes by which solutions were obtained. Finally I will present how these scenarios have informed our ongoing project, the Migen project [2].

Objects for reflection and manipulation in a shared classroom space

There are technologies where each student in a class can build a particular case or part of a mathematical object, and these different instances can be brought together in a common workspace. Students can therefore view their own production alongside that of their peers, so all responses become an object of collective reflection and to be manipulated to draw attention to similarities and differences. This affordance appears to have – so far from mainly anecdotal evidence - a marked impact on mathematical learning. As Hivon et al (2009) argue (in the case of a class of students working with *TI Navigator*): "Each student becomes detached from his/her production as a distance is created between student and the expression of his/her creation and this distance seemed to improve collective reflection on practice. The student becomes involved in the class activity in a different way as the tool maintains this distance between a student and the results proposed to the class and to the teacher". (Trouche & Hivon, 2009).

Designing to share objects at a distance

Turning to the issue of sharing at a distance, we have undertaken two projects that both set out to exploit intersite connectivity (as well as face-to-face collaboration) to promote synchronous and asynchronous sharing, discussion and co-development of mathematical ideas. The overarching objective of both studies was to foster appreciation of the structures and processes *underlying* a set of mathematical ideas through carefully designed collaborative activities. The first project, the *Playground project* sought to design systems in which children aged between 4 and 8 years, could design, build and share simple video games, (see for example, Hoyles, Noss, & Adamson, 2002).

As part of the study we noted an interesting shift when children moved from face-to-face collaboration to collaborating across remote sites. This shift was characterised by a move from socially derived rules to govern the games in the former scenario to system rules (computational expressions) in the latter. This shift seemed to be a result of the necessity to formalise in the absence of all the normal richness of interaction that characterises face-to-face collaboration, where the narrative of the game was fore grounded and rules frequently only tacitly agreed. At a distance such tacit agreements were not available, and the narrative had to be translated into a form that the computer could accept (for elaboration, see Noss, R., Hoyles, C., Gurtner, J-L., Adamson, R. & Lowe, S, 2002).

The absence of face-to-face collaboration does not in any sense guarantee the shift towards formalisation. That it arose at all, undoubtedly owes much to the activity structures, relationships between children, and of course, the presence of the researchers. Nevertheless, it is interesting to speculate whether, by a more focused and prolonged emphasis on remote collaboration with suitably designed computational systems, new kinds of formalised discourse might be engendered in a wider range of learning environments.

In a later project, *WebLabs*, (described earlier) (www.lkl.ac.uk/kscope/weblabs), we attempted to scaffold interactions at a distance by devising a web-based system, *WebReports*, that allowed students to post their ideas—*and their working models* — so that students working in other classrooms could download the models, run and interpret them, reflect on them before sending

[2] Funded under the ESRC/EPSRC, TLRP-TEL programme Grant reference RES-139-25-0381. In what follows about the MiGen system, I acknowledge the central contribution of my collaborators Richard Noss, Manolis Mavrikis and Eirini Geraniou.

comments and possibly amended models (see for example, Simpson, Hoyles, & Noss, 2005). This work built on the importance for learning of externalising cognitive processes and sharing these externalised representations: for example, Scardamalia & Bereiter had argued that an electronic and networked discussion board would foster conversations between students and thus would "contribute to the development of a "knowledge building community" (Scardamalia & Bereiter, 1996). Our key idea was that learners could not only discuss, conjecture with and comment upon each others' ideas, but they could also inspect and edit each others' working models, the computer programs, so that the processes underlying outcomes were made to some extent more visible. The grain size of what to make visible remains however is a huge challenge.

I now turn to briefly describe our latest research, the MiGen project.

The MiGen project

The MiGen project set out to design a pedagogical and technical system to support students in developing a propensity to strive for algebraic generalisation, a mathematical way of thinking fundamental to making progress in the subject. A key design aim is to find ways to help students "see the general through the particular"; while working on the specific to develop an awareness of what this would imply for a general case. Our guiding methodology is derived from constructionism (Harel & Papert, 1991) and based on the design principle that by building objects (on the screen), students will more easily be enabled to grasp their meaning and – crucially - express any relationships within and between them.

At the core of the MiGen system is a microworld, the *eXpresser*, in which students can construct figural patterns using coloured square tiles and express their structure by defining the building blocks of the patterns and any relationships between them. However, underlying the surface goal of building patterns is our main objective: namely, that through interaction in the system, students develop "*algebraic ways of thinking*" that underpin algebraic generalisation, within which we characterise three key components:

1. perceiving structure and exploiting its predictive power;
2. seeing the general in the particular, including identifying variants and invariants;
3. recognising and articulating generalisations, including expressing them symbolically.

This paper sets out how our research is beginning to evaluate whether, engaging with the system, students will not only complete the tasks successfully, but also develop these algebraic ways of thinking.

Building models in eXpresser

First, we provide a brief description of eXpresser and the accompanying activities to help the reader gain an appreciation of the environment3. For more detail, see also Noss, Hoyles, Mavrikis, Geraniou, Gutierrez & Pearce, (2009) and Mavrikis, Noss, Hoyles and Geraniou(submitted paper). In eXpresser students are presented with a model and asked to construct it using one or more pattern (see Figure 1). The model is animated, with the Model Number changing randomly. The animation serves to emphasise the generality expected (see Noss et al. 2009 where this is discussed in more detail): i.e. the task is not to count the tiles. Rather it is to find a rule that would give the number of tiles for any given model number.

[3] The interested reader can interact with eXpresser and the available tutorials, at http://www.migen.org.

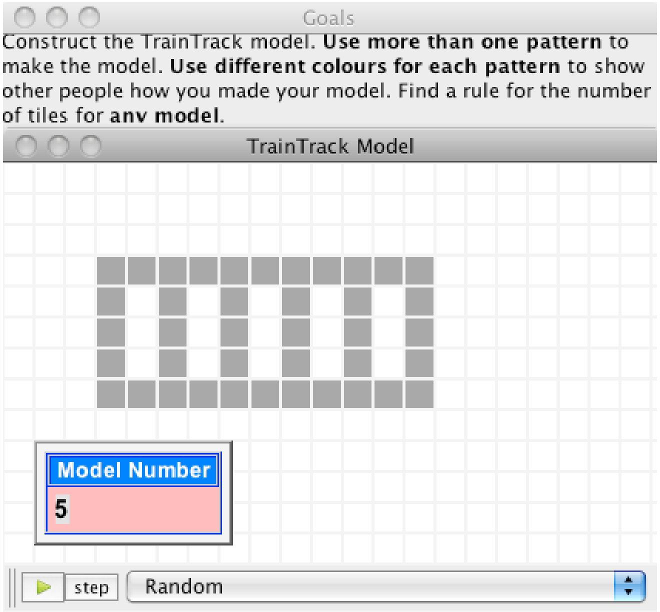


Figure 1. The ‘Train Track’ task in eXpresser. The model to be constructed is animated with the ‘Model Number’ (the number of 'holes' in this case) changing in random steps every few seconds.

In setting about constructing the model, students first have to express how they visualise its structure as sets of patterns. Each pattern takes the form of repeated building blocks that are appropriately placed on the canvas. Students then make explicit their rules to calculate the number of tiles in each pattern. When the rule is correct, the pattern becomes coloured. Finally, students are encouraged to use their rules to obtain the total number of tiles needed in the model; the sum of the tiles needed for each of the constituent patterns. Of course, there is always more than one way to do this, as we see later.

Figure 2 shows a snapshot of the eXpresser with the Train-Track task completed. 'My Model' has been built on the left canvas this time by combining two patterns, one coloured green and the other red4. A pattern in eXpresser comprises a repeated element, called a building block (shown in A), which is created by grouping several tiles together5.

[4] We we will refer to ‘red’ and ‘green’ throughout the paper. In greyscale these colours appear as dark and light grey respectively.

[5] It is worth noting that single tiles or whole patterns can also be considered building blocks themselves thus leading to complex patterns of patterns.

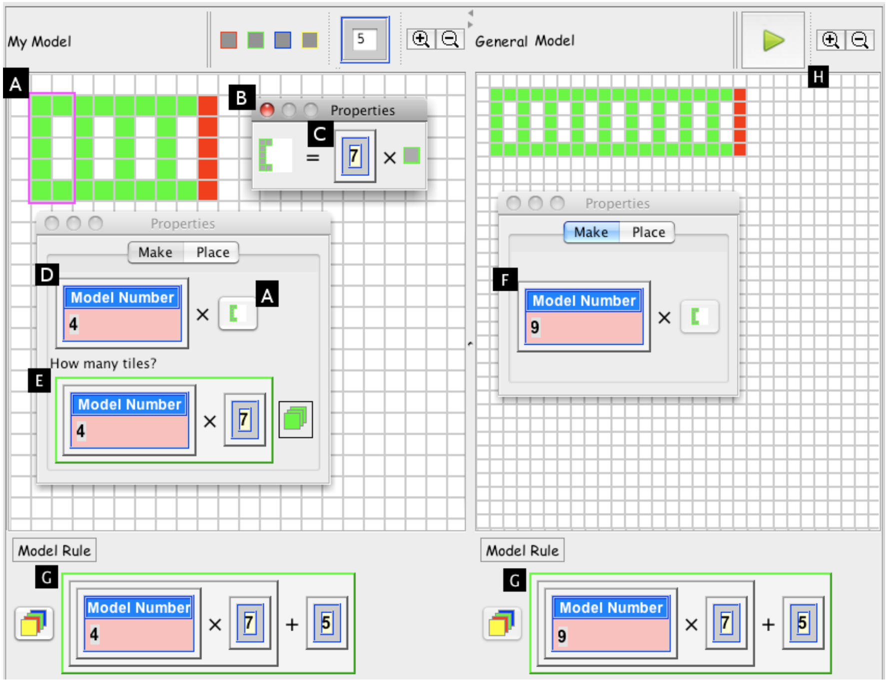


Figure 2. Constructing a model and expressing its rules in eXpresser.

(A) Building block to be repeated to make a pattern. (B) Expression of construction of the building block (C) The number of tiles of the building block (D) The number of repetitions of the building block in this case 4 i.e. the value of the 'variable' `Model Number'. (E) Number of tiles required for the pattern, with general rule. (F) Any variable used in 'My Model' takes a random value in the 'General Model'. (G) For the General Model to be coloured a general rule is required that expresses the total number of tiles in the whole model. (H) Patterns can be animated using the PLAY button which randomly sets the value of the variables in the General Model, with the model remaining accurate and coloured if and only if the rule is correct.

When making a pattern, students have to specify the translations across and down for each repetition of the building block, as well as an initial number of repetitions. In Figure 2, a C-shaped building block (A) has been created and it is repeated by placing each repetition two squares across and zero places down[6]. When the C-shaped building block (A) is made, its properties are shown in an expression (B). In Figure 2, the building block is repeated as many times as the value of a variable called 'Model Number' (D), in this case 4. As students build their constructions in 'My Model', a second canvas is seen alongside (the 'General Model'). This mirrors exactly My Model until the student has introduced a variable into their model.

Patterns will be coloured by calculating and then allocating the exact number of coloured tiles to its construction. In the case of the pattern made of C-shapes, using the expression for construction (B) and the number of times the building block is repeated, the rule for the total number of tiles in the green pattern is 'Model Number × 7', in Figure 2 (E). When variables have been introduced in My Model, eXpresser will randomly change their value in the General Model. In

[6] This is specified during construction and can be shown but is hidden in the figure for simplicity.

Figure 2, the value of the Model Number in the General Model is randomly set to 9, resulting in a different instance of the model (F). The General Model is coloured only when students express *correct* general rules in the 'Model Rule' area of the screen (G). Students cannot interact directly with the General Model. They are, however, encouraged to click the play button (H) to animate their general model to test its generality.

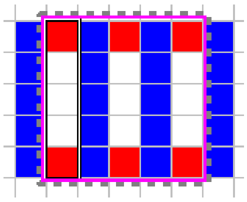
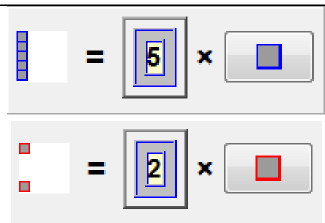
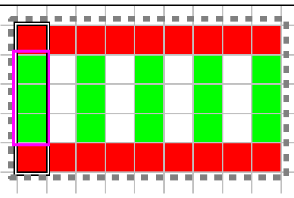
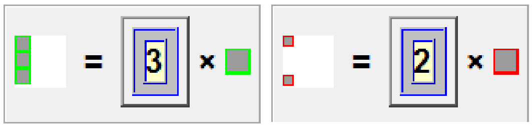
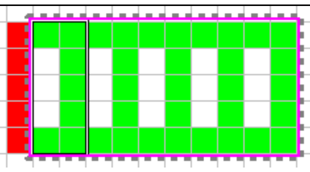
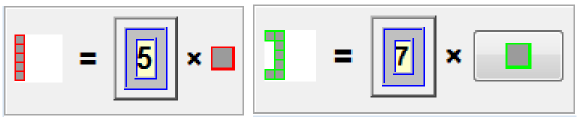
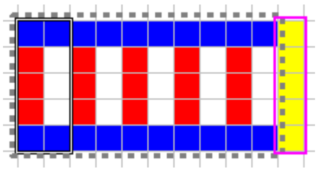
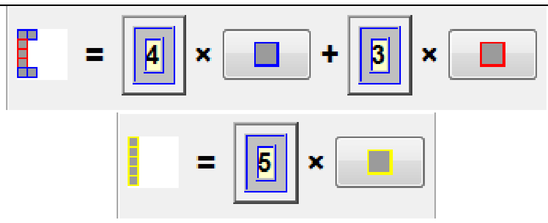
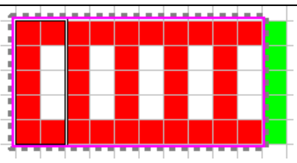
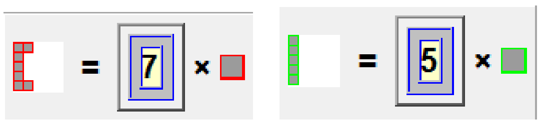
To support students during their interaction, the microworld has a toolbar that students can activate for support (see Figure 3). The support toolbar contains two components. One is a suggestion button (Figure 3A), which only lights up if the system observes an action that implies that help is warranted. Rather than interrupting the student directly, the icon simply lights up to indicate that a suggestion is available: the students are free to ignore these suggestions or evaluate and follow them. The other component, a 'request-maker', comprises a list of drop-down menus that allow students to construct a sentence asking for help. Without constructing this sentence, the help button (Figure 3B) is not enabled. The request maker provides the system with an indication of the student's need, and an incentive for the student to reflect on what they are trying to do by engaging further with the discourse of eXpresser.

Figure 3. The HELP toolbar alerts students to suggestions (A), and allows them to construct a specific request for help (B).

Reflecting and sharing models and rules

As we have seen, when students are presented with a model by the system, they have to think about its structure in terms of its constituent patterns, in order to be able to construct it. The choice of patterns and structures is left to the students who tend to visualise the same models very differently, leading to different conceptualisations of structure and rules. Table 1 shows the different ways with which Alicia, Conor, Fiona, Maria chose to construct the “Train Track” pattern.

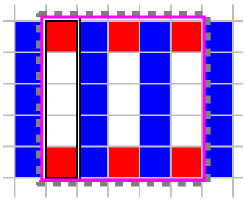
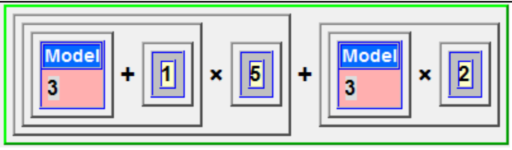
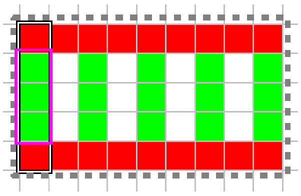
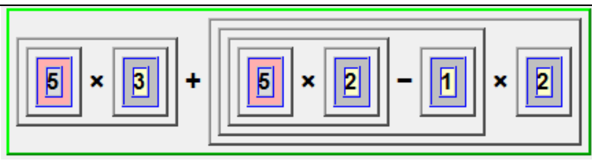
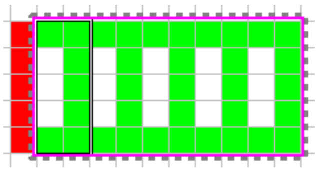
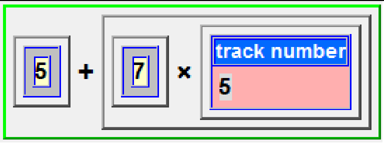
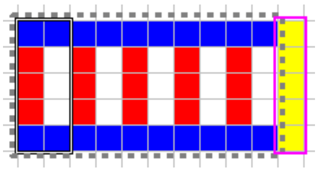
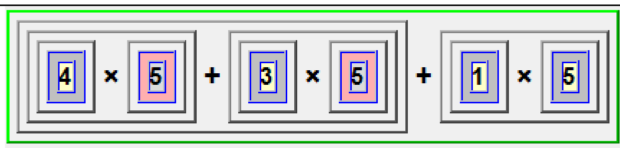
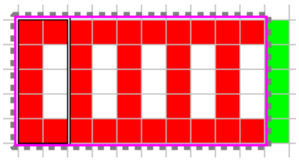
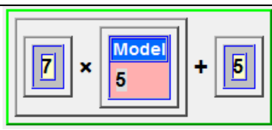
Table 1 Different conceptualisations of the structure of TrainTrack model.

Student	Model showing different constituent patterns	Building Block(s) with expressions for construction
Alicia		
Conor		
Greg		
Fiona		
Maria		

Alternative ways of seeing and expressing the relationships serve in our activity sequence as a basis for potentially fruitful discussions between students, giving them the opportunity to appreciate the qualities of different models in terms of their complexity and generality. We have found that allowing students to collaborate and reflect on their actions in their efforts to justify their solutions to their peers has proved an invaluable step towards developing their algebraic ways of thinking.

The analysis of their productions and discussions is ongoing but I can report that this process has proved to be important in helping students 'read' their eXpresser rules and we foresee being able to align these with corresponding algebra rules. More detail will be available in Geraniou et al., in preparation). Examples of such rules in eXpresser can be seen in Table 2.

Table 2. Students' models (cf. Table 1) and their corresponding eXpresser and algebraic rules.

Student	Model showing different constituent patterns	eXpresser rules and corresponding algebraic rules
Alicia		 $(x+1)\times 5+x\times 2$
Conor		 $x\times 3\times(x\times 2-1)\times 2$
Greg		 $5+(7\times x)$
Fiona		 $(4\times x+3\times 5)+1\times 5$
Maria		 $7\times x+5$

These relatively simple linguistic elements allow students to express themselves quite naturally, emerging as a direct consequence of the model construction, and as an answer to the challenge of colouring the pattern i.e. finding the total number of tiles. By making available to students a language that allows them to express the relationships they perceive, students can begin to experience the power of symbols without reference to conventions, but situated in the microworld: to express situated abstractions (Noss & Hoyles, 1996) within the constraints and discourse of the system.

Conclusions

In this paper, I have considered the question of connectivity, and suggested some ways in which there is potential for mathematical learning: in the possibility of bringing students' constructions and their formal expression as rules, together as objects so *both* are visible for reflection and comparison – in the same classroom or at a distance.

There are many challenges still to be faced, in MiGen but more broadly. One is to research the exact balance between intelligent support and student autonomy, and what indeed can and should be left to the teacher. Another has been designing and developing an extensible, scalable client-server architecture to support multiple concurrent users in a classroom setting. In MiGen, this will enable teachers to view all the models and their expresser rules, and then with the help of the system pair students based on differences in their models and rules, to discuss the correctness and equivalence of their rules. Thus students will be able to log on in their allocated pairs and draw down their models and rules in a combined space on which to reflect, discuss and compare.

I end by emphasising a key idea in this paper and that is design. The obvious but often overlooked fact is that technology *per se* is unlikely to influence mathematical development in any significant ways. It is how it is *designed* to support learning and how it is embedded in *activities designed* with specific learning objectives that it critical and challenging.

The research challenges are considerable, not least because of the rapid advances of the technology. But just in case I am accused of technocentrism, I reiterate that none of these developments will happen without more design research to tease out the ways the tools shape mathematics and its learning, and reciprocally, to better understand how we as teachers and researchers can shape the evolving technology.

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